being time dependent, with the rotation axis along z and the wire in the xy plane. The transformation equations explicitly contain the time.

$$x = r\cos\omega t, \qquad (\omega = \text{angular velocity of rotation})$$

$$y = r\sin\omega t. \qquad (r = \text{distance along wire from rotation axis})$$

While we could then find T (here the same as L) by the same procedure used to obtain (1.71), it is simpler to take over (1.75) directly, expressing the constraint by the relation $\dot{\theta} = \omega$:

$$T = \tfrac{1}{2}m\left(\dot{r}^2 + r^2\omega^2\right).$$

Note that T is not a homogeneous quadratic function of the generalized velocities, since there is now an additional term not involving $\dot{r}$. The equation of motion is then

$$m\ddot{r} = mr\omega^2 = 0$$

or

$$\ddot{r} = r\omega^2,$$

which is the familiar simple harmonic oscillator equation with a change of sign. The solution $r = e^{\omega t}$ shows that the bead moves exponentially outward because of the centripetal acceleration. Again, the method cannot furnish the force of constraint that keeps the bead on the wire. Equation (1.26) with the angular momentum, $\mathbf{L} = mr^2\omega^2 e^{\omega t}$, provides the force $\mathbf{F} = \mathbf{N}/r$, which produces the constraint force, $F = mr\omega^2 e^{\omega t}$, acting perpendicular to the wire and the axis of rotation.

DERIVATIONS

1. Show that for a single particle with constant mass the equation of motion implies the following differential equation for the kinetic energy:

$$\frac{dT}{dt} = \mathbf{F}\cdot\mathbf{v},$$

while if the mass varies with time the corresponding equation is

$$\frac{d(mT)}{dt} = \mathbf{F}\cdot\mathbf{p}.$$

2. Prove that the magnitude R of the position vector for the center of mass from an arbitrary origin is given by the equation

$$M^2R^2 = M\sum_i m_i r_i^2 - \frac{1}{2}\sum_{i,j} m_i m_j r_{ij}^2.$$