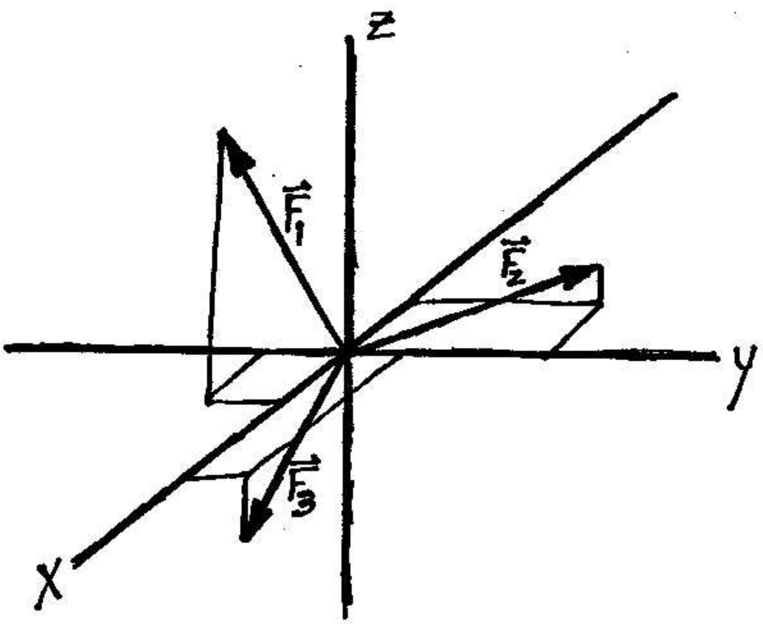
Math Hints

If direction is NOT important to a quantity, then it is a scalar (just a number). Examples: charge, mass, speed, distance, time, energy, etc.

If direction IS important to a quantity, then it is a vector (needs both a magnitude and a direction to point). Examples: force, electric & magnetic fields, velocity, displacement, momenutm, torque, etc.

Resolve vectors into components (numbers) in the $\hat{\mathbf{i}}, \hat{\mathbf{j}}, \hat{\mathbf{k}}$ unit vector directions.



$$\vec{\mathbf{F}}_1 = F_{1x}\hat{\mathbf{i}} + F_{1y}\hat{\mathbf{j}} + F_{1z}\hat{\mathbf{k}}$$
$$\vec{\mathbf{F}}_2 = F_{2x}\hat{\mathbf{i}} + F_{2y}\hat{\mathbf{j}} + F_{2z}\hat{\mathbf{k}}$$
$$\vec{\mathbf{F}}_3 = F_{3x}\hat{\mathbf{i}} + F_{3y}\hat{\mathbf{j}} + F_{3z}\hat{\mathbf{k}}$$
$$\vec{\mathbf{F}}_T = F_{Tx}\hat{\mathbf{i}} + F_{Ty}\hat{\mathbf{j}} + F_{Tz}\hat{\mathbf{k}}$$

