For any 3X3 matrix $\overline{\overline{R}}$ we have its characteristic equation

$$|\overline{\overline{R}} - \lambda \overline{\overline{I}}| = 0$$

$$\Rightarrow \quad \lambda^3 - \lambda^2 \, Tr.(\overline{\overline{R}}) + \lambda \, Tr.(\overline{\overline{C}}) - |\overline{\overline{R}}| = 0 \longrightarrow (A)$$

where $det.(\overline{\overline{R}}) \equiv ||\overline{\overline{R}}|| \equiv |R|$

$$Tr.(\overline{\overline{R}}) \equiv \sum_{i=1}^{3} R_{ii}.$$

$\overline{\overline{C}} \equiv$ matrix of co-factors of $\overline{\overline{R}} \ni$

$$\overline{\overline{R}}^{-1} |\overline{\overline{R}}| = \overline{\overline{C}}^T$$

If $\overline{\overline{R}}$ is orthogonal then $\overline{\overline{R}}^T = \overline{\overline{R}}^{-1}$

$\Rightarrow \quad \overline{\overline{C}} = |\overline{\overline{R}}| \, \overline{\overline{R}}$. If $|\overline{\overline{R}}| = 1$

then $\overline{\overline{C}} = \overline{\overline{R}}$

Result (A) then becomes

$$\lambda^3 - (\lambda^2 - \lambda) \, Tr(\overline{\overline{R}}) - 1 = 0$$

$\Rightarrow$ by factoring out $(\lambda - 1)$ that

$$\lambda = 1 \quad \text{or} \quad \lambda^2 - \lambda[Tr.(\overline{\overline{R}}) - 1] + 1 = 0$$

$$\Rightarrow \quad \lambda = \frac{[Tr(\overline{\overline{R}}) - 1] \pm \sqrt{[Tr(\overline{\overline{R}}) - 1]^2 - 4}}{2}$$

Note $\quad \bar{A} \times \bar{B} = \sum_{i,j,k} \epsilon_{ijk} A_j B_k \hat{e}_i$

where $\hat{e}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $\hat{e}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$, $\hat{e}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

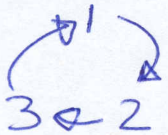
i.e. $\hat{e}_{ij} = \delta_{ij}$

ϵ_{ijk} is the Levi-Civita symbol

$\epsilon_{ijk} = 0$, if any 2 indices are equal

$= 1$, if i, j, k are cyclic

$= -1$ " " " anti-cyclic



$\left.\begin{matrix} 123 \\ 231 \\ 312 \end{matrix}\right\} = \text{cyclic}$

$\left.\begin{matrix} 132 \\ 321 \\ 213 \end{matrix}\right\} = \text{anti-cyclic}$

For a 3X3 matrix $\bar{A}$

$$\det.(\bar{A}) \equiv |\bar{A}| = \sum_{i,j,k=1}^{3} \epsilon_{ijk} a_{1i} a_{2j} a_{3k}$$

In n-dimensions we get

$$|\bar{A}| = \left[\frac{1}{n!}\right] \sum_{\substack{i_1, i_2 \\ \dots i_n = 1}}^{n} \epsilon_{i_1 \dots i_n} \epsilon_{j_1 \dots j_n} a_{i_1 j_1} \dots a_{i_n j_n} .$$

Also in 3-dimensions

$$\bar{A} \cdot (\bar{B} \times \bar{C}) = \sum_{i,j,k} \epsilon_{ijk} A_i B_j C_k$$

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