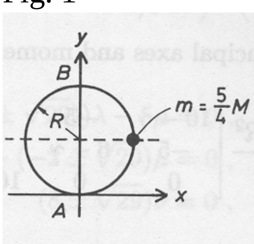
Final Examination for PHYS 6220/7220, Fall 2015

1. A particle of mass m approaches a center of force from a far away distance with initial speed v_0 and impact parameter b. The center of force exerts a force on the particle corresponding to the potential $V(r) = -k/r^n$, where r is the distance of the particle from the center of force, k is a positive constant of appropriate dimensions and n is a positive integer. Express all answers in terms of the known constants, m, k, v_0, b and n.
(i) Find an implicit equation to determine distance of closest approach c. (**3 points**)
(ii) State all the cases for the value of n when analytic closed form solutions can be obtained for c. Solve explicitly for c whenever possible in these cases. (**5 points**)
(iii) State any special conditions that should be satisfied by the known constants in solving part (ii) for physical solutions to exist for c. (**2 points**)

2. A thin disk of radius R and mass M lying the XY plane has a point mass m = (5M)/4 attached to its edge (as shown in Fig. 1). The moment of inertia matrix $\mathbf{I}_D$ of the disk alone about its center of mass has elements given by $\mathbf{I}_{D\ i,j} = \left(\frac{MR^2\delta_{i,j}}{4}\right)(1+\delta_{i,3})$ where $\delta_{i,j}$ is the Kronecker delta function. The coordinate system used to calculate $\mathbf{I}_D$ is parallel to the one shown in the Fig. 1.

Fig. 1



(a) Find the moment of inertia matrix $\mathbf{I}_{DA}$ only of the disk, about point A, in the coordinate system shown in Fig. 1. (**2 point**)
(b) Find the moment of inertia matrix $\mathbf{I}_{PA}$ only of the point mass, about point A, in the coordinate system shown in Fig. 1. (**1 point**)
(c) Find the total moment of inertia matrix $\mathbf{I}_{TA}$, of the disk and point mass, about point A in the coordinate system shown in Fig. 1. (**1 point**)
(d) Write the appropriate equation D = 0, where D is the determinant containing the eigenvalue λ of $\mathbf{I}_{TA}$. (**1 point**)
(e) By a careful observation of the result in (d) find one value of λ. (**1 point**)
(f) Then using the result from part (e) find the other two values. Keep irrational numbers in their square root form. Do not convert any answers to decimals. (**2 points**)
(g) Find the unit vectors along the principal axes of this system about point A. (**3 points**)

3. A particle of mass m and charge e moves in a potential $V(r) = [m\Omega^2 r^2]/2$, where r is the radius of the particle and Ω is a constant of appropriate dimensions. It is simultaneously subjected to a constant electric field **E** pointing along the positive X axis and a constant magnetic field **B** pointing along the Z direction. Their corresponding electromagnetic potentials are $\phi = -Ex$ and $\mathbf{A} = (B/2)(-y, x, 0)$.
(a) Write the Lagrangian for this system. (**3 points**)
(b) Write the Euler Lagrange equations for all the generalized coordinates, q_1, q_2 and q_3. (**3 points**)

(c) One of the equations in part (b) will yield the equation for a well-known problem. Identify this generalized coordinate and solve its equation to get its most general solution. Let this generalized coordinate be labeled q_3 for convenience. (**1 point**)

(d) For one of the other coordinates called q_1, for specificity, perform the transformation $q_{1,n} \equiv q_1 - c$ where c is a constant that depends only on the given quantities, m, e, E, B and Ω. Specify the value of c and show that in the new coordinate, $q_{1,n}$, the new Euler-Lagrange equation is simplified. Without this simplification part (e) and (f) will not get solved. (**1 point**)

(e) For the generalized coordinates $q_{1,n}$ and q_2, substitute an oscillatory solution of the form $q_{1,n} = C_1\exp(i\omega t)$ and $q_2 = C_2\exp(i\omega t)$ in the equations obtained in parts (b) and (d). Here, $i = \sqrt{-1}$. The C_k (k = 1, 2), are arbitrary unknown constants and t stands for time. The unknown parameter ω is yet to be determined. (**1 point**)

(f) From the two equations obtained after the substitution in part (e) state the condition which will give the unknown parameter ω that was introduced. Give a reason for the choice of your condition. Solve for ω using this condition. (**1 point**)

(g) Clearly describe how you would proceed further to obtain the most general oscillatory solution to this problem. You need not perform the steps but merely explain the procedure. (**1 point**)