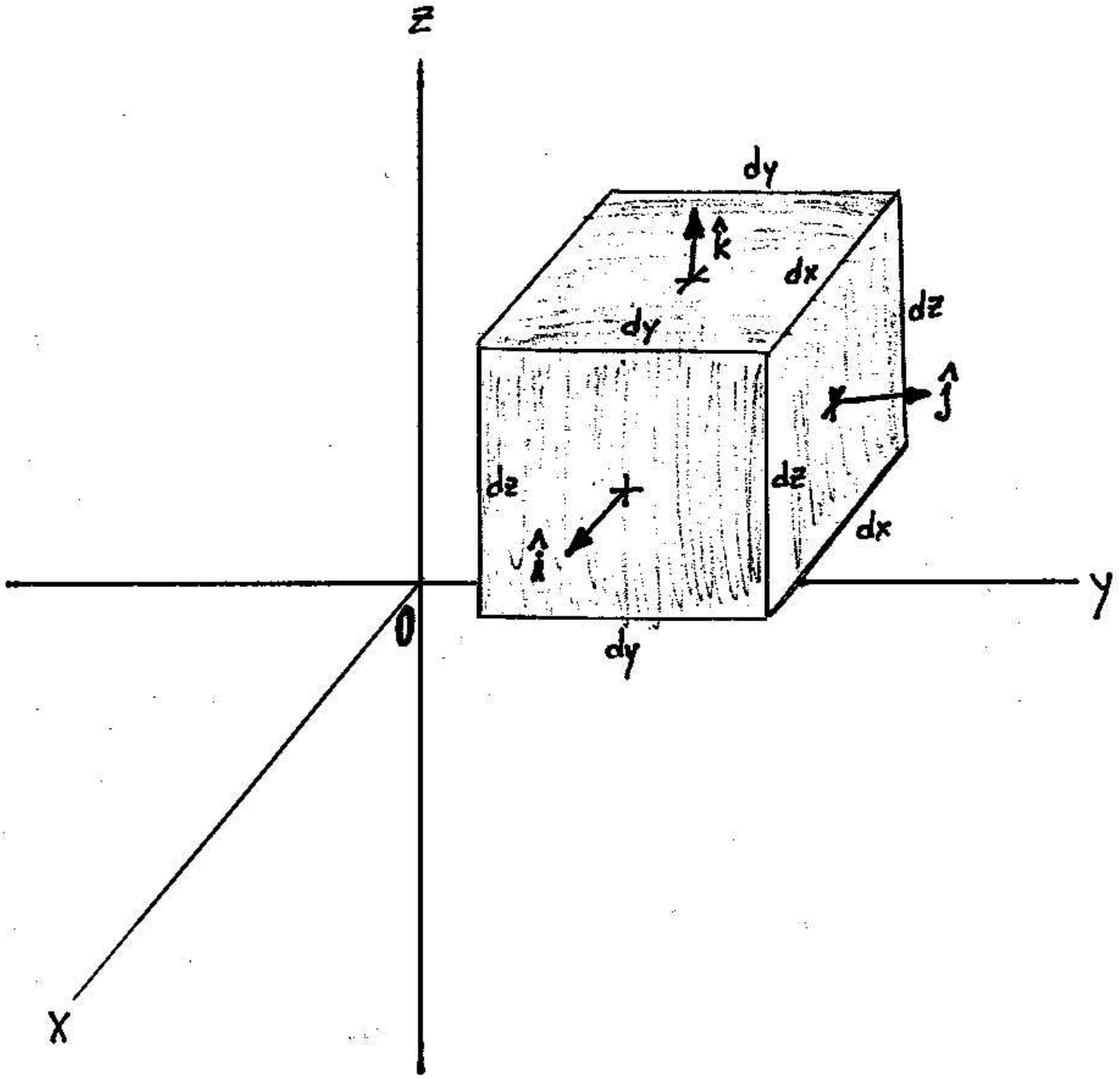
where $F_{Tx} = F_{1x} + F_{2x} + F_{3x}$,

$F_{Ty} = F_{1y} + F_{2y} + F_{3y}$,

and $F_{Tz} = F_{1z} + F_{2z} + F_{3z}$.

