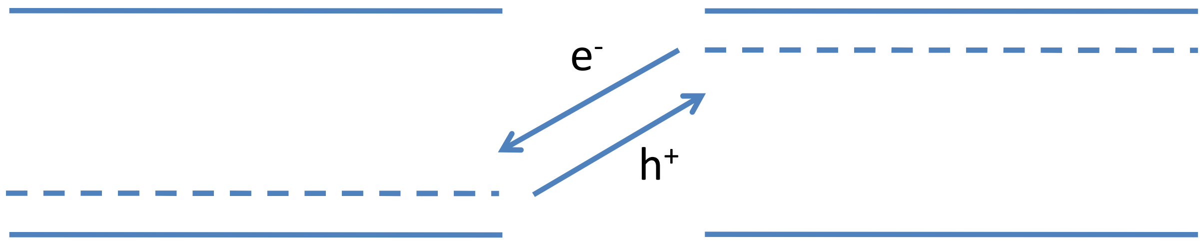
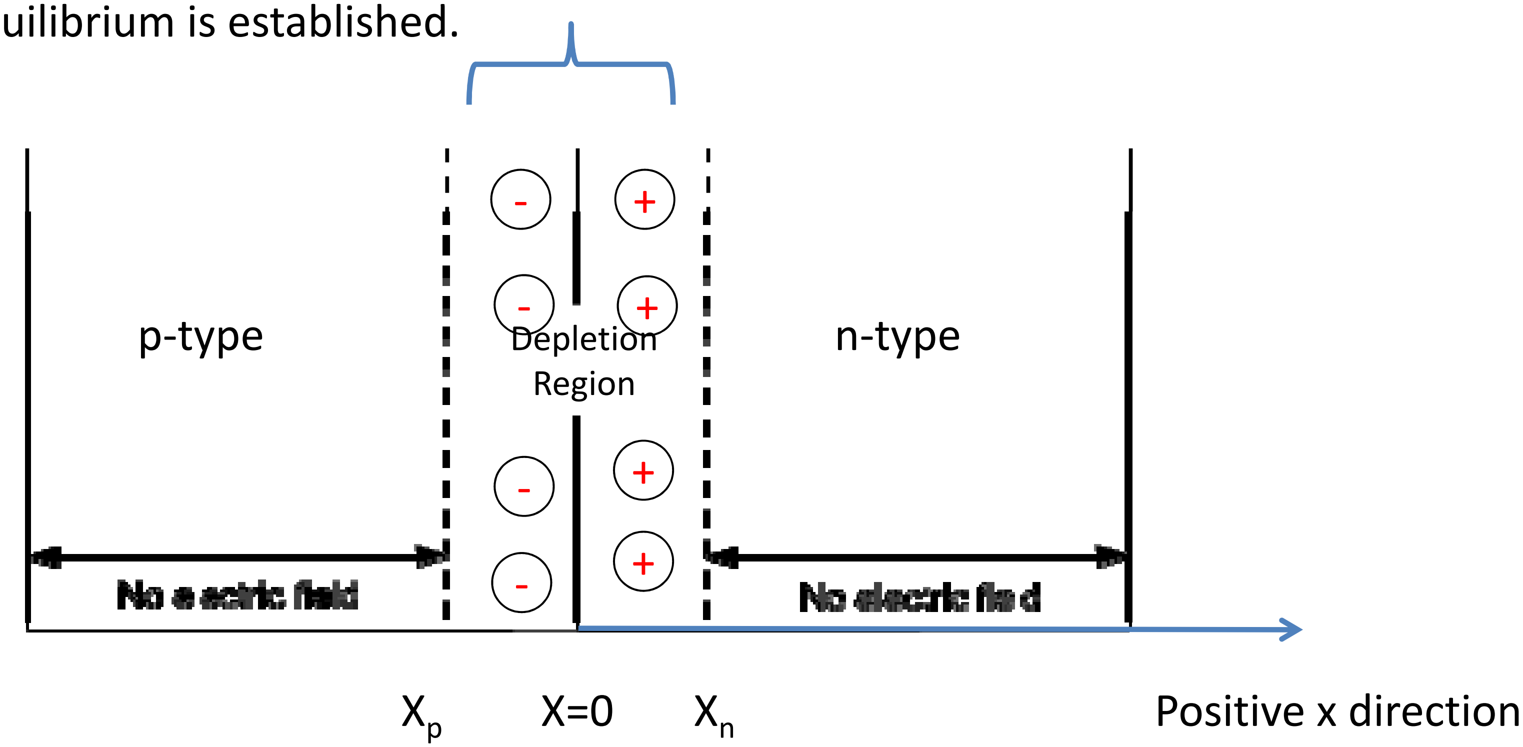
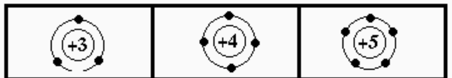
Before/during contact

