Final Examination for PHYS 6220/7220, 9th December 2024

First **Last**

Student Name:

Instructions:

1) This test is worth a total of 25 points which will be scaled to a weight of 20% of the final letter grade.

2) Use more pages as needed for question 13.

1. Define the conditions under which a rigid body is called a symmetrical top. [**1 point**]

2. Define the conditions under which a rigid body is called a rotor. [**1 point**]

3. Write an expression for the energy E, in terms of Euler angles, of a symmetrical top moving under the influence of gravity with one point fixed? [**1 point**]

4. Write an expression for I_{yy}, the yy component of the inertia matrix, for a continuous body. [**1 point**]

5. Write an expression for I_{yz}, the yz component of the inertia matrix, for a rigid body made of discrete points. [**1 point**]

6. The perpendicular axis theorem for the moment of inertia of a rigid body is applicable to what type of body? [**1 point**]

7. When is a set of axes on a rigid body called its principal axes? [**1 point**]

8. Write an expression for the kinetic energy of a rigid body in terms of its angular velocity $\boldsymbol{\omega}$ and its inertia matrix $\mathbf{I}^{M}$. [**1 point**]

9. If the eigen-value of the $\mathbf{T}^{-1}\mathbf{V}$ matrix is zero, what kind of motion will occur in a small oscillations problem? Answer the same question but now when the eigen-value is negative instead of zero. [**2 points**]

10. Write an expression for the element V_{ij} of the **V** matrix in a small oscillations problem, in terms of the potential $V(\{q_i\})$? [**1 point**]

11. Write the most general solution for a single simple harmonic oscillator with frequency ω. [**1 point**]

12. In a set of N coupled simple harmonic oscillators what is the maximum possible number of normal mode frequencies? [**1 point**]

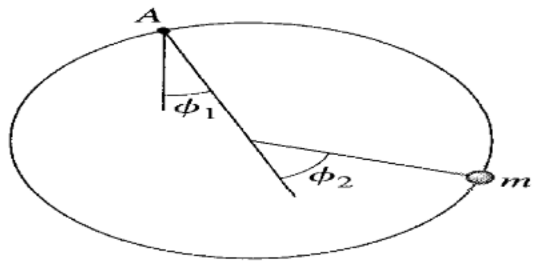

Figure 1. Bead in a hoop

13. A bead of mass m is threaded on a circular hoop with radius b and of mass m as well. The hoop is suspended at point A and is free to rotate in its own vertical plane as shown in Figure 1. The acceleration due to gravity is g pointing downward. To obtain linearized Lagrangian equations, the kinetic energy is given by
$T = \frac{mb^2}{2}(3\dot{\varphi}_1^2 + \dot{\varphi}_2^2 + 2\dot{\varphi}_1\dot{\varphi}_2)$. The total potential energy is given by
$V = \frac{mgb}{2}(2\varphi_1^2 + \varphi_2^2)$.

(1) Consider small oscillations in φ_1 and φ_2. Write the appropriate equations for the kinetic energy matrix $\boldsymbol{T}$ and the potential energy matrix $\boldsymbol{V}$. **[2 points]**

(2) Write the appropriate matrix equations to find the square of the eigenfrequencies ω^2. It will have two values ω_1^2 and ω_2^2. Find both ω_1^2 and ω_2^2. **[2 points]**

(3) From these frequencies, find the normalized eigenvectors $\boldsymbol{a_1} = \begin{bmatrix} a_{11} \\ a_{12} \end{bmatrix}$ and $\boldsymbol{a_2} = \begin{bmatrix} a_{21} \\ a_{22} \end{bmatrix}$. **[4 points]**

(4) From these eigenvectors, draw a figure showing the normal modes of motion with arrows of appropriate magnitude and direction. **[2 points]**

(5) Find the most general solution for $\varphi_1(t)$ and $\varphi_2(t)$. **[2 points]**