Final Examination for PHYS 6220/7220, 8th December 2025

First **Last**

Student Name:

Instructions:

1) **This test is worth a total of 30 points which will be scaled to a weight of 24% of the final letter grade.**

2) **Use more pages as needed for question 13.**

1. Define the conditions under which a rigid body is called a symmetrical top. [**1 point**]

2. Define the conditions under which a rigid body is called a rotor. [**1 point**]

3. Write an expression for the energy E, in terms of Euler angles, of a symmetrical top moving under the influence of gravity with one point fixed? [**1 point**]

4. Write an expression for I_{yy}, the yy component of the inertia matrix, for a continuous body. [**1 point**]

5. Write an expression for I_{yz}, the yz component of the inertia matrix, for a rigid body made of discrete points. [**1 point**]

6. The perpendicular axis theorem for the moment of inertia of a rigid body is applicable to what type of body? [**1 point**]

7. When is a set of axes on a rigid body called its principal axes? [**1 point**]

8. Write an expression for the kinetic energy of a rigid body in terms of its angular velocity $\boldsymbol{\omega}$ and its inertia matrix $\mathbf{I}^M$. [**1 point**]

9. If the eigen-value of the $\mathbf{T}^{-1}\mathbf{V}$ matrix is zero, what kind of motion will occur in a small oscillations problem? Answer the same question but now when the eigen-value is negative instead of zero. [**2 points**]

10. Write an expression for the element V_{ij} of the $\mathbf{V}$ matrix in a small oscillations problem, in terms of the potential $V(\{q_i\})$? [**1 point**]

11. Write the most general solution for a single simple harmonic oscillator with frequency ω. [**1 point**]

12. In a set of N coupled simple harmonic oscillators what is the maximum possible number of normal mode frequencies? [**1 point**]

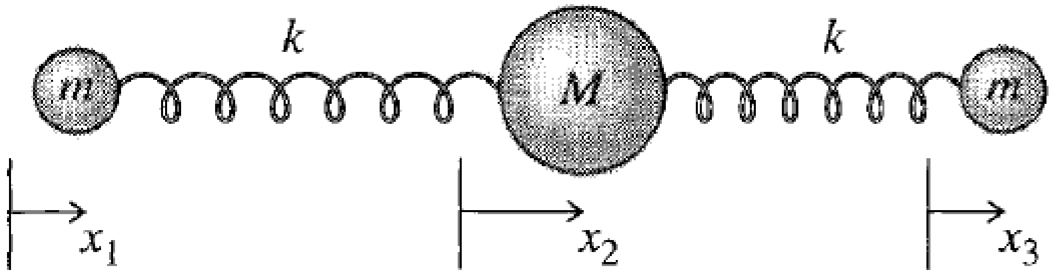

Figure 1. Linear triatomic molecule.

13. Consider the system shown in Figure 1, with two identical particles each of mass m connected by two identical springs to a single atom of mass M. Assume that the system is confined to moving in one dimension. Let λ = M/m. The total potential energy is given by $V = \frac{k}{2}((x_1 - x_2)^2 + (x_2 - x_3)^2)$.

(1) Write an expression for the total kinetic energy (T) in terms of λ. **[1 point]**

(2) Consider small oscillations in $x_1,\ x_2$, and x_3. Write the appropriate equations for the kinetic energy matrix $\boldsymbol{T}$ and the potential energy matrix $\boldsymbol{V}$. **[2 points]**

(3) Set m = k =1, in solving (3) – (6). Write the appropriate matrix equations to find the square of the eigenfrequencies ω^2. It will have three values $\omega_1^2,\ \omega_2^2,$ and ω_3^2. Find $\omega_1^2,\ \omega_2^2,$ and ω_3^2. **[3 points]**

(4) From these frequencies, find the normalized eigenvectors $\boldsymbol{a_1} = \begin{bmatrix} a_{11} \\ a_{21} \\ a_{31} \end{bmatrix}$, $\boldsymbol{a_2} = \begin{bmatrix} a_{12} \\ a_{22} \\ a_{32} \end{bmatrix}$ and $\boldsymbol{a_3} = \begin{bmatrix} a_{13} \\ a_{23} \\ a_{33} \end{bmatrix}$. **[6 points]**

(5) From these eigenvectors, draw a figure showing the normal modes of motion with arrows of appropriate magnitude and direction. **[2 points]**

(6) Find the most general solution for $x_1(t),\ x_2(t)$, and $x_3(t)$. **[3 points]**