Examination II for PHYS 6220/7220, Fall 2011

1. A one dimensional simple harmonic oscillator has mass m, and generalized canonical coordinates q and p, and angular vibrational frequency ω, and Hamiltonian H(q, p).

(a) Evaluate [u, H] where $u = -i\omega t + \ln(p + i m\omega q)$ and $i \equiv \sqrt{-1}$. (**2 points**)

(b) Use result in part (a) to obtain du/dt. Comment on your result. (**2 points**)

(c) Express H as H = H(u) and other known constants. (**1 point**)

2. A collision cross section σ_c is defined as the area normal to the parallel incident beam of particles in which all particles are absorbed by the center of the central potential. No particles falling outside this area are absorbed. Consider a beam of particles of initial speed V_0 incident on a planet of mass M and radius R. Assume that any particle that lands on the surface of the planet sticks to it and does not bounce. The magnitude of the escape velocity from the planet is V_{esc}. The maximum collision impact parameter s_{max} is defined such that all particles having an impact parameter $s \leq s_{max}$ land on the planet's surface while others do not.

(a) Write a simple expression relating σ_c to s_{max}. (**1 point**)

(b) Draw the trajectory of a particle with $s = s_{max}$. Clearly mark the planet and its radius in your drawing. What is the direction of the velocity of the particle when it lands on the surface of the planet? (**2 point**)

(c) From the results in part (a) and (b) and appropriate conservation laws obtain an expression for σ_c purely in terms of the initial speed V_0, V_{esc} and R. (**4 points**)

(d) If you could not answer part (c) guess its solution based on standard physical principles. Explain your reasoning. Make appropriate checks such as limits of different parameters to verify the correctness of the answer in part (c) or of your guess. (**2 points**)

3. A 3 x 3 square matrix **A** is given by its matrix elements, $a_{11} = a_{22} = 9/10$, $a_{33} = 4/5$, $a_{12} = a_{21} = 1/10$, $a_{31} = -a_{13} = a_{23} = -a_{32} = (3\sqrt{2})/10$. You may express all answers for angles as unambiguous inverse trigonometric functions of numerical constants.

(a) Does this matrix **A** correspond to a physical rotation? If the answer is yes then give reasons for your answer.

(b) Does this matrix **A** correspond to a reflection in a plane? If the answer is yes then give reasons for your answer? (parts a and b together **3 points**)

(c) If it corresponds to a rotation find the unit normal along the axis of rotation and the angle of rotation. If this rotation is to be accomplished by three Euler angle rotations what are the values of the Euler angles?

(d) If it corresponds to a reflection find the unit normal to the plane. (parts c and d together **5 points**)

(e) If it corresponds neither to a rotation or a reflection justify your answer. (**1 point**)