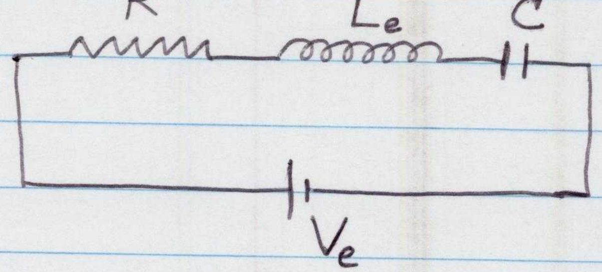
Identify the analogy of the principle

$$\delta \int L\,dt = 0$$ for electrical

circuits. Consider the circuit shown



$L_e \equiv$ inductance here
$C \equiv$ capacitance
$R \equiv$ resistance
$V_e \equiv$ emf of battery.

We want $V_e = IR + L\dot{I} + \frac{q}{c}$

$I \equiv \dot{q}$, $q(t)$ is the charge flowing through the circuit at time t,

$\therefore$ we need $L_e\ddot{q} + \frac{q}{C} - V_e = -R\dot{q}$

$$= \frac{d}{dt}(L_e\dot{q}) - \frac{\partial}{\partial q}\left(qV_e - \frac{q^2}{2C}\right) = -\frac{d}{d\dot{q}}\left(\frac{R\dot{q}^2}{2}\right)$$

$$= \frac{d}{dt}\left[\frac{\partial}{\partial \dot{q}}\left(\frac{L_e\dot{q}^2}{2}\right)\right] - \frac{\partial}{\partial q}\left(qV_e - \frac{q^2}{2C}\right) = -\frac{d}{d\dot{q}}\left(\frac{R\dot{q}^2}{2}\right)$$

Hence we identify $T \equiv \frac{1}{2}L_e\dot{q}^2$

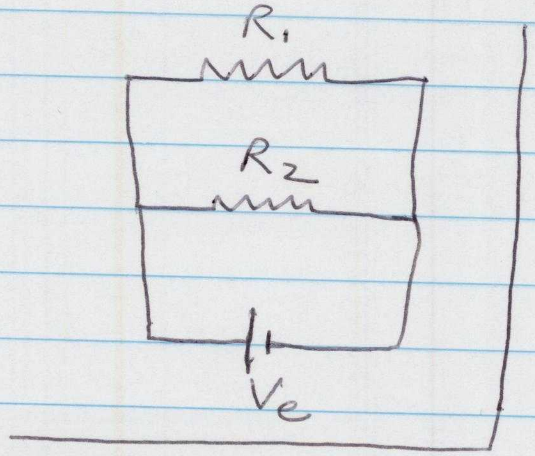
$$V = -qV_e + \frac{q^2}{2C}, \quad \mathcal{F} = \frac{R\dot{q}^2}{2}$$

Also one co-ordinate q_i for each loop i.
n loops means there are q_i charges, $i = 1, n$.

$$\therefore L \equiv T - V = \frac{L_e}{2}\dot{q}^2 - \frac{q^2}{2C} + V_e q$$

Note :→ 1 circuit loop gives one co-ordinate. n-loops give n coordinates.

Consider the following 2-loop circuit.



$$\mathcal{F} = \frac{R_1 \dot{q}_1^2}{2} + R_2 \frac{\dot{q}_2^2}{2}$$

$$V = q_1 V_e + q_2 V_e$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = \frac{-d\mathcal{F}}{d\dot{q}_i}$$

where $i = 1, 2$.

$$\Rightarrow \quad -V_e = -R_1 \dot{q}_1 \quad , \quad -V_e = -R_2 \dot{q}_2$$

$$\dot{q}_1 = V_e / R_1 \quad , \quad \dot{q}_2 = V_e / R_2$$

Total $\dot{q} \equiv \dot{q}_1 + \dot{q}_2 = V_e / R$, where $R \equiv \left(\frac{R_1 R_2}{R_1 + R_2}\right)$

Mechanical Concept → Electrical Concept

- Displacement → Charge (q)
- Velocity → Current ($\dot{q}$)
- Constant Force → (V_e) Potential difference
- Mass → Inductance (L_e)
- Spring Constant → Reciprocal Capacitance ($1/C$)
- k in Rayleigh dissipation function $\mathcal{F}$ → Resistance (R)

Each Closed loop gives a co-ordinate q_i.