Examination I for PHYS 6220/7220, Fall 2015

1. An ant is located at point P_1 with Cartesian coordinates (x_1, y_1, z_1) on a surface which is limited to the region $z > 0$. The surface has its defining equation as $z = s|(x^2 + y^2)^{1/2}|$, where s is a positive constant. The ant wants to remain on the surface and walk to a final destination point $P_2(x_2, y_2, z_2)$. There is no gravity in this problem. Express all answers in given quantities only. This implies determining all constants that may emerge in your solution in terms of given quantities. Answer both parts for all possible points P_1 and P_2 on the surface. This means solving for the general case for the relative position of P_2 with respect to P_1 as well as two other special cases that could arise where the general solution will fail.
(a) Find the exact equation of the curve that the ant should walk to complete its journey in the shortest distance. (**8 points**)
(b) Find the total distance the ant will travel along this curve of shortest distance. (**3 points**)

2. A hemisphere of mass M and radius b rests with its flat surface on a frictionless horizontal plane as shown in Fig. 1. A mass m initially at rest at the top of the frictionless hemisphere loses its position of unstable equilibrium at time $t = 0$ and starts sliding on the surface of the hemisphere under the influence of gravity. Magnitude of the acceleration due to gravity is g. The X and Z axis of an inertial Cartesian frame are as shown. The radial vector to mass m from the center of the hemisphere makes an angle θ with the Z axis, at time t, as shown. Eliminate all constants of integration, using initial conditions, from your solution.

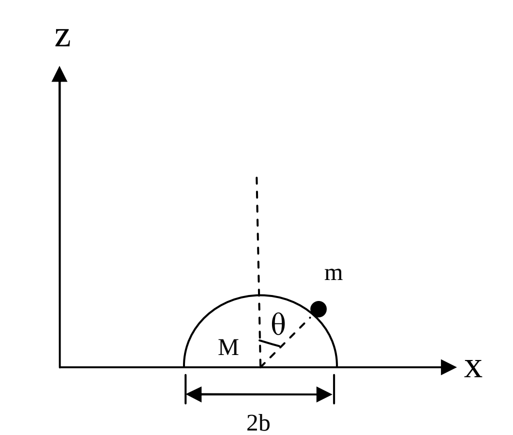

Fig. 1

(a) Define clearly an appropriate set of generalized coordinates, in words. Use these to obtain the Lagrangian of the system. (**2 points**)
(b) Use a Lagrange multiplier λ associated with the constraint that m moves over the hemisphere and write the Euler-Langrange equations of motion. (**2 points**)
(c) Write an expression for λ purely as a function of only one generalized coordinate and its derivatives with time. (**3 points**)
(d) State in words the constants of motion in this problem. Write expressions for these constants in terms of the generalized coordinates and generalized velocities. (**2 points**)
(e) Eliminate the time derivatives in part (c) to express λ purely in terms of one generalized coordinate. (**2 points**)