Final Examination for PHYS 6220/7220, Fall 2014

1. A three-dimensional surface is defined by the equation $z = bx^2$ where b is a positive constant of appropriate dimensions. Two arbitrary points A and B on the surface have coordinates (x_1, y_1, z_1) and (x_2, y_2, z_2), respectively. Express all answers in terms of given quantities only.

(a) Find the equation of the curve which traverses the shortest path on the surface to join A to B. (**5 points**)

(b) Find this shortest distance between these two points along that curve. (**2** points)

2. Two identical simple planar pendulums, each of length b and mass m, are coupled by a spring of constant κ as shown in the figure. When the masses are hanging vertically the spring is in its equilibrium length ℓ. When they oscillate they make angles θ_1 and θ_2 with the vertical as shown. In the limit $\ell/b \ll 1$ we may approximate the extension or contraction in the spring to be $b(\sin(\theta_1) - \sin(\theta_2))$. The magnitude of the acceleration due to gravity is g.

(a) Write the Lagrangian of the system in terms of appropriate generalized coordinates. (**3 points**)

(b) Find the matrices for **T** and **V**. (**2 points**)

(c) Find the frequencies of normal modes of small oscillations. (**2 points**)

(d) Find the corresponding eigenvectors. (**2 points**)

(e) Find the most general solution to this small oscillations problem. (**1 point**)

(f) Depict and describe the normal modes of oscillations. (**1 point**)