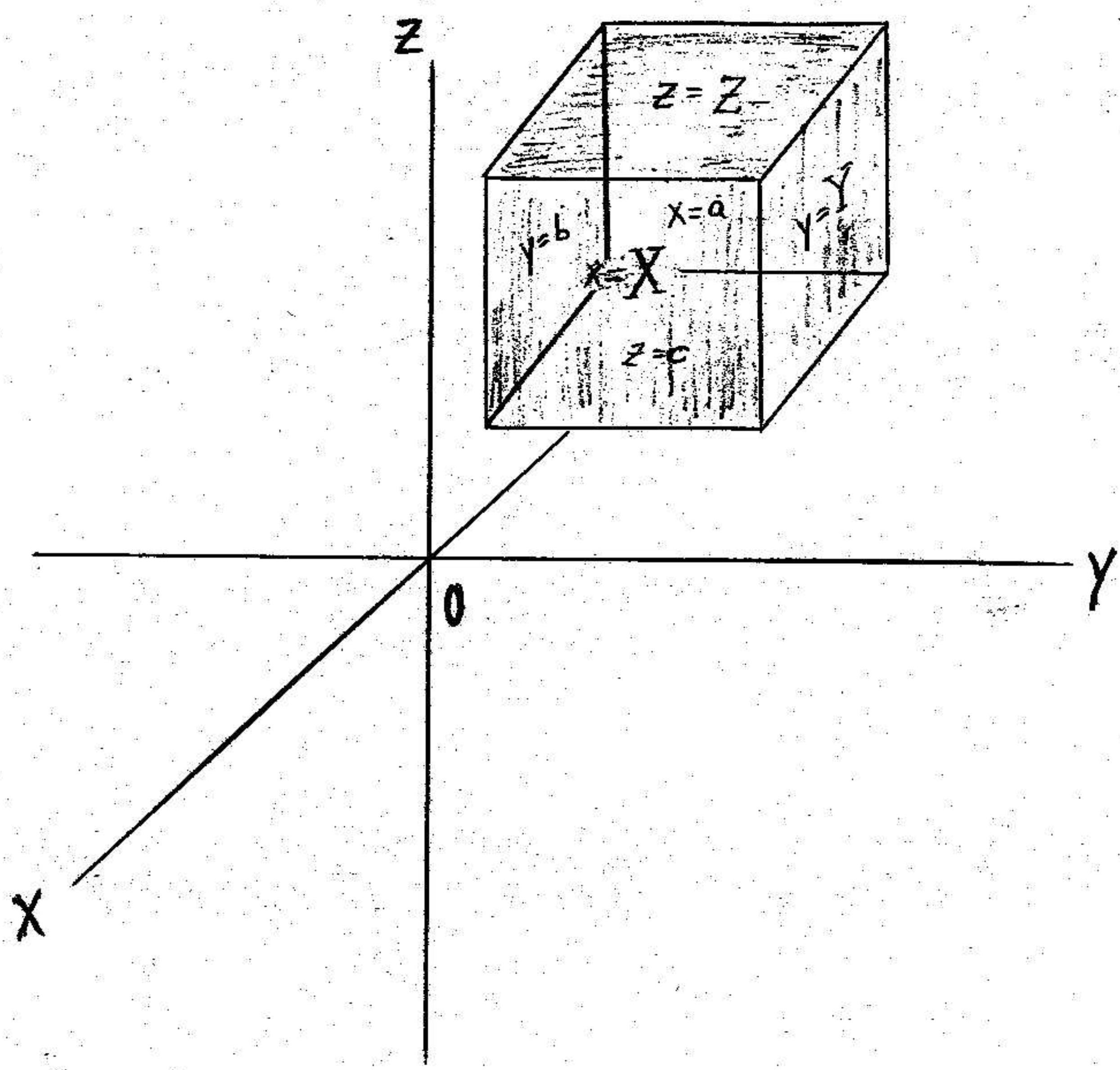
To mathematically build a box:

Start from a point, then go straight a length dx. Repeat for dy and dz. Differential area elements (orientation will be important!) will be [dx dy]$\hat{k}$, [dx dz]$\hat{j}$, etc., and the differential volume element will be [dx dy dz].



