The standard path for Lagrangian Problems

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Many students who have studied the theory behind solving mechanics problems using the Lagrangian or Hamiltonian method find the methods very confusing at best. At worst they are simply unable to even get started on the problem. To guide such students we have designed an 18 step algorithm which will provide succor to get past conceptual hurdles. These are listed below.

1. State the initial conditions and given quantities in writing, initially in sentences and once more expertise is attained in phrases or mere symbols.
2. Draw a sketch of the various parts of the system. Label as many special points as you can. Err on the side of too many labeled points but never on the side of too few.
3. Define an inertial frame in which the problem will be solved. State it in words. If you need to work in a non-intertial frame you should still always have an intertial frame defined.
4. Select the origin and Cartesian axes in this frame. Note all 3 Cartesian axes should always be defined even if the eventual problem may be solved in some other coordinates.
5. State the number of particles N and hence the 3N degrees of freedom (dof).
6. Write holonomic constraints for each identifiable constraint. Do not skip this step. If you get stuck here then struggle with it till you get a set of k holonomic constraints.
7. With each holonomic constraint there will be a reduction in the dof. There will now be 3N-k dofs. Write these down in words or symbols for coordinates if possible.
8. Using the help of symmetries choose the generalized set of coordinates $\{q_i, I = 1, 2, \ldots 3N –k\}$. Care should be taken in this choice so that the resulting equations in step 11 below will be simple. If you do not get a good choice now, work through step 11 and come back to this step to make a better choice of coordinates.
9. Write T, V, and then L. Note while writing T and V that you may first write them in a form which is convenient like in Cartesian or other coordinates. Then transform these expressions in generalized coordinates and velocities.
10. At this step pause and check various limits and dimensions of T and V. Correct any errors you may find.
11. Write the Euler- Lagrange Equations from the Lagrangian L.
12. Solve these equations to the extent possible.
13. Plug in initial conditions if any special ones are given.
14. Take a pause after the solution to check limits and dimensions.

15. Check that your constraints are satisfied by your solution.
16. Search for constants of the motion if any.
17. Comment on your results, in terms of any new or old physics you may have observed in this problem. Any connections to other problems in mechanics or other branches of science.
18. Make notes on what new physics or math you have learned in the process of solving this problem for future reference and study.