2. Show that the transformation for a system of one degree of freedom,

$$Q = q \cos \alpha - p \sin \alpha,$$

$$P = q \sin \alpha + p \cos \alpha,$$

satisfies the symplectic condition for any value of the parameter α. Find a generating function for the transformation. What is the physical significance of the transformation for $\alpha = 0$? For $\alpha = \pi/2$? Does your generating function work for both of these cases.

3. In Section 8.4 some of the problems of treating time as one of the canonical variables are discussed. If we are able to sidestep these difficulties, show that the equations of transformation in which t is considered a canonical variable reduce to Eqs. (9.14) if in fact the transformation does not affect the time scale.

4. Show directly that the transformation

$$Q = \log\left(\frac{1}{q}\sin p\right), \quad P = q \cot p$$

is canonical.

5. Show directly that for a system of one degree of freedom the transformation

$$Q = \arctan \frac{\alpha q}{p}, \quad P = \frac{\alpha q^2}{2}\left(1 + \frac{p^2}{\alpha^2 q^2}\right)$$

is canonical, where α is an arbitrary constant of suitable dimensions.

6. The transformation equations between two sets of coordinates are

$$Q = \log(1 + q^{1/2}\cos p),$$

$$P = 2(1 + q^{1/2}\cos p)q^{1/2}\sin p.$$

(a) Show directly from these transformation equations that Q, P are canonical variables if q and p are.

(b) Show that the function that generates this transformation is

$$F_3 = -(e^Q - 1)^2 \tan p.$$

7. (a) If each of the four types of generating functions exist for a given canonical transformation, use the Legendre transformation to derive relations between them.

(b) Find a generating function of the F_4 type for the identify transformation and of the F_3 type for the exchange transformation.

(c) For an orthogonal point transformation of q in a system of n degrees of freedom, show that the new momenta are likewise given by the orthogonal transformation of an n-dimensional vector whose components are the old momenta plus a gradient in configuration space.

8. Prove directly that the transformation