Some Mathematical Notations

$a \equiv b$, a is identically equal to b

$A \Rightarrow B$, A implies B

$A \Leftrightarrow B$, A implies B and B implies A.

$\exists$, there exists or there exist

$\ni$, such that

$\therefore$, therefore

$\because$, because

iff, if and only if

$\{a_i\}$, the set of all elements $a_1, a_2, \ldots$ upto the terminal value of i.

δ_{mn}, Kronecker δ function for discrete indices m and n. It equals 1 when $m=n$ and 0 when $m \neq n$.

$\delta(x-y)$, Dirac delta function for continuous arguments x and y.

$\Phi[f(x)]$, Φ is a functional of f. f is a function of x.

• Matrices will always be denoted by capital letters with two bars above as follows: $\overline{\overline{A}}$, $\overline{\overline{P}}$, and $\overline{\overline{Y}}$ are all matrices.

• a_{ij}, p_{ij}, and y_{ij} are the i, j-th elements of matrices $\overline{\overline{A}}$, $\overline{\overline{P}}$, and $\overline{\overline{Y}}$ respectively.

• The determinant of a matrix is denoted as follows: $\det(\overline{\overline{A}}) \equiv |\overline{\overline{A}}|$

• Multiplication of two matrices is denoted as follows: $\overline{\overline{A}}_{n\times m} \times \overline{\overline{B}}_{m\times p} \equiv \overline{\overline{C}}_{n\times p}$. Note: to multiply two matrices, the number of columns of the first matrix (m) must be equal to the number of rows in the second matrix (m).

• Unit vectors are represented by: $\hat{i} \equiv \hat{e}_x \equiv \hat{e}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}_{3\times 1}$, $\hat{j} \equiv \hat{e}_y \equiv \hat{e}_2 \equiv \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}_{3\times 1}$, and $\hat{k} \equiv \hat{e}_z \equiv \hat{e}_3 \equiv \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}_{3\times 1}$.

• A vector is denoted as: $\overline{a} \equiv \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix}_{n\times 1}$

• The transpose of a vector is denoted as: $\overline{a}^T \equiv \begin{bmatrix} a_1 & a_2 & \dots & a_n \end{bmatrix}_{1\times n}$

• The multiplication of a row vector by a column vector is denoted as: $\overline{b}^T_{1\times n}\overline{d}_{n\times 1} \equiv f_{1\times 1} \equiv \overline{b}^T \cdot \overline{d} \equiv f$, which is a scalar.

• The multiplication of a column vector by a row vector is denoted as: $\overline{g}_{n\times 1}\overline{h}^T_{1\times n} \equiv \overline{\overline{L}}_{n\times n}$, which is an $n \times n$ matrix, also called a dyad.