Final Test for PHYS 6220/7220, Fall 2013

1. A square plate, of side of length b, lies in the XY plane with its sides parallel to the axes. The origin O is at the center of the plate. It has uniform areal mass density ρ. A square cavity of side c is introduced in it at point P whose coordinates are (b/4, b/4). Two parallel sides of the cavity make an angle θ with the X axis. Note that $c < b/2$. In both cases below the matrices are calculated in the frame located at O.
(a) Find the moment of inertia matrix of the plate with the cavity for the case $\theta = 0$. (**4 points**)
(b) For the case $0 < \theta < \pi/2$, describe clearly the procedure to follow to obtain the moment of inertia matrix of the plate with the cavity. You need not obtain the matrix. (**1 point**)

2. A particle of mass m and magnitude of initial angular momentum ℓ moves in a central force field such that $r = (r_0) \exp(k\theta)$, where r is its distance from the center of force and the origin is taken at the center of force. The angle its position vector makes with the positive X axis is defined to be θ. Constants r_0 and k have appropriate dimensions. Assume that at t = 0, $r(0) = r_0$. Express all answers in terms of m, k, r_0 and ℓ only.
(a) Find the form of the central potential V = V(r). (**2 points**)
(b) Find r = r(t). (**1 point**)
(c) Find $\theta = \theta(t)$. (**1 point**)

3. A block of mass m_1 has attached to it a string of fixed length ℓ and of negligible mass. At the other end of the string is a freely hanging bob of mass m_2. The block is restricted to move only along a fixed horizontal straight line as shown in Fig. 1. The horizontal straight line and the bob always lie in one fixed plane. The acceleration due to gravity has magnitude g.
(a) Define clearly an appropriate set of generalized coordinates. Use these to obtain the Lagrangian of the system. (**2 points**)
(b) Use a Lagrange multiplier λ associated with the constraint that the string is of fixed length and write the Langrange equations of motion. (**2 points**)
(c) Write an expression for λ purely as a function of only one generalized coordinate and its velocity. (**2 points**)
(d) State in words the constants of motion in this problem. Write expressions for these constants in terms of the generalized coordinates and generalized velocities. (**2 points**)
(e) Indicate further (do not solve) how you would express λ purely in terms of just one coordinate by eliminating the velocity from the expression. (**1 point**)