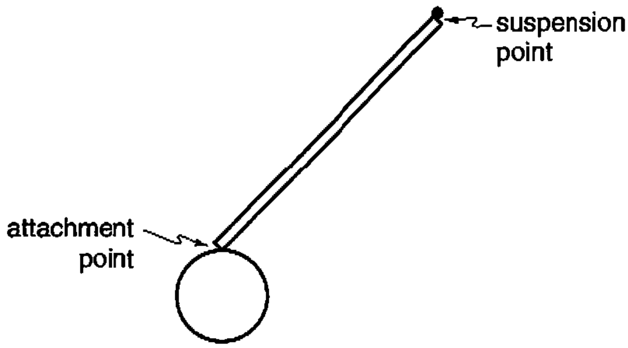

20. A plane pendulum consists of a uniform rod of length l and negligible thickness with mass m, suspended in a vertical plane by one end. At the other end a uniform disk of radius a and mass M is attached so it can rotate freely in its own plane, which is the vertical plane. Set up the equations of motion in the Lagrangian formulation.

21. A compound pendulum consists of a rigid body in the shape of a lamina suspended in the vertical plane at a point other than the center of gravity. Compute the period for small oscillations in terms of the radius of gyration about the center of gravity and the separation of the point of suspension from the center of gravity. Show that if the pendulum has the same period for two points of suspension at unequal distances from the center of gravity, then the sum of these distances is equal to the length of the equivalent simple pendulum.

22. A uniform rod slides with its ends inside a smooth vertical circle. If the rod subtends an angle of 120° at the center of the circle, show that the equivalent simple pendulum has a length equal to the radius of the circle.

23. An automobile is started from rest with one of its doors initially at right angles. If the hinges of the door are toward the front of the car, the door will slam shut as the automobile picks up speed. Obtain a formula for the time needed for the door to close if the acceleration f is constant, the radius of gyration of the door about the axis of rotation is r_0, and the center of mass is at a distance a from the hinges. Show that if f is 0.3 m/s^2 and the door is a uniform rectangle 1.2 m wide, the time will be approximately 3.04 s.

24. A wheel rolls down a flat inclined surface that makes an angle α with the horizontal. The wheel is constrained so that its plane is always perpendicular to the inclined plane, but it may rotate about the axis normal to the surface. Obtain the solution for the two-dimensional motion of the wheel, using Lagrange's equations and the method of undetermined multipliers.

25. (a) Express in terms of Euler's angles the constraint conditions for a uniform sphere rolling without slipping on a flat horizontal surface. Show that they are nonholonomic.

(b) Set up the Lagrangian equations for this problem by the method of Lagrange multipliers. Show that the translational and rotational parts of the kinetic energy are separately conserved. Are there any other constants of motion?

26. For the axially symmetric body precessing uniformly in the absence of torques, find analytical solutions for the Euler angles as a function of time.