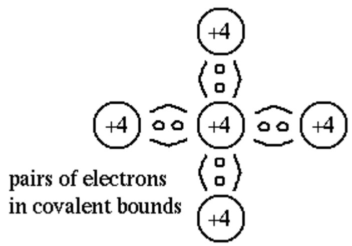


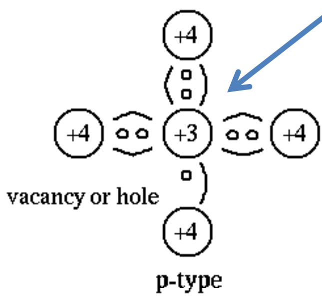
Depletion approximation: electric field is confined to space charge region, fixed charge associated with dopants is left “uncovered” after electronic equilibrium is established.



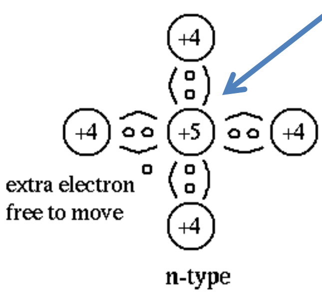


5 B BORON	6 C CARBON	7 N NITROGEN
13 Al ALUMINUM	14 Si SILICON	15 P PHOSPHORUS
31 Ga GALLIUM	32 Ge GERMANIUM	33 As ARSENIC
49 In INDIUM	50 Sn TIN	51 Sb ANTIMONY



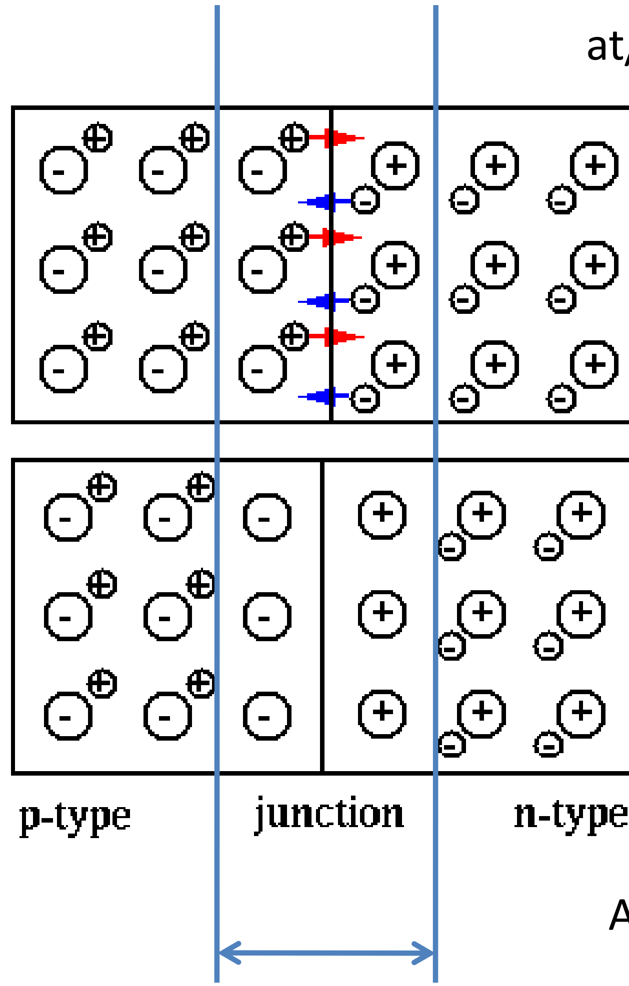


Charge of -1 relative to the other atoms (N_A^-)



