Final Examination for PHYS 4210/5210, 9th December 2024

First **Last**

Student Name:

Instructions:

1) **This test is worth a total of 25 points which will be scaled to a weight of 20% of the final letter grade.**

1. In a small oscillations problem, the constants are k_1 and k_2 are positive and have appropriate dimensions. The kinetic energy K and potential energy V are given by expressions shown below. The two angular variables are φ_1 and φ_2. The dot over the variables signifies a time derivative. To obtain linearized Lagrangian equations, the kinetic energy is given by
$T = \frac{k_1}{2}(3\dot{\varphi}_1^2 + \dot{\varphi}_2^2 + 2\dot{\varphi}_1\dot{\varphi}_2)$. The total potential energy is given by
$V = \frac{k_1 k_2}{2}(2\varphi_1^2 + \varphi_2^2)$.

(1) Consider small oscillations in φ_1 and φ_2. Write the appropriate expressions for the kinetic energy matrix $\boldsymbol{M}$ and the potential energy matrix $\boldsymbol{K}$. **[2 points]**

(2) Write the appropriate matrix equations to find the square of the eigenfrequencies ω^2. It will have two values ω_1^2 and ω_2^2. Find both ω_1^2 and ω_2^2. **[2 points]**

(3) From these frequencies, find the normalized eigenvectors $\boldsymbol{a_1} = \begin{bmatrix} a_{11} \\ a_{12} \end{bmatrix}$ and $\boldsymbol{a_2} = \begin{bmatrix} a_{21} \\ a_{22} \end{bmatrix}$. **[2 points]**

(4) From these eigenvectors, draw a figure showing the normal modes of motion with arrows of appropriate magnitude and direction. **[2 points]**

(5) Find the most general solution for $\varphi_1(t)$ and $\varphi_2(t)$. **[2 points]**

2. Consider a damped one-dimensional oscillator with mass m, resistive force -b$\dot{x}$ and the Hooke's law force is -kx where x is the position of the particle.
   (1) Write Newton's second law equation for this particle. Do not solve it. **[1 point]**
   (2) Consider the circuit shown in Figure 1, where the capacitor is C, the resistor is R and the inductor is L. The charge flowing through the circuit at time t is $q(t)$ and the current is $I \equiv \dot{q}(t)$. Applying Kirchhof's second rule for circuits we conclude that $L\ddot{q} + R\dot{q} + q/C = 0$. Compare the answer in part (1) to the above equation and write the correspondence between m, b, k, x, and $\dot{x}$ and the relevant quantities in the circuit described here. **[3 points]**

(3) Write the most general solution to $q(t)$ in the circuit. You need not derive the solution but just write it. Your answer should contain only the constants L, R, C, time t and any arbitrary constants that occur. **[2 points]**

3. A railroad car is accelerating with a constant linear acceleration **b** horizontally to the right. From its ceiling hangs a simple pendulum of mass m and length ℓ. The acceleration due to gravity is **g** pointing downward.
   (1) Find the angle θ the pendulum makes, when in equilibrium, with a vertical line at the pivot point. State if the direction of θ is clockwise or not. **[3 points]**
   (2) Find the angular speed ω of small oscillations around the equilibrium. **[2 points]**

4. A uniform thin wire lies along the X axis between the limits $|x| \leq L/2$. It is now bent downward into an arc of a circle with radius b, leaving the midpoint fixed at the origin. The X axis is tangent to the bent wire at the origin. The total mass of the wire is M. The bent wire lies in the XY plane.
   (1) Find the coordinates of its center of mass (x_{CM}, y_{CM}). **[2 points]**
   (2) Check that the two limits L/R tends to 0 and L/R tends to 2π give physically expected values for y_{CM}. **[2 points]**

Figure 1