Final Examination for PHYS 6220/7220, Fall 2011

1. A particle of mass m and angular momentum ℓ moves in a central force field and has an equation of the orbit given by $r = a(1 + \cos(\theta))$, where a is a positive constant of appropriate dimensions. Answer all questions in terms of given quantities.
(a) Find the form of the potential V(r). (**2 points**)
(b) A particle of mass M approaches this center of force from far away with initial speed V_0. What is the critical value of the impact parameter b above which the particle fails to reach the origin? Ignore the gravitational influence of the first particle of mass m. (**3 points**)

2. An ant is located at the cylindrical coordinates (R, ϕ_1, z_1) on a tree trunk. We can approximate the trunk as a right circular cylinder of radius R. The ant notices a drop of honey at the point (R, ϕ_2, z_2) on the trunk. It wants to go to the drop by walking the shortest distance on the surface of the trunk. All answers should only involve the given quantities.
(a) Find the equation of the path it should follow. (**3 points**)
(b) Describe special cases that may arise and analyze them separately if needed. (**2 points**)
(c) If your general solution in part (a) fails for either of the special cases in part (b) give reasons for the failure. (**1 point**)

3. Two Cartesian coordinate systems S and S**'** share a common origin and Z axis. S**'** is always at rest with respect to S. The axes of system S**'** can be obtained by rotating system S about the Z axis by an angle θ. The rotation matrix that takes the frame S to S**'** is called **R**. Both frames are fixed onto a planar lamina of undetermined shape such that it lies in the XY plane of S. In the frame S the lamina has an angular velocity of rotation $\boldsymbol{\omega}$ and moment of inertia matrix $\mathbf{I_M}$. In the frame S**'** the lamina has an angular velocity of rotation $\boldsymbol{\omega}$**'** and moment of inertia matrix $\mathbf{I_M}$**'**. One point on the lamina is held fixed at the origin of S.
(a) Express the kinetic energy T**'** of the lamina in terms of the primed quantities only. (**1 point**)
(b) Express the kinetic energy of the lamina T in terms of the unprimed quantities only. (**1 point**)
(c) What is the relationship of T to T**'**? (**1 point**)
(d) Write a relation between $\boldsymbol{\omega}$**'** and $\boldsymbol{\omega}$. (**1 point**)
(e) Combine the results of (c) and (d) to obtain a relationship between $\mathbf{I_M}$**'** and $\mathbf{I_M}$. (**1 point**)
(f) Three components of $\mathbf{I_M}$ are given by, $I_{M11} = a$, $I_{M22} = b$, and $I_{M12} = c$. Express all other components of $\mathbf{I_M}$ in terms of these three. (**1 point**)
(g) Construct $\mathbf{I_M}$**'** in terms of a, b, c and the angle θ. (**3 points**)
(h) If S' is known to be the principal axes of the lamina then express θ in terms of a, b, and c? (**1 point**)
(i) Comment on what physical result has been achieved through this problem. (**1 point**)