Charge of +1 relative to the other atoms (N_D^+)

at/during contact

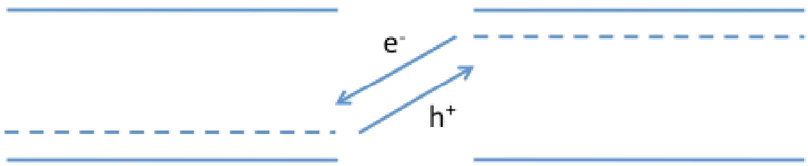


At equilibrium

Space charge region (SCR) or Depletion region

R. Wang: http://fourier.eng.hmc.edu/e84/lectures/ch4/node1.html

Before/during contact

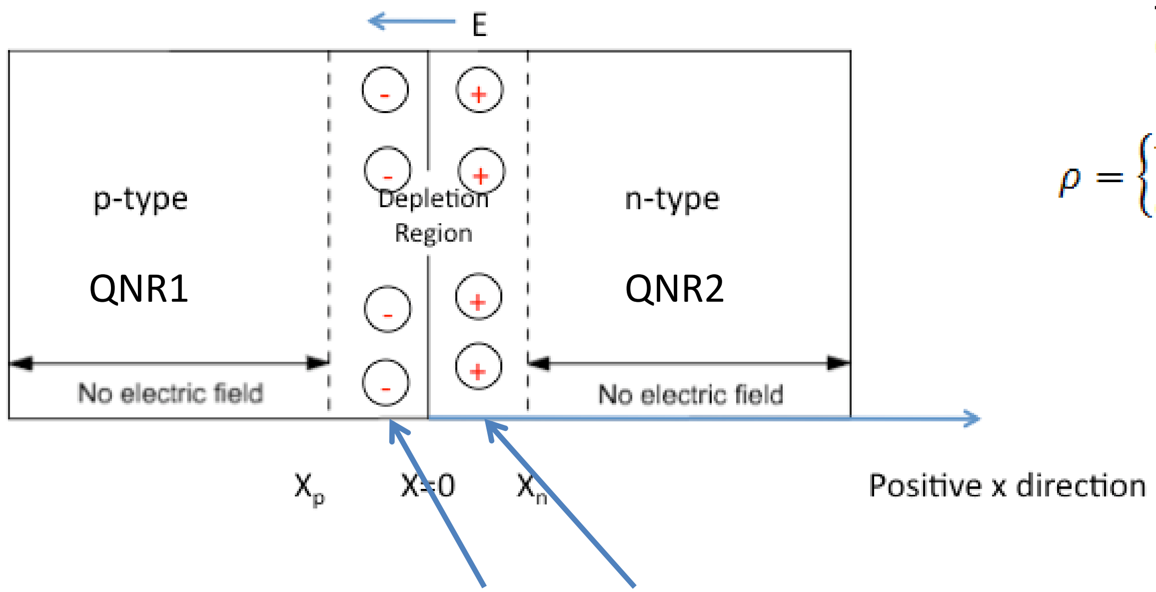


What is the electric field?

$$\frac{\partial \vec{E}}{\partial x} = \frac{\rho}{\varepsilon} = \frac{q}{\varepsilon}(-N_A + N_D)$$

$$\rho = \begin{cases} -qN_A, & when -x_p \le x \le 0 \\ qN_D, & when\ 0 \le x \le x_n \end{cases}$$

At equilibrium, Electric field develops to oppose further charge



- net charge density is zero outside SCR
- p(x) and n(x) = 0 within SCR
- all dopants are ionized, so $N_A^- = N_A$ and $N_D^+ = N_D$

Charge neutrality: $X_p N_A^- = X_n N_D^+$

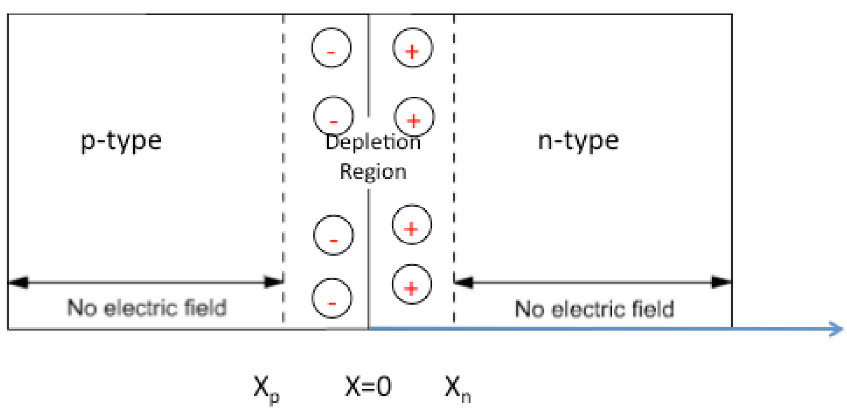
$$\frac{\partial \hat{E}}{\partial x} = \frac{\rho}{\varepsilon} = \frac{q}{\varepsilon}(-N_A + N_D) \qquad \rho = \begin{cases} -qN_A, & when -x_p \leq x \leq 0 \\ qN_D, & when\ 0 \leq x \leq x_n \end{cases}$$

$$E = \begin{cases} \int -\frac{qN_A}{\varepsilon}dx = -\frac{qN_A}{\varepsilon}x + C_1, & for -x_p \leq x < 0 \\ \int \frac{qN_D}{\varepsilon}dx = \frac{qN_D}{\varepsilon}x + C_2, & for\ 0 \leq x < x_n \end{cases}$$

