EXERCISES

10. A planet of mass M is in an orbit of eccentricity $e = 1 - \alpha$ where $\alpha \ll 1$, about the Sun. Assume the motion of the Sun can be neglected and that only gravitational forces act. When the planet is at its greatest distance from the Sun, it is struck by a comet of mass m, where $m \ll M$ traveling in a tangential direction. Assuming the collision is completely inelastic, find the minimum kinetic energy the comet must have to change the new orbit to a parabola.

11. Two particles move about each other in circular orbits under the influence of gravitational forces, with a period τ. Their motion is suddenly stopped at a given instant of time, and they are then released and allowed to fall into each other. Prove that they collide after a time $\tau/4\sqrt{2}$.

12. Suppose that there are long-range interactions between atoms in a gas in the form of central forces derivable from a potential

$$U(r) - \frac{k}{r^m},$$

where r is the distance between any pair of atoms and m is a positive integer. Assume further that relative to any given atom the other atoms are distributed in space such that their volume density is given by the Boltzmann factor:

$$\rho(r) = \frac{N}{V} e^{-U(r)/kT},$$

where N is the total number of atoms in a volume V. Find the addition to the virial of Clausius resulting from these forces between pairs of atoms, and compute the resulting correction to Boyle's law. Take N so large that sums may be replaced by integrals. While closed results can be found for any positive m, if desired, the mathematics can be simplified by taking $m = +1$.

13. **(a)** Show that if a particle describes a circular orbit under the influence of an attractive central force directed toward a point on the circle, then the force varies as the inverse-fifth power of the distance.

(b) Show that for the orbit described the total energy of the particle is zero.

(c) Find the period of the motion.

(d) Find $\dot{x}$, $\dot{y}$, and v as a function of angle around the circle and show that all three quantities are infinite as the particle goes through the center of force.

14. **(a)** For circular and parabolic orbits in an attractive $1/r$ potential having the same angular momentum, show that the perihelion distance of the parabola is one-half the radius of the circle.

(b) Prove that in the same central force as in part (a) the speed of a particle at any point in a parabolic orbit is $\sqrt{2}$ times the speed in a circular orbit passing through the same point.

15. A meteor is observed to strike Earth with a speed v, making an angle ϕ with the zenith. Suppose that far from Earth the meteor's speed was v' and it was proceeding in a direction making a zenith angle ϕ', the effect of Earth's gravity being to pull it into