Examination I for PHYS 6220/7220, Fall 2011

1. A Hamiltonian for a particle of mass m is of the form $H(x, p) = (k/2)x^2 + p^2/(2m) - ap$, where a and k are constants of appropriate dimensions.
(a) Write Hamilton's equations of motion for this problem. (**2 points**)
(b) Solve these equations for x(t) and p(t). (**2 points**)
(c) Find the Lagrangian of the system. (**2 points**)
(d) Describe the physical system for which this Hamiltonian has been written. (**1 point**)

2. Consider pendulum of mass m with a fixed length of string a. Unlike an ordinary pendulum however it is not restricted to be in one plane.
(a) Define clearly an appropriate set of generalized coordinates. Use these to obtain the Lagrangian of the system. Draw one or more figures to depict your coordinates. Define any angular coordinates with a figure, giving their ranges and clearly stating how they are measured. (**1 point**)
(b) Use a Lagrange multiplier λ associated with the constraint that the string is of fixed length and write the Langrange equations of motion. (**1 point**)
(c) Write an expression for λ as a function of the generalized coordinates and their velocities. (**1 point**)
(d) State in words the constants of motion in this problem. Write expressions for these constants in terms of the generalized coordinates and generalized velocities. (**2 points**)
(e) Express λ purely in terms of generalized coordinates by eliminating the velocities from the expression in part (c). (**2 point**)
(f) Interpret your result from part (e) in terms of the possible motion of the particle. Analyze the motion for all possible values of λ and what effect it has on the particle motion. (**2 point**)

3. In an unknown universe there are new laws of mechanics. The Lagrangian for a system of n particles is given by $L = L(\{q_i\}, \{\dot{q}_i\}, \{\ddot{q}_i\}, t)$ instead of the usual $L = L(\{q_i\}, \{\dot{q}_i\}, t)$. The Lagrange equations of motion for this new Lagrangian are given by $\frac{d^2}{dt^2}\left(\frac{\partial L}{\partial \ddot{q}_i}\right) - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) + \frac{\partial L}{\partial q_i} = 0.$ Assume that partial derivatives with respect to different variables commute. However a complete derivative should be proved to commute with a partial derivative when such a result is needed. Do not use Einstein summation convention while solving this problem.

Consider function $F = F(\{\dot{q}_i\}, t)$.

(a) Write an expression for $\dot{F}$. (**1 point**)

(b) Use result from part (a) to evaluate $\frac{\partial \dot{F}}{\partial \ddot{q}_i}$. (**2 points**)

(c) Use result from part (b) to show that $\dot{F}$ obeys the new Lagrange's equation. (**1 point**)