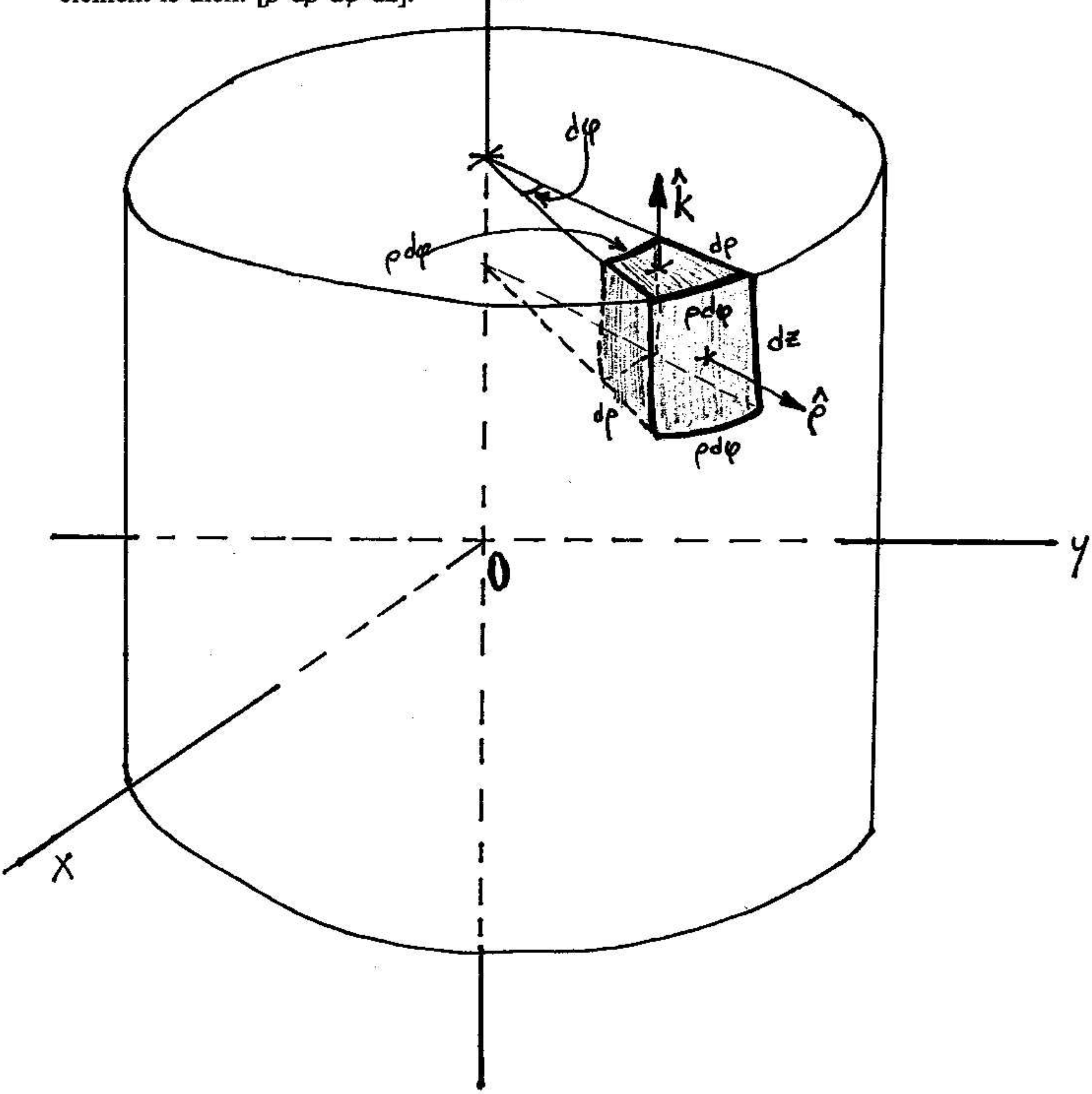
<u>To mathematically build a box</u>:

$$BOX = \int_c^Z \int_b^Y \int_a^X f(x,y,z)\, dx\, dy\, dz$$



To mathematically build a cylinder:

Note that the wall is curved, but the ends are flat. Start from a point, then go straight a length $d\rho$. Swing the $d\rho$ line through an arc length $(\rho d\varphi)$. The differential end area element is then $[\rho\ d\rho\ d\varphi]\hat{k}$ after regrouping terms. The differential wall area element $[\rho\ d\varphi\ dz]\hat{\rho}$ is built by generating dz along the wall and then swinging that line through an arc length $(\rho d\varphi)$. The differential volume element is then: $[\rho\ d\rho\ d\varphi\ dz]$.



