solution, the obvious iteration relation is

$$\psi_n = \omega t + e \sin \psi_{n-1}.$$

Using this iteration procedure, find the analytic form for an expansion of ψ in powers of e at least through terms in e^3.

26. Earth's period between successive perihelion transits (the "anomalistic year") is 365.2596 mean solar days, and the eccentricity of its orbit is 0.0167504. Assuming motion in a Keplerian elliptical orbit, how far does the Earth move in angle in the orbit, starting from perihelion, in a time equal to one-quarter of the anomalistic year? Give your result in degrees to an accuracy of one second of arc or better. Any method may be used, including numerical computation with a calculator or computer.

27. In hyperbolic motion in a $1/r$ potential, the analogue of the eccentric anomaly is F defined by

$$r = a(e \cosh F - 1),$$

where $a(e - 1)$ is the distance of closest approach. Find the analogue to Kepler's equation giving t from the time of closest approach as a function of F.

28. A *magnetic monopole* is defined (if one exists) by a magnetic field singularity of the form $\mathbf{B} = b\mathbf{r}/r^3$, where b is a constant (a measure of the magnetic charge, as it were). Suppose a particle of mass m moves in the field of a magnetic monopole and a central force field derived from the potential $V(r) = -k/r$.

(a) Find the form of Newton's equation of motion, using the Lorentz force given by Eq. (1.60). By locking at the product $\mathbf{r} \times \dot{\mathbf{p}}$ show that while the mechanical angular momentum is not conserved (the field of force is noncentral) there is a conserved vector

$$\mathbf{D} = \mathbf{L} - \frac{qb}{c}\frac{\mathbf{r}}{r}.$$

(b) By paralleling the steps leading from Eq. (3.79) to Eq. (3.82), show that for some $f(r)$ there is a conserved vector analogous to the Laplace–Runge–Lenz vector in which $\mathbf{D}$ plays the same role as $\mathbf{L}$ in the pure Kepler force problem.

29. If all the momentum vectors of a particle along its trajectory are translated so as to start from the center of force, then the heads of the vectors trace out the particle's *hodograph*, a locus curve of considerable antiquity in the history of mechanics, with something of a revival in connection with space vehicle dynamics. By taking the cross product of $\mathbf{L}$ with the Laplace–Runge–Lenz vector $\mathbf{A}$, show that the hodograph for elliptical Kepler motion is a circle of radius mk/l with origin on the y axis displaced a distance A/l from the center of force.

30. What changes, if any, would there be in Rutherford scattering if the Coulomb force were attractive, instead of repulsive?

31. Examine the scattering produced by a repulsive central force $f = kr^{-3}$. Show that the differential cross section is given by

$$\sigma(\Theta)\, d\Theta = \frac{k}{2E}\frac{(1-x)\, dx}{x^2(2-x)^2 \sin \pi x},$$

where x is the ratio of Θ/π and E is the energy.