# Examination II for PHYS 6220/7220, Fall 2012

1. A system of two degrees of freedom is described by a Hamiltonian $H = q_1p_1 - q_2p_2 - aq_1^2 + bq_2^2$, where a and b are constants. Two functions are defined by $F_1 = (p_1 - aq_1)/q_2$ and $F_2 = q_1q_2$.

(i) Compute $[F_1, H]$ and $[F_2, H]$. (**4 points**)

(ii) Analyzing the results from part (i) further, what can you say about these two functions? (**2 points**)

2. A particle of mass m approaches a center of force from a far away distance with initial speed $v_0$ and impact parameter b. The center of force exerts a force on the particle corresponding to the potential $V(r) = -k/r^n$, where r is the distance of the particle from the center of force, k is a positive constant and n is a positive integer. Express all answers in terms of the known constants, m, k, $v_0$, b and n.

(i) Find an implicit equation to determine distance of closest approach c. (**3 points**)
(ii) State all the cases for the value of n when analytic closed form solutions can be obtained. Solve explicitly for c whenever possible in these cases. (**5 points**)
(iii) State any special conditions that should be satisfied by the known constants in part (ii). (**2 points**)

3. Two successive rotations are performed on a rigid body with a common fixed point on the body for both rotations. Each rotation is through $\pi$ radians. The two rotation axes are defined by unit vectors in the laboratory Cartesian coordinate system given by $\mathbf{n_1} = (1,0,1)/\sqrt{2}$, and $\mathbf{n_2}$ = (0, 0, 1) respectively.

(a) Find all elements of the matrix **A** corresponding to the first rotation. (**1 point**)
(b) Find all elements of the matrix **B** corresponding to the second rotation. (**1 point**)
(c) If the resulting net displacement of the body is represented by a matrix **R** then find all its elements. (**2 points**)
(d) Find the resulting angle of rotation as if only one effective rotation was performed on the body through only one axis. (**1 point**)
(e) What are the Euler angles ($\theta$, $\phi$, $\psi$) that would be needed if the matrix **R** were to be obtained by three rotations by these angles rather than a single rotation. (**3 points**)