with symmetrical expressions for η_2 and η_4. The secular determinant will then factor into determinants of lower rank.]

(b) Solve this problem using computer techniques.

7. In the linear triatomic molecule, suppose that motion in the y and z directions is governed by the potentials

$$V_y = \frac{k}{2}(y_2 - y_1)^2 + \frac{k}{2}(y_3 - y_2)^2,$$
$$V_z = \frac{k}{2}(z_2 - z_1)^2 + \frac{k}{2}(z_3 - z_2)^2.$$

Find the eigenfrequencies for small vibrations in three dimensions and describe the normal modes. What symmetries do the zero frequencies represent? You may want to use the kind of intermediate coordinates suggested in Exercise 6.

8. The equilibrium configuration of a molecule is represented by three atoms of equal mass at the vertices of a 45° right triangle connected by springs of equal force constant. Obtain the secular determinant for the modes of vibration in the plane and show by rearrangement of the columns that the secular equation has a triple root $\omega = 0$. Reduce the determinant to one of third rank and obtain the nonvanishing frequencies of free vibration.

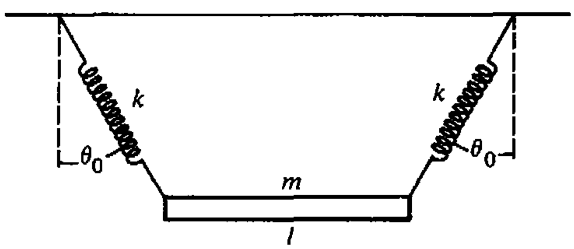
9. Show directly that the equations of motion of the preceding problem are satisfied by (a) a uniform translation of all atoms along the x axis, (b) a uniform translation along the y axis, and (c) a uniform rotation about the z axis.

10. (a) Three equal mass points have equilibrium positions at the vertices of an equilateral triangle. They are connected by equal springs that lie along the arcs of the circle circumscribing the triangle. Mass points and springs are constrained to move only on the circle, so that, for example, the potential energy of a spring is determined by the arc length covered. Determine the eigenfrequencies and normal modes of small oscillations in the plane. Identify physically any zero frequencies.

(b) Suppose one of the springs has a change in force constant δk, the others remaining unchanged. To first order in δk, what are the changes in the eigenfrequencies and normal modes?

(c) Suppose what is changed is the mass of one of the particles by an amount δm. Now how do the normal eigenfrequencies and normal modes change?

11. A uniform bar of length l and mass m is suspended by two equal springs of equilibrium length b and force constant k, as shown in the diagram.



Find the normal modes of small oscillation in the plane.