To integrate a function in the cylindrical system:

If the cylindrical function has a "slice" missing, then change 2π to whatever angle it should be:

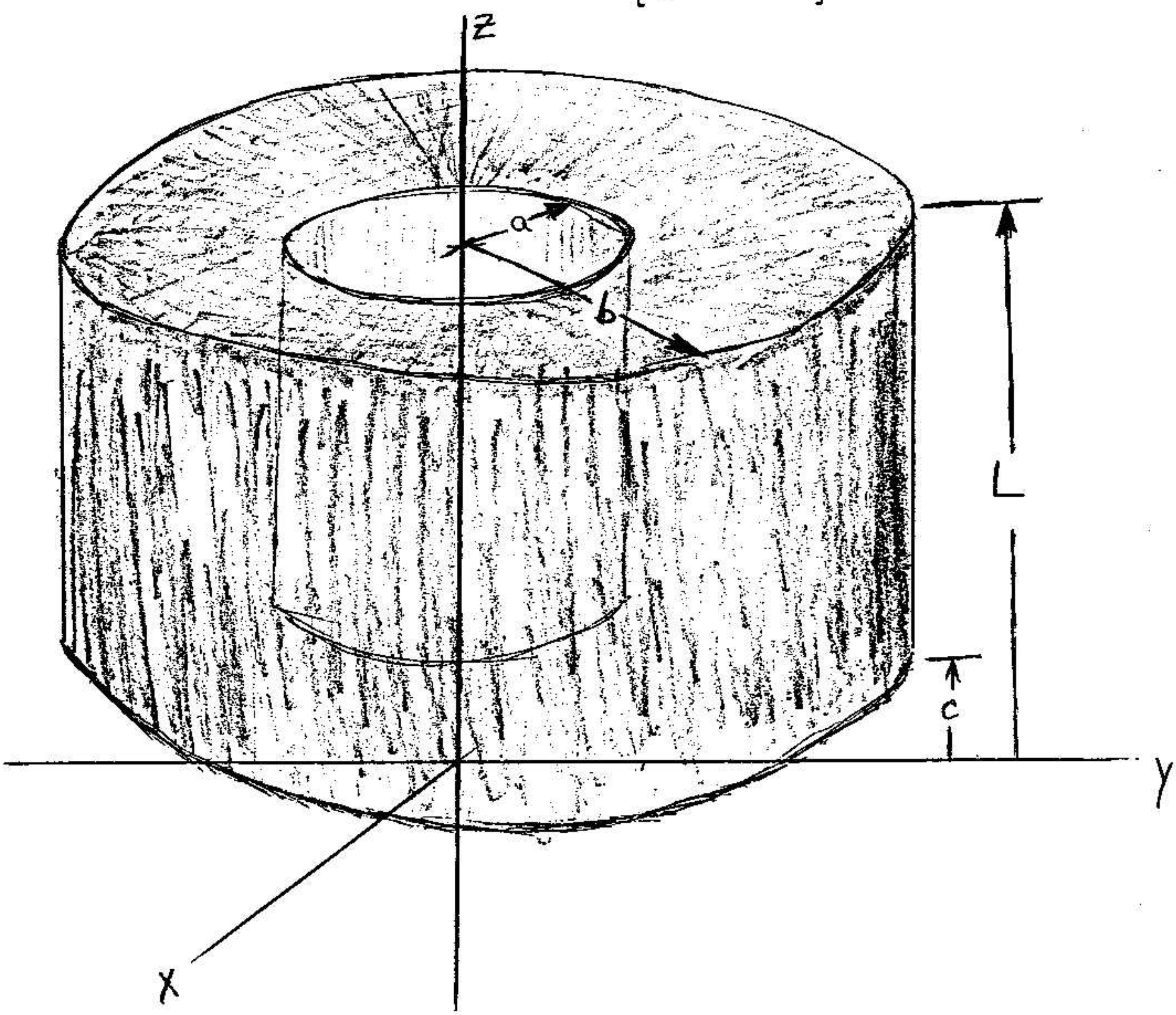
$$\text{CYLINDER} = \int_c^L \int_0^{2\pi} \int_a^b f(\rho, \varphi, z)\, \rho\, d\rho\, d\varphi\, dz$$

If the cylinder is solid, then put a = 0.

