

For any 3×3 matrix \bar{R} we have its characteristic equation:

$$|\bar{R} - \lambda \bar{I}| = 0$$

$$\Rightarrow \lambda^3 - \lambda^2 \text{Tr}(\bar{R}) + \lambda \text{Tr}(\bar{C}) - |\bar{R}| = 0 \rightarrow \textcircled{A}$$

where $\det(\bar{R}) \equiv ||\bar{R}|| \equiv |\bar{R}|$

$$\text{Tr}(\bar{R}) \equiv \sum_{i=1}^3 R_{ii}$$

$\bar{C} \equiv$ matrix of co-factors of $\bar{R} \Rightarrow$

$$\bar{R}^{-1} |\bar{R}| = \bar{C}^T$$

If \bar{R} is orthogonal then $\bar{R}^T = \bar{R}^{-1}$

$$\Rightarrow \bar{C} = |\bar{R}| \bar{R}. \text{ If } |\bar{R}| = 1$$

$$\text{then } \bar{C} = \bar{R}$$

Result \textcircled{A} then becomes

$$\lambda^3 - (\lambda^2 - \lambda) \text{Tr}(\bar{R}) - 1 = 0$$

\Rightarrow by factoring out $(\lambda - 1)$ that

$$\lambda = 1 \text{ or } \lambda^2 - \lambda [\text{Tr}(\bar{R}) - 1] + 1 = 0$$

$$\Rightarrow \lambda = \frac{[\text{Tr}(\bar{R}) - 1] \pm \sqrt{[\text{Tr}(\bar{R}) - 1]^2 - 4}}{2}$$

Note $\bar{A} \times \bar{B} = \sum_{i,j,k} \epsilon_{ijk} A_j B_k \hat{e}_i$

where $\hat{e}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $\hat{e}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$, $\hat{e}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

i.e., $\hat{e}_{ij} = \delta_{ij}$

ϵ_{ijk} is the Levi-Civita symbol

$\epsilon_{ijk} = 0$, if any 2 indices are equal

$= 1$, if i, j, k are cyclic

$= -1$ " " " anti-cyclic

$$\left. \begin{array}{c} \curvearrowright \\ 1 \curvearrowright \\ 3 \curvearrowright 2 \end{array} \right\} \begin{array}{l} 123 \\ 231 \\ 312 \end{array} \} = \text{cyclic}$$

$$\left. \begin{array}{l} 132 \\ 321 \\ 213 \end{array} \right\} = \text{anti-cyclic}$$

For a 3×3 matrix \bar{A}

$$\det(\bar{A}) \equiv |\bar{A}| = \sum_{i,j,k=1}^3 \epsilon_{ijk} a_{1i} a_{2j} a_{3k}$$

In n -dimensions we get

$$|\bar{A}| = \frac{1}{n!} \sum_{\substack{i_1, i_2, \dots, i_n \\ \dots \\ i_n=1}} \epsilon_{i_1 \dots i_n} \epsilon_{j_1 \dots j_n} a_{i_1 j_1} \dots a_{i_n j_n}$$

Also in 3-dimensions

$$\bar{A} \cdot (\bar{B} \times \bar{C}) = \sum_{i,j,k} \epsilon_{ijk} A_i B_j C_k$$

Copyright © 1983, by John Wiley & Sons, Inc.

All rights reserved. Published simultaneously in Canada.

Reproduction or translation of any part of this work beyond that permitted by Sections 107 and 108 of the 1976 United States Copyright Act without the permission of the copyright owner is unlawful. Requests for permission or further information should be addressed to the Permissions Department, John Wiley & Sons.

Library of Congress Cataloging in Publication Data:

Moore, E. Neal (Edwin Neal), 1934-
Theoretical mechanics.

Includes index.

1. Mechanics. I. Title.

QA805.M74 1983 531 83-6841
ISBN 0-471-87488-4

Printed in the United States of America

10 9 8 7 6 5 4 3 2 1