Third Examination for PHYS 6220/7220, Fall 2012

1. A sphere of uniform mass density ρ and radius b_1 has its geometric center at point O. A spherical cavity of radius b_2 centered at point P is now introduced in it. The distance between the center of the sphere and the cavity is $|\mathbf{OP}| = c$, such that $(c + b_2) < b_1$. All answers should be expressed in given quantities ρ, b_1, b_2, and c.
(a) Compute the inertia matrix through an appropriately chosen set of axis passing through O. Describe carefully the choice of axes. (**3 points**)
(b) Find the center of mass point Q of this rigid body. (**1 point**)
(c) Write the Euler Lagrange equations describing the motion of this rigid body where point O is held fixed and the body lies in a uniform gravitational field of magnitude g. (**3 points**)
(d) State all constants in the problem and reduce the problem to a one dimensional problem. (**4 points**)
(e) What changes occur in solving for the motion of the rigid body in part (c) and (d) if the fixed point is chosen to be Q instead of O. Describe this motion qualitatively. (**2 points**)

2. Two particles each of mass m are constrained to move on a circle of radius b. They are connected by a spring of spring constant k and equilibrium length c such that $0 < c < 2b$. There is no gravity in the problem. Define an appropriate frame of reference and all the generalized coordinates with a figure.
(a) Compute the kinetic energy T of the system. (**1 point**)
(b) Write an expression for the potential energy V of the system purely as a function of generalized coordinates. (**1 point**)
(c) Find the point of equilibrium for this system. State it in words and make comments about its uniqueness. (**1 point**)
(d) Construct the appropriate Lagrangian for small oscillations about the equilibrium point by constructing the **T** and **V** matrices. (**2 points**)
(e) Find the eigen-frequencies for these oscillations. (**2 points**)
(f) Find the corresponding eigenvectors. (**2 points**)
(g) Find the most general solution. (**1 point**)
(h) Depict the normal modes with arrows. (**1 point**)