If for instance, $f(\rho, \varphi, z)$ is only a function of ρ, then

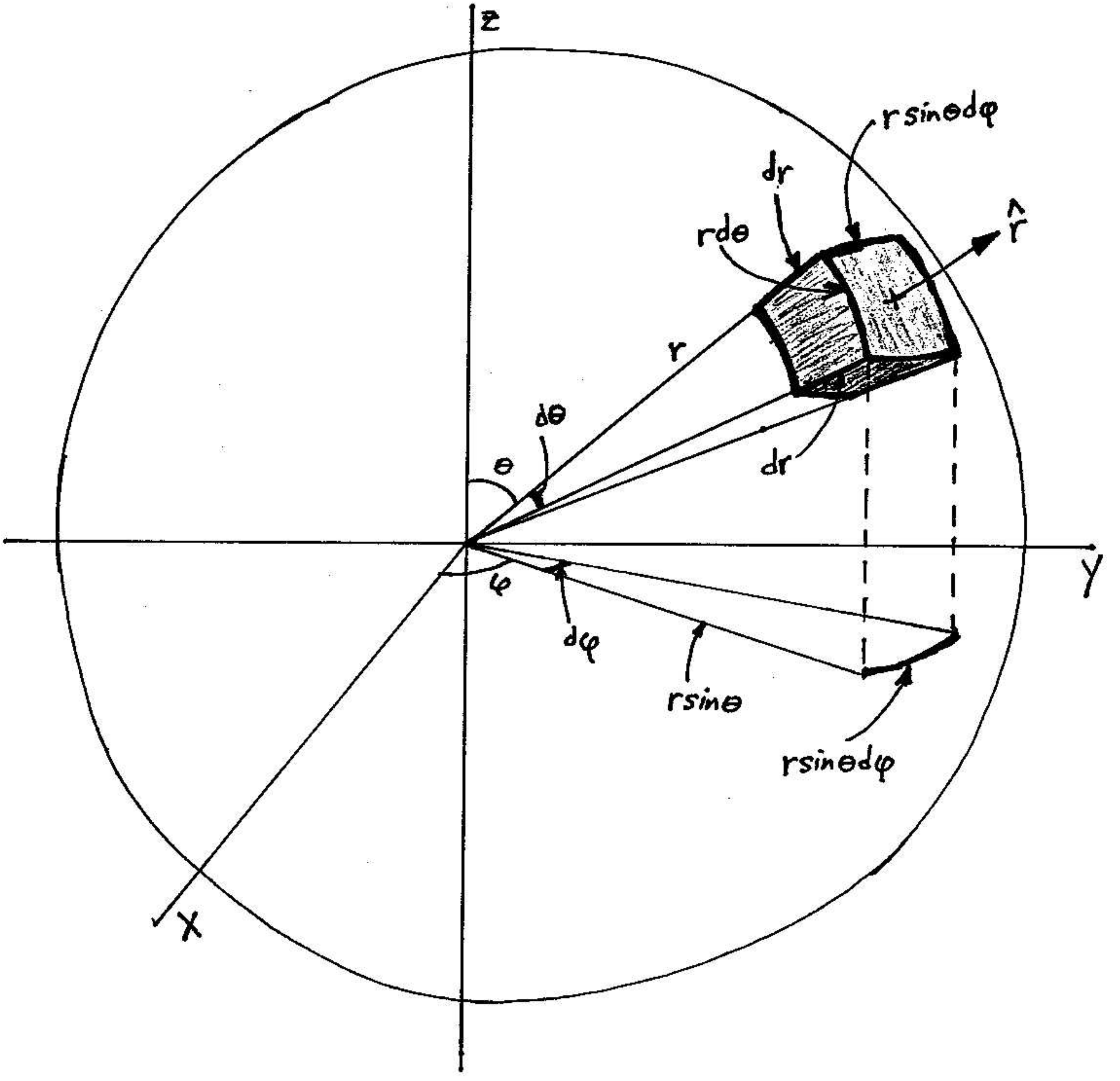
$$\text{CYLINDER} = \left[\int_c^L dz\right]\left[\int_0^{2\pi} d\varphi\right]\left[\int_a^b f(\rho)\, \rho\, d\rho\right]$$

