

What are the equations of motion? Examine particularly the two cases  $a = 0 = c$  and  $b = 0, c = -a$ . What is the physical system described by the above Lagrangian? Show that the usual Lagrangian for this system as defined by Eq. (1.57') is related to  $L'$  by a point transformation (cf. Derivation 10). What is the significance of the condition on the value of  $b^2 - ac$ ?

19. Obtain the Lagrange equations of motion for a spherical pendulum, i.e., a mass point suspended by a rigid weightless rod.
20. A particle of mass  $m$  moves in one dimension such that it has the Lagrangian

$$L = \frac{m^2 \dot{x}^4}{12} + m \dot{x}^2 V(x) - V_2(x),$$

where  $V$  is some differentiable function of  $x$ . Find the equation of motion for  $x(t)$  and describe the physical nature of the system on the basis of this equation.

21. Two mass points of mass  $m_1$  and  $m_2$  are connected by a string passing through a hole in a smooth table so that  $m_1$  rests on the table surface and  $m_2$  hangs suspended. Assuming  $m_2$  moves only in a vertical line, what are the generalized coordinates for the system? Write the Lagrange equations for the system and, if possible, discuss the physical significance any of them might have. Reduce the problem to a single second-order differential equation and obtain a first integral of the equation. What is its physical significance? (Consider the motion only until  $m_1$  reaches the hole.)
22. Obtain the Lagrangian and equations of motion for the double pendulum illustrated in Fig. 1.4, where the lengths of the pendula are  $l_1$  and  $l_2$  with corresponding masses  $m_1$  and  $m_2$ .
23. Obtain the equation of motion for a particle falling vertically under the influence of gravity when frictional forces obtainable from a dissipation function  $\frac{1}{2}kv^2$  are present. Integrate the equation to obtain the velocity as a function of time and show that the maximum possible velocity for a fall from rest is  $v = mg/k$ .
24. A spring of rest length  $L_0$  (no tension) is connected to a support at one end and has a mass  $M$  attached at the other. Neglect the mass of the spring, the dimension of the mass  $M$ , and assume that the motion is confined to a vertical plane. Also, assume that the spring only stretches without bending but it can swing in the plane.
- Using the angular displacement of the mass from the vertical and the length that the string has stretched from its rest length (hanging with the mass  $m$ ), find Lagrange's equations.
  - Solve these equations for small stretching and angular displacements.
  - Solve the equations in part (a) to the next order in both stretching and angular displacement. This part is amenable to hand calculations. Using some reasonable assumptions about the spring constant, the mass, and the rest length, discuss the motion. Is a resonance likely under the assumptions stated in the problem?
  - (For analytic computer programs.) Consider the spring to have a total mass  $m \ll M$ . Neglecting the bending of the spring, set up Lagrange's equations correctly to first order in  $m$  and the angular and linear displacements.
  - (For numerical computer analysis.) Make sets of reasonable assumptions of the constants in part (a) and make a single plot of the two coordinates as functions of time.