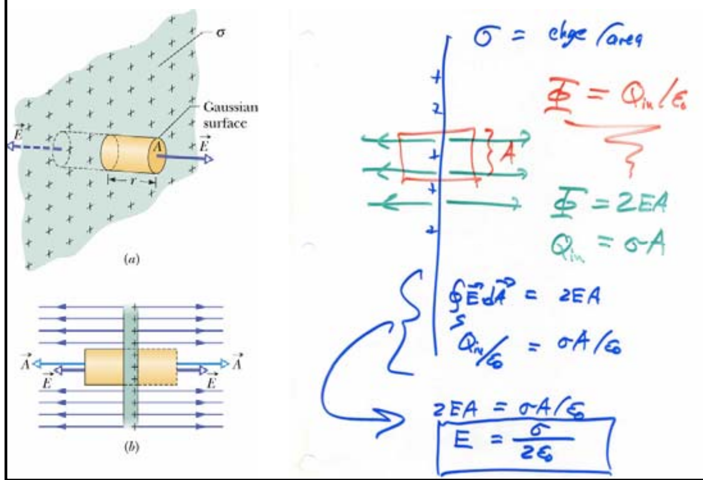
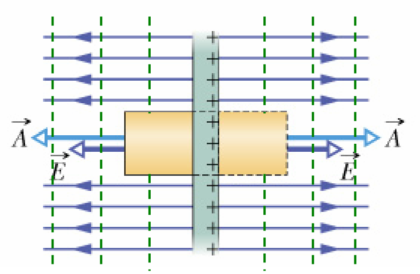
Infinite, Thin, Nonconductive Sheet of Charge



Equipotential surface of charge infinite nonconducting sheet



Potential due to many charges

V due to many charges is just the number summary of potentials by each charges

$$V = \sum_{i=1}^{n} V_i$$

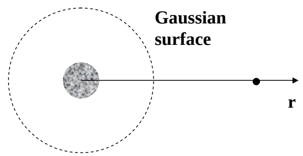
$$V = \frac{q}{4\pi\varepsilon_o}\left[-\frac{1}{2d}-\frac{1}{d}+\frac{1}{d}+\frac{1}{d}\right]$$

$$= \frac{q}{8\pi\varepsilon_o}$$

$$= \frac{(8.99\times10^{9}\,N\cdot m^2/C^2)(4.00\times10^{-15}\,C)}{2(5.00\times10^{-2}\,m)}$$

$$= 3.6\times10^{-4}\,V$$

Shell Theorems



$$V = k\frac{q}{r}$$

To know V, we need to know q.

Using Gauss' law, $q = \varepsilon_o\Phi = +495.8$ nC.

$$V = \frac{q}{4\pi\varepsilon_o r} = \frac{\varepsilon_o\Phi}{4\pi\varepsilon_o r}$$

$$= \frac{\Phi}{4\pi r} = \frac{5.6\times10^4}{4\pi\cdot 0.12}$$

$$= 3.71\times10^4\,V$$

Potential due to continuous charges

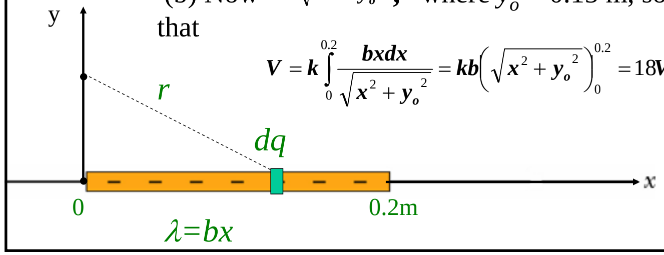
$V = k\int \frac{dq}{r}$ $\qquad$ $dq = \lambda\, dx = bx\, dx$

(a) Here $r = x > 0$, so that

$$V = \frac{1}{4\pi\varepsilon_0}\int_0^{0.20}\frac{bx\,dx}{x} = \frac{b(0.20)}{4\pi\varepsilon_0} = 36V$$