$$= (2\pi)(L - c)\left[\int_a^b f(\rho)\, \rho\, d\rho\right]$$



To mathematically build a sphere:

Note that the surface is curved in two dimensions. Start from a point, then go radially out a length dr. Swing the dr line through an arc length ($rd\theta$). This generates a differential side area element of: [$r\,dr\,d\theta$]$\hat{\varphi}$ after regrouping terms. The other differential side area element [($r \sin\theta$)$dr\,d\varphi$]$\hat{\theta}$ is generated by sweeping the projection ($r \sin\theta$)dr through an arc length ($rd\varphi$). The differential surface area element is generated by sweeping r and ($r \sin\theta$) through the two angles [($r^2 \sin\theta$) $d\theta\, d\varphi$]$\hat{r}$. The differential volume element is found by multiplying the differential surface area by dr: [($r^2 \sin\theta$) $dr\, d\theta\, d\varphi$].



To integrate a function in the spherical system:

If the problem calls for only half a sphere (i.e., hemisphere), then change either 2π to π in the φ integration or π to $\pi/2$ in the θ integration depending on the orientation of the hemisphere. (Note that other orientations of a hemisphere are possible and will cause different limits than the ones mentioned above.)

$$\text{SPHERE} = \int_0^{2\pi}\int_0^{\pi}\int_a^{b} f(r,\theta,\varphi)\, r^2 \sin\theta \, dr\, d\theta\, d\varphi$$

