Examination I for PHYS 6220/7220, Fall 2012

1. For many physical situations one encounters a general linear differential equation of constraint in the set of coordinates $\{x_i, i = 1, 2, \ldots n\}$ of the form $\sum_{i=1}^{n} g_i(x_1, x_2, \ldots x_n)dx_i = 0$. A constraint of this type may be integrated if an integrating function $f = f(x_1, x_2, \ldots x_n)$ is found such that it obeys $\frac{\partial(f g_i)}{\partial x_j} = \frac{\partial(f g_j)}{\partial x_i}, \quad \forall i, j$. In a particular problem we obtain the following set of coordinates: x, y, θ, and ϕ. These follow a constraint equation:
$dx - (b\sin\theta)\, d\phi = 0$.
(a) For this equation identify the value of n and all the functions g_i, (i = 1, 2, …n). (**2 points**)
(b) Evaluate all the relevant partial differential equations to find the factor f, if it exists and thus integrate the equation. If it does not exist then prove rigorously that f = 0. (**4 points**)

2. A thin solid disk of mass m_1 and radius b_1 can move without friction on a horizontal surface. One entire side of the disk always stays in contact with the surface. Another thin solid disk of mass m_2 and radius b_2 is pinned through its center to a point off the center of the first disk, by a distance c, so that it can rotate without friction on the first disk. One entire side of the second disk always stays in contact with the upper surface of the first disk. Note $c < b_1$ and $b_2 < b_1$.
(a) Define clearly a frame of reference and a set of generalized coordinates for the system with a figure. State how all coordinates are measured in words as well. (**3 points**)
(b) Derive the Lagrangian for the system. (**4 points**)
(c) Derive all the Euler-Lagrange equations for the system. (**4 points**)
(d) Solve all the Euler-Lagrange equations. (**4 points**)
(e) State all constants of motion in the problem. Explain why these constants are as expected. (**2 points**)

3. A Hamiltonian is given by $H(x, p) = (cp)/(ax)$, where a and c are positive constants and x > 0 and p > 0.
(a) Solve the Hamilton equations of motion to obtain x = x(t) and p = p(t). (**3 points**)
(b) Obtain the Lagrangian for the system $L = L(x, \dot{x}, t)$. Comment on your preference to use the Hamiltonian or Lagrangian formulation to solve this problem. (**2 points**)