Third Examination for PHYS 6220/7220, Fall 2011

1. A one dimensional rod is constrained to lie in the XY plane. Its end points are denoted by points P and Q. Point P which is closer to the origin maintains a fixed distance b from the origin O. Point P moves continuously in a circle with fixed angular frequency ω anti-clockwise around the Z axis. An ant of mass m moves from point P to point Q on the rod, with constant speed v_0 with respect to the rod. It is located at point P when it starts its motion. The rod lies along the X axis when the ant starts its motion. Express all answers in terms of b, v_0, ω and unit vectors along the coordinate axes. Parts (a) and (b) describe two independent motions.
(a) If the points P, Q, and O are constrained to be collinear at all times then compute the total force on the ant, caused by the non-intertial motion of the rod, as a function of time. Draw an appropriate figure clearly defining and marking all axes, points, and unit vectors needed for your solution. (**3 points**)
(b) If the points P and Q are constrained to lie parallel to the X axis at all times compute the total force on the ant, caused by the non-intertial motion of the rod, as a function of time. Draw an appropriate figure clearly defining and marking all axes, points, and unit vectors needed for your solution (**3 points**)

2. A particle of mass m moves under gravity on a smooth surface the equation of which is $z = x^2 + y^2 - xy$. The Z axis is taken along the vertical pointing upwards. The magnitude of the acceleration due to gravity is g.
(a) Write an expression for the potential energy of the particle purely as a function of x and y, V = V(x, y). (**1 point**)
(b) Find the point (x_0, y_0) at which the potential is an extremum. (**1 point**)
(c) Compute the kinetic energy $T = T(x, y, \dot{x}, \dot{y})$. (**1 point**)
(d) Construct the appropriate Lagrangian for small oscillations about the point (x_0, y_0). Find the eigenfrequencies for these oscillations. (**2 points**)
(e) Find the corresponding eigenvectors. (**2 points**)
(f) Find the most general solution. (**1 point**)
(g) Depict the normal modes with arrows. (**1 point**)