Examination II for PHYS 6220/7220, Fall 2013

1. Two right handed Cartesian reference frames F and F' have a common origin O. Frame F has axes X, Y and Z while F' has axes X', Y' and Z'. Unit vectors along the six axes are **i, j, k, i', j'** and **k'** respectively. The positive Z axis makes an angle $\pi/4$ with the positive Z' axis. The projection of vector **i** on vector **i'**, is a null vector. The cosine of the angle between the negative Y axis and the negative Z' axis is $\left(1/\sqrt{2}\right)$. Let the matrix that transforms the frame F to F' be called **A**.

(a) State the properties you expect **A** to have. Give reasons for your answer. (**2 points**)

(b) Find all elements of the matrix **A**. (**9 points**)

(c) If the transformation from frame F to F' was carried out by Euler angles what would be their precise values. (**3 points**)

(d) If the transformation from F' to F were carried out as a single rotation about an axis, find the angle of rotation and the unit vector along the axis of rotation. (**4 points**)

2. A particle of mass m, with magnitude of the initial angular momentum being $\boldsymbol{\ell}$, moves in a potential $V(r) = -V_0[\exp(-(r/a)^2)]$, where V_0 and a are positive constants of appropriate dimensions. There exists a critical value of the angular momentum $\boldsymbol{\ell}_c$. This leads to four possible cases for the nature of the effective potential $V_e(r)$. These are: (i) $\boldsymbol{\ell} = 0$, (ii) $0 < \boldsymbol{\ell} < \boldsymbol{\ell}_c$, (iii) $\boldsymbol{\ell} = \boldsymbol{\ell}_c$ and (iv) $\boldsymbol{\ell} > \boldsymbol{\ell}_c$. All answers should only involve the given constants in the problem, m, V_0, a and possibly $\boldsymbol{\ell}$. Graphs generated on a computer will receive no credit.

(a) Find the critical value, $\boldsymbol{\ell}_c$, of the angular momentum. State, with an explanation, whether bounded orbits are possible below or above $\boldsymbol{\ell}_c$. (**3 points**).

(b) Draw qualitative graphs of the effective potential for each case listed above. (**4 points**)

(c) Explain how the nature of the graph was obtained in each case. (**4 points**)

(d) In each graph in part (b) mark clearly the physically allowed limiting low and high values of the total energy of the particle E_{lo} and E_{hi}, respectively. Write expressions for them where applicable. (**4 points**)

(e) Give an expression for the radius of circular orbits that may arise in case (ii). (**2 point**s)

(f) If the orbit at this radius is stable find the frequency of small oscillations around this circular orbit if the particle is slightly perturbed. If the orbit is unstable state any limits that exist on the value of the radius. (**2 points**).