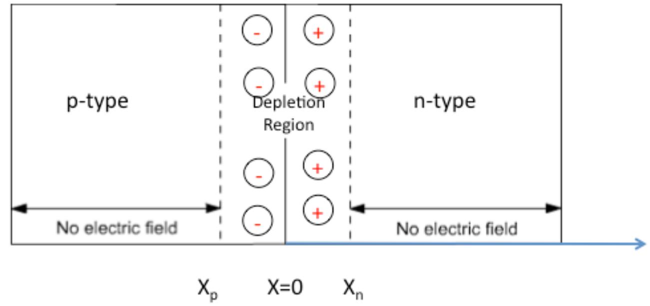
Bringing back the Boundary Conditions
(E goes to zero at boundaries of DR)

$$E(x = -x_p) = 0 \quad \Longrightarrow \quad C_1 = \frac{-qN_A}{\varepsilon}x_p$$

$$E(x = x_n) = 0 \Longrightarrow C_2 = -\frac{qN_D}{\varepsilon}x_n$$



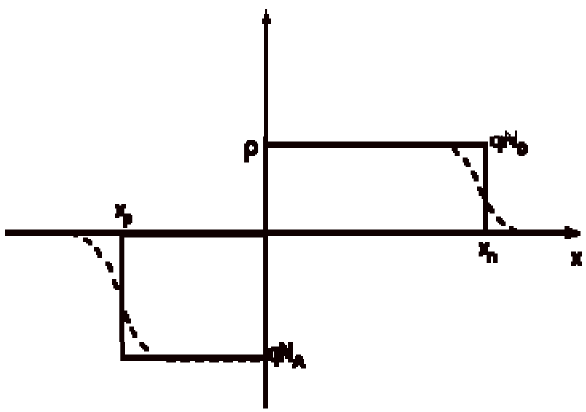
http://www.pveducation.org



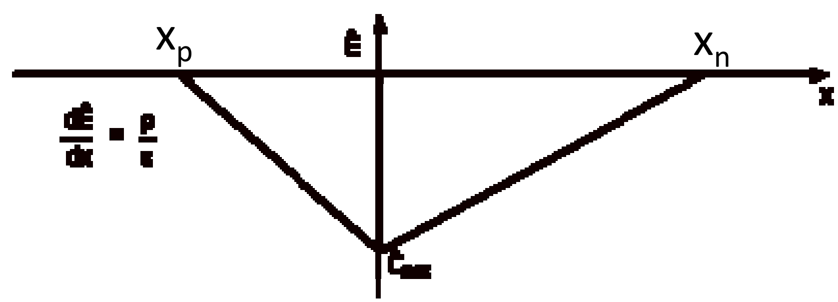
$$E = \begin{cases} \int -\frac{qN_A}{\varepsilon}dx = -\frac{qN_A}{\varepsilon}x + \frac{-qN_A}{\varepsilon}x_p & for -x_p \le x < 0 \\ \int \frac{qN_D}{\varepsilon}dx = \frac{qN_D}{\varepsilon}x - \frac{qN_D}{\varepsilon}x_n & for\ 0 \le x < x_n \end{cases}$$

The two parts of the solution must give the same value for E when x = 0

so $-N_A x_p = -N_D x_n$, which is the charge neutrality relation written earlier



Plots shown for $N_A \neq N_D$

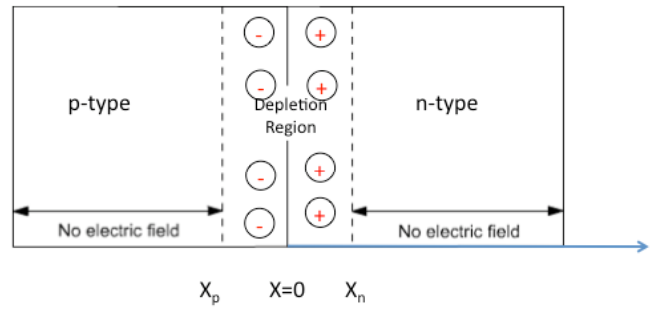


$|E|_{max}$ at x = 0

http://www.pveducation.org

What is the potential distribution?