(b) Now $r = \sqrt{x^2 + y_o^2}$, where $y_o = 0.15$ m, so that

$$V = k\int_0^{0.2}\frac{bxdx}{\sqrt{x^2+y_o^2}} = kb\left(\sqrt{x^2+y_o^2}\right)_0^{0.2} = 18V$$



Electrostatics

Exam on Wednesday: Chapters 21-24

- **Review lectures.**
- **More Examples**
- **Review textbook homework problems.**

About the exam

- **Closed-book exam**
- **One 3” × 5” note card with equations, problems, …..allowed**
- **Format :**
  - **4 multiple choice questions for 50%**
  - **Computer-scanned answer sheet**
  - **Two problems for 50%**
  - **Bring your calculators and pencils**
- **All four chapters covered about equally**
- **Focus on key ideas and fundamental laws**
- **Quiz and homework problems may reappear.**
- **Cover page is posted**

Key ideas

- **Electric charge:** **conserved and quantized**
- **Electric field:**
  - **force per unit charge, field lines, adding vectors**
- **Flux:** **amount of field passing through an area**
- **Electric potential:**
  - **energy per unit charge, integral of field**
  - **new unit: electron volt (eV)**
- **Dipole moment:** **paired + and – charges**
- **Shell Theorems**: **inside and outside the charged shell**

Fundamental Laws

Coulomb's Law (point charge):

1. $F = kQq/r^2$
2. $E = kQ/r^2$
3. $V = kQ/r$

Gauss's Law:

The outward flux of the electric field through any closed surface equals the net enclosed charge divided by ε_0.

Relations between potential and field:

The potential difference between A and B is the work required to carry a unit positive charge from A to B.

$$\Delta V = -\int E_x dx \qquad E_x = -\frac{dV}{dx} \quad etc.$$

Physics constants

- **Elementary Charge: e = 1.6 × 10^{-19} C**
- **k = 9 × 10^9 SI unit**
- **1 eV = 1e ·1V = 1.6 × 10^{-19} J - Energy unit**

Terminology

Words whose precise definitions you must know:

Field (definition, direction, vector summing,)

Field Lines (three properties, direction, distribution around single and couple point charge, large plate)

Flux (definition, use in Gauss' law)

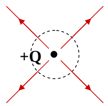
Dipole moment (definition, direction, highest and lowest potential energy external field)

Potential (definition, potential energy, relationship with E)

Potential difference, Equipotential surface (and body)

And of course the SI units for all these things.

$\vec{E}$ field in 3 geometries (by Gauss' Law)



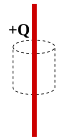
1. Point charge

$\vec{E}$ lines spread out in 3-D

E ∝ $1/r^2$

$$\Phi = 4\pi r^2 E = Q/\varepsilon_0$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$



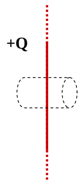
2. Line charge

$\vec{E}$ lines spread out in 2-D

E ∝ 1/r

$$\Phi = 2\pi r l E = \lambda l/\varepsilon_0$$

$$E = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{r}$$



3. Plane charge

$\vec{E}$ lines in only 1-D

E constant

$$\Phi = 2AE = \sigma A/\varepsilon_0$$

$$E = \frac{\sigma}{2\varepsilon_0}$$

Conductor

Definition: materials in which a significant number of charged particles (electrons in metals) are free to move

Fact 1: **In a steady state the electric field inside a good conductor must be zero.**

Fact 2: **In a steady state, any net charge on a good conductor must be entirely on the surface.**

Fact 3: **Electric field lines at the surface of conductor is always perpendicular to the surface.**

– Equipotential Body

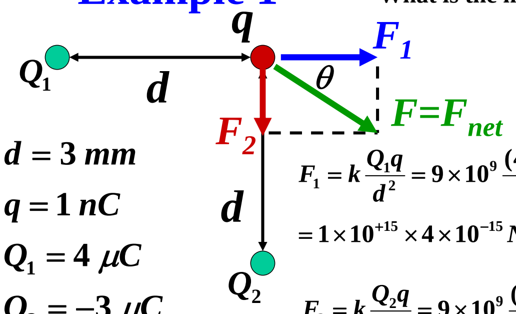
Application of Physical laws and concepts

1) **F and E by point charges:**

   When more than two point charges present, Coulomb's law for each pair of charges → superposition of all the vectors on the charge by all the others.

