Test III for PHYS 6220/7220, Fall 2013

1. A thin uniform circular disc of mass m, with radius of length b rotates at a constant angular speed ω about an axis through the center of the disc tilted by an angle θ with respect to the normal to the disc. The angle θ remains constant at all times. The body axes are denoted by x, y and z and the laboratory axis by x', y' and z'. The common origin O of both systems is at the center of the disc. The z axis is normal to the plate and the z' axis is the axis of rotation. The x and y axes are chosen such that the three axes x, z and z' are coplanar at all times. Let the unit vectors along the body axes be **i**, **j** and **k** and those along the laboratory axes be **i'**, **j'** and **k'**. Express all answers in terms of the given quantities only.

(a) Compute the moment of inertia matrix of the disc in the body frame. Comment on your result. (**4 points**)

(b) Express the angular velocity $\boldsymbol{\omega}$ of rotation in the laboratory frame and the body frame. (**1 point**)

(c) Compute the angular momentum of the disc in the body frame at all times. Compute the torque on the disc in the body frame. (**2 points**)

(d) Compute the torque on the disc in the laboratory frame. (**1 point**)

(e) Compute the angular momentum of the disc in the laboratory frame. (**2 points**)

3. A graduate student who had just finished PHYS 6220 had studied small oscillations very well. In an electromagnetism course she encountered two circuits for which, using Kirchoff's laws, she arrived at the following equations for charges q_1 and q_2 on two capacitors.

$\left(L\,\ddot{q}_1\right)+\left(q_1/C\right)+\left(M\,\ddot{q}_2\right)=0$ and $\left(L\,\ddot{q}_2\right)+\left(q_2/C\right)+\left(M\,\ddot{q}_1\right)=0$, where L, C, and M are constants of appropriate dimensions and $M < L$. She knew that the two charges could vary independently in time. The problem was to solve for the most general solution to the time dependence of the two charges $q_1(t)$ and $q_2(t)$. Follow the steps that she took by answering the following.

(a) She first identified a kinetic energy and potential energy term treating the two charges as the generalized coordinates. Find these. (**2 points**)

(b) She then wrote the Lagrangian for the system. She solved for part(a) in a manner that the given equations were the Euler-Lagrange equations for her problem. Write the Lagrangian and confirm that the given equations can be derived from it naturally. (**1 point**)

(c) She then constructed the **T** and **V** matrices to make the problem appear as a small oscillations problem. Write these matrices. (**2 points**)

(d) Using the results of part(c) she solved for the eigen-frequencies of the problem. Find these. (**2 points**)

(e) Using the results of part(d) she solved for the eigen-vectors of the problem. Find these. (**2 points**)

(f) She then proceeded to write the most general solution for the original equations she was to solve. Obtain this solution. (**1 point**)

(g) Draw a circuit diagram for the problem. (**1 point**)