Examination 2 for PHYS 6220/7220, 12th November 2025

First **Last**

Student Name:

Instructions:

1) **This test is worth a total of 20 points which will be scaled to a weight of 20% of the final letter grade.**

2) **Use more pages as needed for question 11.**

1. How is the time dependence of H related to that of L? [**1 point**]

2. How is [A, B] related to [B, A]? [**1 point**]

3. Write equations for the fundamental Poisson brackets? [**1 point**]

4. If a particle is only under the influence of a central force what vector physical quantity of the particle is conserved? What scalar physical quantity is conserved? [**2 points**]

5. In a central force problem, it is known that at a certain time the radial coordinate r of the particle is at its maximum. What physical quantity is zero at that instance? [**1 point**]

6. Write the differential equation for the orbit of a particle, in a central force field. [**1 point**]

7. Write the integral equation for the orbit of a particle, in a central force field. [**1 point**]

8. For a central potential $V(r) = -k/r$, $k > 0$, write the equation of the orbit when its energy is negative. State the two special shapes that occur. [**2 points**]

9. What is the maximum number for the degrees of freedom of a rigid body? [**1 point**]

10. Write the definition of an orthogonal matrix. [**1 point**]

11. Consider two Cartesian coordinate systems which both have the same origin but the axes of S$'$ are rotated with respect to S. Coordinate system S has unit vectors along its XYZ axes as:

$$\hat{\imath} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \hat{\jmath} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \text{and } \hat{k} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

Coordinate system S$'$ has unit vectors along its X'Y'Z' axes as:

$$\hat{\imath}' = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \hat{\jmath}' = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \text{and } \hat{k}' = \frac{1}{\sqrt{2}}\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}.$$

(1) Find the rotation matrix $\overline{\overline{R_1}}$ corresponding to the rotation from S to S$'$. **[3 points]**

(2) Find the rotation matrix $\overline{\overline{R_2}}$ corresponding to the rotation from S$'$ to S. **[3 points]**

(3) An arbitrary vector is given by its components in the S$'$ frame as $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$. Derive its components in the S frame. **[2 points]**