Example 1

What is the net force on *q*?



$d = 3\ mm$

$q = 1\ nC$

$Q_1 = 4\ \mu C$

$Q_2 = -3\ \mu C$

$$F_1 = k\frac{Q_1 q}{d^2} = 9\times10^9\frac{(4\times10^{-6})(1\times10^{-9})}{(3\times10^{-3})^2} = 1\times10^{+15}\times4\times10^{-15}\,N = \underline{4\ N}$$

$$F_2 = k\frac{Q_2 q}{d^2} = 9\times10^9\frac{(3\times10^{-6})(1\times10^{-9})}{(3\times10^{-3})^2} = 1\times10^{+15}\times3\times10^{-15}\,N = \underline{3\ N}$$

$$\vec{F} = \vec{F}_1 + \vec{F}_2$$

$$F = \sqrt{F_1{}^2 + F_2{}^2} = \sqrt{3^2 + 4^2} = \underline{5\ N}$$

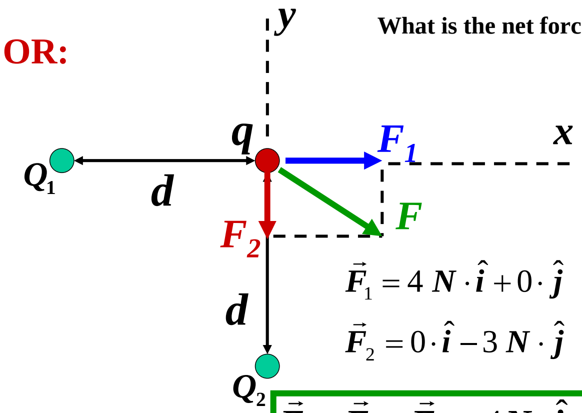
$$\tan\theta = 3/4$$

$$\theta = 37°$$

Example 1 (cont'd)

What is the net force on *q*?

OR:



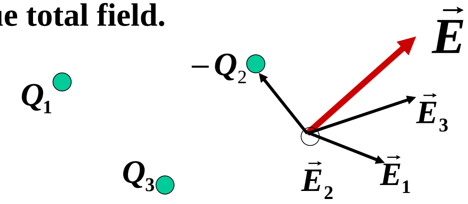
$$\vec{F}_1 = 4\ N\cdot\hat{i} + 0\cdot\hat{j}$$

$$\vec{F}_2 = 0\cdot\hat{i} - 3\ N\cdot\hat{j}$$

$$\boxed{\vec{F} = \vec{F}_1 + \vec{F}_2 = 4N\cdot\hat{i} - 3\ N\cdot\hat{j}}$$

Example 2 for Adding fields

- **Principle of superposition**
- **Electric fields due to different sources combine vector addition to form the one true total field.**



$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \cdots$$

Application of Physical laws and concepts

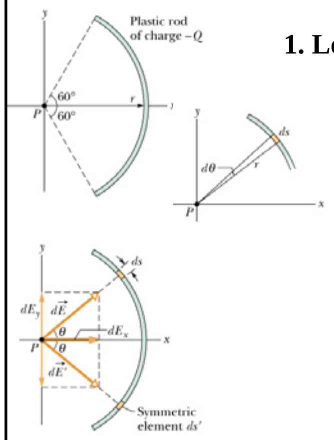
1) F and E by point charges:

When more than two point charges present, Coulomb's law for each pair of charges → superposition of all the vectors on the charge by all the others.

2) **Calculate E due to a Continuous Charge Distribution:**

treat charge elements dQ as point charges → Coulomb's law look for expression of dE → look for symmetry → summing via integration.

Example 3



$$E = k\frac{Q}{r^2}$$ **Q – source charge**

1. Look for *dE* generated by *dQ*

$$\because dQ = \lambda ds = \lambda r \cdot d\theta$$

$$\therefore dE = k\frac{dQ}{r^2} = k\lambda \cdot \frac{d\theta}{r}$$

2. Look for symmetry to simply calculation

$$dE_x = dE\cos\theta$$

$$= \frac{k\lambda}{r}\cos\theta \cdot d\theta$$

$$E = \int_{\theta_1}^{\theta_2} dE_x = \frac{k\lambda}{r}\int_{\theta_1}^{\theta_2}\cos\theta \cdot d\theta = \ldots$$

Application of Physical laws and concepts

1) F and E by point charges:

When more than two point charges present, Coulomb's law for each pair of charges → superposition of all the vectors on the charge by all the others.

