Examination I for PHYS 6220/7220, Fall 2014

1. A flat plate of mass m_1 is free to slide on a frictionless horizontal surface. A particle of mass m_2 is constrained to move in a horizontal circle of radius b centered at a fixed point P on the plate. The system as a whole has zero initial linear momentum.
(a) Draw a detailed figure, defining a proper frame of reference and generalized coordinates. Depict the generalized coordinates on the figure and also describe them in words. (**2 points**)
(b) Write an expression for the Lagrangian of the system. Make proper use of a Lagrange's undetermined multiplier (λ) to find the force between the two masses associated with the fixed radius b. (**2 points**)
(c) Write all the relevant Euler-Lagrange equations. (**2 points**)
(d) Integrate all the equations completely. (**3 points**)
(e) Using result in (d) find the simplest expression for the multiplier. (**1 points**)
(f) Give expressions for all the constants of motion in this problem. (**1 points**)

2. A charged particle of charge e is subject to an electro-magnetic field. The vector potential **A** that generates this field is given in Cartesian coordinates by **A** = (0, xB, 0) where B is a positive constant. The scalar potential Φ is zero.
(a) Write the Lagrangian for this system. (**1 point**)
(b) Write the Hamiltonian for the system. (**1 point**)
(c) Find the constants of motion that appear in the Hamiltonian. (**1 point**)
(d) Use the results in part (c) to reduce the dimensionality of the problem. (**1 point**)
(e) Now write Hamilton's equations for this reduced dimensionality problem. (**1 point**)
(f) Solve the equations in part (e). (**1 point**)
(g) Using results from parts (c)-(f) obtain the complete time dependence of all three Cartesian coordinates. (**2 points**)
(h) Describe the shape of the trajectory found after completing part (g). Prove quantitatively that your described shape is correct. (**1 point**)