path of least curvature. By Jacobi's principle such a path must be a geodesic, and the geometrical property of minimum curvature is one of the well-known characteristics of a geodesic. It has been pointed out that variational principles in themselves contain no new physical content, and they rarely simplify the practical solution of a given mechanical problem. Their value lies chiefly as starting points for new formulations of the theoretical structure of classical mechanics. For this purpose, Hamilton's principle is especially fruitful, and to a lesser extent, so also is the principle of least action.

DERIVATIONS

1. (a) Reverse the Legendre transformation to derive the properties of $L(q_i, \dot{q}_i, t)$ from $H(q_i, p_i, t)$, treating the $\dot{q}_i$ as independent quantities, and show that it leads to the Lagrangian equations of motion.
   (b) By the same procedure find the equations of motion in terms of the function

$$L'(p, \dot{p}, t) = -\dot{p}_i q_i - H(q, p, t).$$

2. It has been previously noted that the total time derivative of a function of q_i and t can be added to the Lagrangian without changing the equations of motion. What does such an addition do to the canonical momenta and the Hamiltonian? Show that the equations of motion in terms of the new Hamiltonian reduce to the original Hamilton's equations of motion.

3. A Hamiltonian-like formulation can be set up in which $\dot{q}_i$ and $\dot{p}_i$ are the independent variables with a "Hamiltonian" $G(\dot{q}_i, \dot{p}_i, t)$. [Here p_i is defined in terms of $q_i, \dot{q}_i$ in the usual manner.] Starting from the Lagrangian formulation, show in detail how to construct $G(\dot{p}_i, \dot{p}_i, t)$, and derive the corresponding "Hamilton's equation of motion."

4. Show that if λ_i are the eigenvalues of a square matrix, then if the reciprocal matrix exists it has the eigenvalues λ_i^{-1}.

5. Verify that the matrix $\mathbf{J}$ has the properties given in Eqs. (8.38c) and (8.38e) and that its determinant has the value $+1$.

6. Show that Hamilton's principle can be written as

$$\delta \int_1^2 [2H(\boldsymbol{\eta}, t) + \boldsymbol{\eta}\mathbf{J}\dot{\boldsymbol{\eta}}]\, dt = 0.$$

7. Verify that both Hamiltonians, Eq. (8.45) and Eq. (8.47), lead to the same motion as described by Eq. (8.44).

8. Show that the modified Hamilton's principle, in the form of Eq. (8.71), leads to Hamilton's equations of motion.

9. If the canonical variables are not all independent, but are connected by auxiliary conditions of the form

$$\psi_k(q_i, p_i, t) = 0,$$