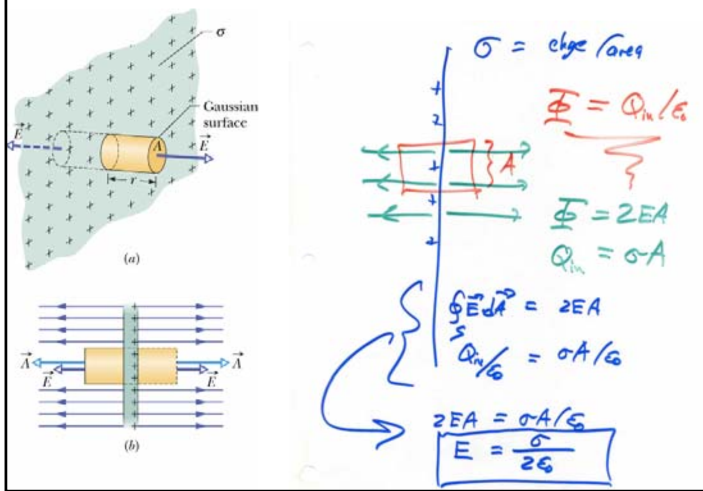
2) Calculate E due to a Continuous Charge Distribution:

treat charge elements dQ as point charges → Coulomb's law look for expression of dE → look for symmetry → summing via integration.

3) **Gauss' Law:** $\oint_s \vec{E} \cdot d\vec{A} = Q_{enc}/\varepsilon_0$ **to calculate E**

select Gaussian surface by symmetry to simplify calculation → Q_{enc} determine total flux → find out E

Infinite, Thin, Nonconductive Sheet of Charge



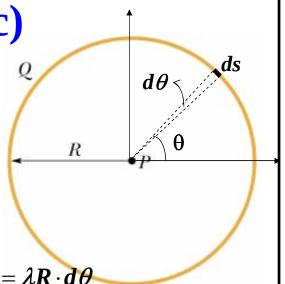
Application of Physical laws and concepts

1) F and E by point charges:
   When more than two point charges present, Coulomb's law for each pair of charges → superposition of all the vectors on the charge by all the others.
2) Calculate E due to a Continuous Charge Distribution:
   treat charge elements dQ as point charges → Coulomb's law
   look for expression of dE → look for symmetry → summing via integration.
3) Gauss' Law: $\oint \vec{E} \cdot d\vec{A} = Q_{enc} / \varepsilon_0$ to calculate E
   select Gaussian surface by symmetry to simplify calculation → Q_{enc} determine total flux → find out E
4) **unlike F and E, potential *V* due to many charges is just the number summary of potentials by each charges** $V = \sum_{i=1}^{n} V_i$

Example 5 – Q. 24-6 (c)

What is E and V at position the center?

Through symmetry, it is obvious that $E_p = 0$. But does it mean



$V_P = 0$? **Not necessary!**

$$V = \sum_{i=1}^{n} V_i, \quad dV = k\frac{dQ}{R}, \quad dQ = \lambda ds = \lambda R \cdot d\theta$$

$$V = \int_0^{2\pi} k\frac{\lambda R \cdot d\theta}{R} = k\lambda \int_0^{2\pi} d\theta = 2\pi k\lambda$$

since $\lambda = Q/2\pi R \;\Rightarrow\; V = 2\pi k \cdot Q/2\pi R = kQ/R$

or $V = \int_0^{2\pi} k\frac{dQ}{R} = \frac{k}{R}\int_0^{2\pi} dQ = \frac{kQ}{R} \neq 0$

Electrostatics

Exam On Wednesday: Chapters 21-25

- **Review lectures.**
- **Review quizzes.**
- **Review textbook homework problems.**