**14.** The Lagrangian for a system can be written as

$$L = a\dot{x}^2 + b\frac{\dot{y}}{x} + c\dot{x}\dot{y} + fy^2\dot{x}\dot{z} + g\dot{y} - k\sqrt{x^2 + y^2},$$

where $a, b, c, f, g$, and $k$ are constants. What is the Hamiltonian? What quantities are conserved?

**15.** A dynamical system has the Lagrangian

$$L = \dot{q}_1^2 + \frac{\dot{q}_2^2}{a + bq_1^2} + k_1 q_1^2 + k_2\dot{q}_1\dot{q}_2,$$

where $a, b, k_1$, and $k_2$ are constants. Find the equations of motion in the Hamiltonian formulation.

**16.** A Hamiltonian of one degree of freedom has the form

$$H = \frac{p^2}{2\alpha} - bqpe^{-\alpha t} + \frac{ba}{2}q^2 e^{-\alpha t}(\alpha + be^{-\alpha t}) + \frac{kq^2}{2},$$

where $a, b, \alpha$, and $k$ are constants.

(a) Find a Lagrangian corresponding to this Hamiltonian.

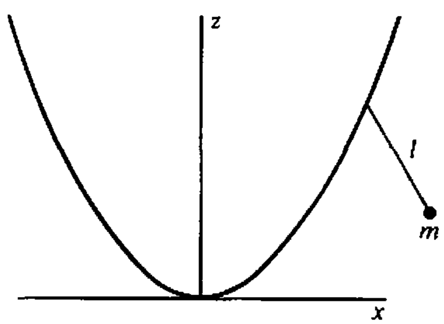
(b) Find an equivalent Lagrangian that is not explicitly dependent on time.

(c) What is the Hamiltonian corresponding to this second Lagrangian, and what is the relationship between the two Hamiltonians?

**17.** Find the Hamiltonian for the system described in Exercise 19 of Chapter 5 and obtain Hamilton's equations of motion for the system. Use both the direct and the matrix approach in finding the Hamiltonian.

**18.** Repeat the preceding exercise except this time allow the *pendulum* to move in three dimensions, that is, a spring-loaded spherical pendulum. Either the direct or the matrix approach may be used.

**19.** The point of suspension of a simple pendulum of length $l$ and mass $m$ is constrained to move on a parabola $z = ax^2$ in the vertical plane. Derive a Hamiltonian governing the motion of the pendulum and its point of suspension. Obtain the Hamilton's equations of motion.




**20.** Obtain Hamilton's equations of motion for a plane pendulum of length $l$ with mass point $m$ whose radius of suspension rotates uniformly on the circumference of a vertical circle of radius $a$. Describe physically the nature of the canonical momentum and the Hamiltonian.