# Examination II for PHYS 6220/7220, Fall 2014

1. A one dimensional simple harmonic oscillator has mass m, and generalized canonical coordinates q and p, and vibrational angular frequency $\omega$, and Hamiltonian H(q, p).
(a) Evaluate [u, H] where $u = -i\omega t + \ln(p + i m\omega q)$ and $i \equiv \sqrt{-1}$. (**2 points**)
(b) Use result in part (a) to obtain du/dt. Comment on your result. (**2 points**)
(c) Express H as H = H(u). (**1 point**)

2. Two successive rotations are performed on a rigid body with a common fixed point on the body for both rotations. Each rotation is through π radians. The two rotation axes are defined by unit vectors in the laboratory Cartesian coordinate system given by
$\mathbf{n_1} = (1,1,0)/\sqrt{2}$, and $\mathbf{n_2}$ = (0, 0, 1) respectively.
(a) Find all elements of the matrix **A** corresponding to the first rotation. (**1 point**)
(b) Find all elements of the matrix **B** corresponding to the second rotation. (**1 point**)
(c) If the resulting net displacement of the body is represented by a matrix **R** then find all its elements. (**2 points**)
(d) Another single rotation about a fixed axis is carried out to bring the body back to its original configuration. Find the needed angle of rotation and the axis of rotation. (**2 point**)
(e) If the rotation in part (d) is carried out by three Euler angle rotations find these angles. (**3 points**)

3. Consider the motion of a particle of mass m and magnitude of angular momentum $\ell$, moving in a potential V(r) = $-[(k/r) + (k'/r^3)]$, where r is the distance of the particle from the origin and k and k′ are positive constants of appropriate dimensions.

(a) Find the radius $r_0$ of a circular orbit for the particle in terms of k, k′, m, and $\ell$. (**2 points**)
(b) Find the critical radius $r_{0c} = r_{0c}(k, k')$ for which circular orbits will be marginally stable. (**2 points**)
(c) State if orbits for which $r_0 > r_{0c}$ are stable. State if orbits for which $r_0 < r_{0c}$ are stable. Give justifications for both answers. (**2 points**)
(d) If a circular orbit exists with radius $r_{0c}$ find it angular speed $\omega = \omega(m, k, k')$. (**2 points**)