

Physics 4210/5210 Theoretical Mechanics Fall 2024 - Exam 1

This exam is out of 24 points

[1] (a) Describe the surface, in words, defined by $f = \text{constant}$ if $f = y - 2x^2$.

[2 points]

(b) Draw a figure depicting the surface $f = 3$, for f defined in part (a).

[2 points]

(c) Find the unit vector in the normal direction to the surface in part (b) at the point $(x_0, y_0, z_0) = (1, 5, 1)$. **[2 points]**

[2] A dumbbell consists of two equal masses, each being m , and a connecting massless rod of length $2b$. It lies along the Y axis on a frictionless table centered at the origin and is at rest at time $t = 0$. The mass below the origin is given a sharp tap in the form of a horizontal force F_0 making an angle α with the positive X axis, with $0 < \alpha < \pi/2$. The tap lasts for a time Δt . Answer all questions after the tap is complete. Express all answers in unit vectors along the axes.

(a) What is the velocity of the center of mass of the dumbbell.

[2 points]

(b) What is the angular velocity of rotation of the dumbbell.

[2 points]

(c) What are the velocities of each mass.

[2 points]

[3] A mass m is constrained to move along the X axis with an initial speed $v_0 > 0$. It is subject to a force dependent on the instantaneous velocity, $\mathbf{v}(t)$, of the particle with magnitude $F(v) = -F_0 \exp(v/c)$, where F_0 and c are positive constants.

(a) Find the speed $v(t)$ for all times, $t > 0$.

[3 points]

(b) At what time t_c does the particle come to rest.

[2 points]

[4] A particle with charge q and mass m moves in a constant electric field $(0, E, 0)$ and a constant magnetic field $(0, 0, B)$. At $t = 0$ its initial velocity is $(v_0, 0, 0)$. Define $w = qB/m$.

(a) Find the acceleration of the particle for all time $t > 0$. In terms of w , E and B only.

[3 points]

(b) Find the velocity of the component for which there is no acceleration.

[2 points]

(c) Prove that if you choose a special relationship between v_0 , E , and B then the acceleration of the other two components is zero.

[2 points]