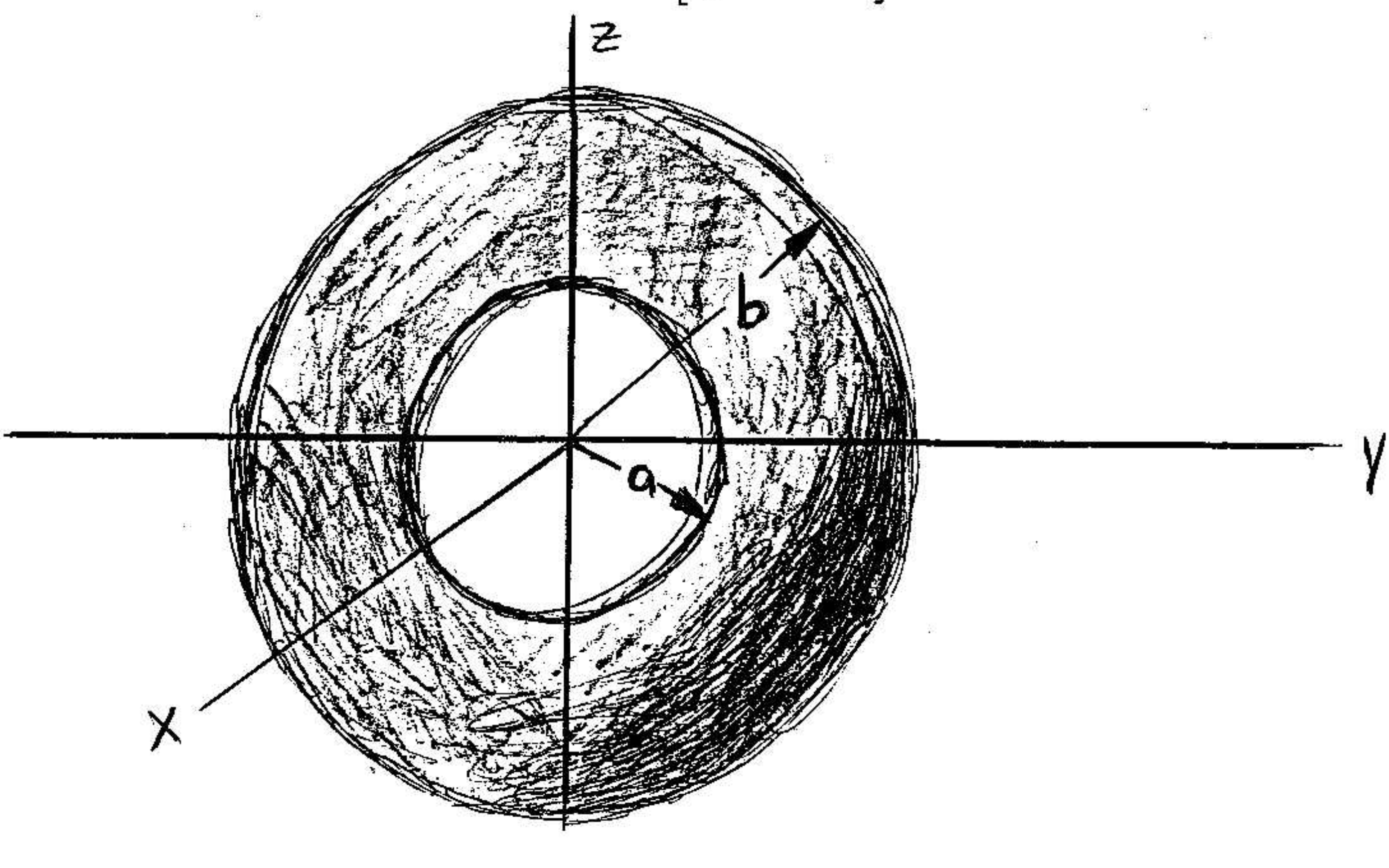
If the sphere is solid, then put a = 0.

If for instance, f(r, θ, φ) is only a function of r, then

$$\text{SPHERE} = \left[\int_0^{2\pi} d\varphi\right]\left[\int_0^{\pi} \sin\theta\, d\theta\right]\left[\int_a^{b} f(r)\, r^2\, dr\right]$$

$$= \left[\,[2\pi]\ \underbrace{[-\cos\theta]\Big|_0^{\pi}}_{2}\right]\left[\int_a^{b} f(r)\, r^2\, dr\right]$$

$$= (4\pi)\left[\int_a^{b} f(r)\, r^2\, dr\right]$$



Math Hints II

In Math Hints I, the ways to set up the surface and volume differentials in cartesian, cylindrical, and spherical geometries were given. The problems in this quarter will basically be in one of those three geometries. The easiest way toward the solution will be to stay in whatever geometry the problem is in. Even though you can mathematically force a square peg in a round hole, it is much easier to use a round peg! What that means for this quarter is that if you set up the integral in the appropriate geometry, the limits of the integrals will be constants and the double (triple) integral will just be a product of two (three) single integrals.

What will seem as countless times this quarter, the solutions of the problems will require integrals over solid or hollow (but thick) boxes, cylinders, and/or spheres. When you stop to think about it, the geometry is the same for a solid object as it is for a hollow object. The only difference between the two is that for a hollow object, the inner dimension is not zero. This will only affect the lower limit of your integral.

When doing the integrals, stop to consider what the function you're integrating over is doing in that variable. If that function depends on that variable, then you MUST integrate the function over that variable. For example, a given problem could have a spherical charge density vary only as a function of r, the radial distance from the origin. In this problem, the charge density doesn't depend on θ or φ, so it doesn't affect the θ and φ integrals. But you must integrate the charge density over r. See the following example.