Homework #4  
5 problems  
Solutions will be posted September 18  

Exercises  

1. Calculate the electric field a distance \( h \) above the center of a disk with radius \( R \) and surface charge density \( \sigma \), using integration. Use polar coordinates, and start by finding \( dq \) and \( \vec{R} \).  

2. In class, we showed that the electric field a distance \( h \) above the center of a ring (radius \( R \), line charge density \( \lambda \)) is  
\[
\vec{E} = \frac{2\pi kh\lambda R}{(R^2 + h^2)^{3/2}} \hat{z}
\]  
(a) Approximate the electric field of the ring in the limit where \( h \gg R \); show that the field becomes the same as that of a point charge with the same charge as the ring.  
(b) Approximate the electric field when \( R \gg h \): as the ring gets larger, does the field increase, decrease, or stay constant?  

3. Write the integral for the electric field a distance \( h > s \) from the center of a cube of side \( s \). Include the proper limits, but don’t try to do the integral.  

4. The figure shows an oddly shaped pipe, which is 2 cm thick everywhere. Water flows in from the left, moving at 10 m/s. Twice as much water exits the pipe through the bottom branch as through the top branch.  
(a) Find the flux into the pipe (that is, from the left side).  
(b) Find the speed of the water as it exits both top and bottom branches.  
(c) Find the amount of water that flows through the pipe in a minute.  
Note: this subject will not appear on Exam 1.  

Problems  

5. Suppose the disk in problem 1 has a surface charge density \( \sigma(r) = \sigma_0 \left( \frac{r}{R} \right)^2 \): the charge is concentrated on the outer ring. Find the electric field at the same point.