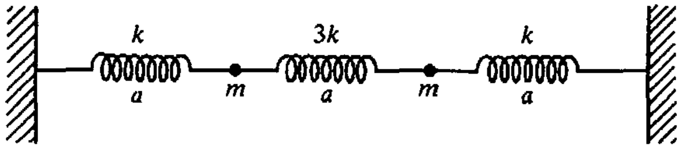
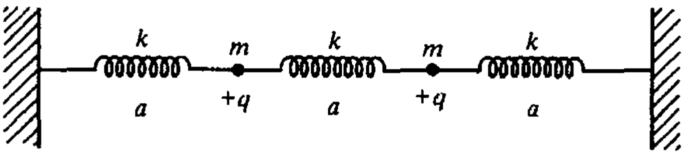
12. Two particles move in one dimension at the junction of three springs, as shown in the figure. The springs all have unstretched lengths equal to a, and the force constants and masses are shown.



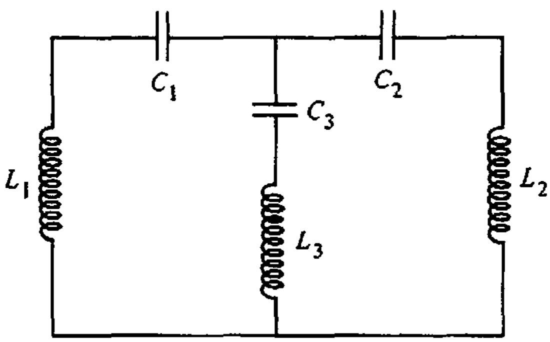
Find the eigenfrequencies and normal modes of the system.

13. Two mass points of equal mass m are connected to each other and to fixed points by three equal springs of force constant k, as shown in the diagram.



The equilibrium length of each spring is a. Each mass point has a positive charge $+q$, and they repel each other according to the Coulomb law. Set up the secular equation for the eigenfrequencies.

14. Find expressions for the eigenfrequencies of the following electrical coupled circuit:



15. If the generalized driving forces Q_i are not sinusoidal, show that the forced vibrations of the normal coordinates in the absence of damping are given by

$$\zeta_i = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \frac{G_i(\omega)}{\omega_i^2 - \omega^2} e^{-i\omega t}\, d\omega,$$

where $G_i(\omega)$ is the Fourier transform of Q_i defined by

$$Q_i(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} G_i(\omega) e^{-i\omega t}\, d\omega.$$

If the dissipation function is simultaneously diagonalized along with T and V, show that the forced vibrations are given by

$$\zeta_i = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \frac{G_i(\omega)(\omega_i^2 - \omega^2 + i\omega\mathcal{F}_i)}{(\omega_i^2 - \omega^2)^2 + \omega^2 \mathcal{F}_i^2} e^{-i\omega t}\, dt,$$