- Integrate the electric field over distance.



$$E = \begin{cases} \int -\frac{qN_A}{\varepsilon}dx = -\frac{qN_A}{\varepsilon}x + \frac{-qN_A}{\varepsilon}x_p & for -x_p \le x < 0 \\ \int \frac{qN_D}{\varepsilon}dx = \frac{qN_D}{\varepsilon}x - \frac{qN_D}{\varepsilon}x_n & for\ 0 \le x < x_n \end{cases}$$

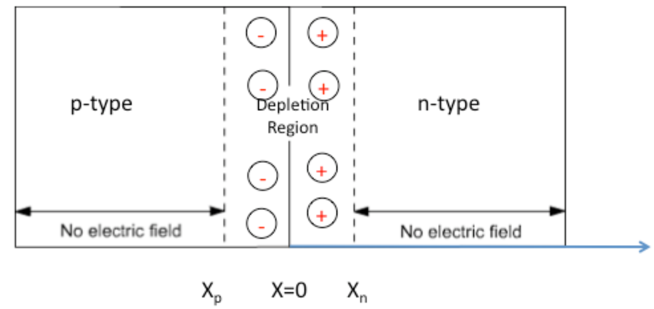
$$V(x) = \begin{cases} \int -E(x)dx = \int \frac{qN_A}{\varepsilon}(x + x_p)dx = \frac{qN_A}{\varepsilon}\left(\frac{x}{2} + x_p\right)x + C_3, & for -x_p \le x < 0 \\ \int -E(x)dx = \int \frac{qN_D}{\varepsilon}(x_n - x)dx = \frac{qN_D}{\varepsilon}\left(x_n - \frac{x}{2}\right)x + C_4, & for\ \ 0 \le x < x_n \end{cases}$$

Since we are interested in the potential difference, we can set the voltage on the p-type side to be zero, such that at $x = -x_p$, $V=0$.

Thus: $C_3 = \frac{qN_A}{2\varepsilon}{x_p}^2$ and $V(x) = \frac{qN_A}{2\varepsilon}(x + x_p)^2, \quad for\ -x_p \le x < 0$

http://www.pveducation.org

C_4 may be determined by using the fact that the potential on the n-type side and p-type side are identical at the interface.



$$V(x) = \begin{cases} = \dfrac{qN_A}{\varepsilon}\left(\dfrac{x}{2} + x_p\right)x + \dfrac{qN_A}{2\varepsilon}{x_p}^2 & for - x_p \le x < 0 \\ = \dfrac{qN_D}{\varepsilon}\left(x_n - \dfrac{x}{2}\right)x + C_4 & for \ \ 0 \le x < x_n \end{cases}$$

Thus, at x = 0:

$$V_p(x=0) = \frac{qN_A}{2\varepsilon}{x_p}^2 = V_n(x=0) = \frac{qN_D}{2\varepsilon}\left(x_n - \frac{x}{2}\right)x + C_4$$

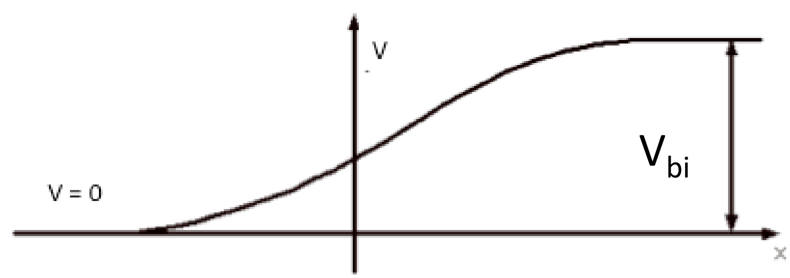
So: $C_4 = \dfrac{qN_A}{2\varepsilon}{x_p}^2$

Overall:

$$V(x) = \begin{cases} \dfrac{qN_A}{2\varepsilon}(x + x_p)^2, & for - x_p \le x < 0 \\ \dfrac{qN_D}{\varepsilon}\left(x_n - \dfrac{x}{2}\right)x + \dfrac{qN_A}{2\varepsilon}{x_p}^2, & for \ \ 0 \le x < x_n \end{cases}$$

Voltage is a max at $x = x_n$:

$$V(x - x_n) - \frac{q}{2\varepsilon}\left(N_D{x_n}^2 + N_A{x_p}^2\right) = V_{bi} \text{ (aka } V_o\text{)}$$



V_{BI} is the difference between the Fermi levels!

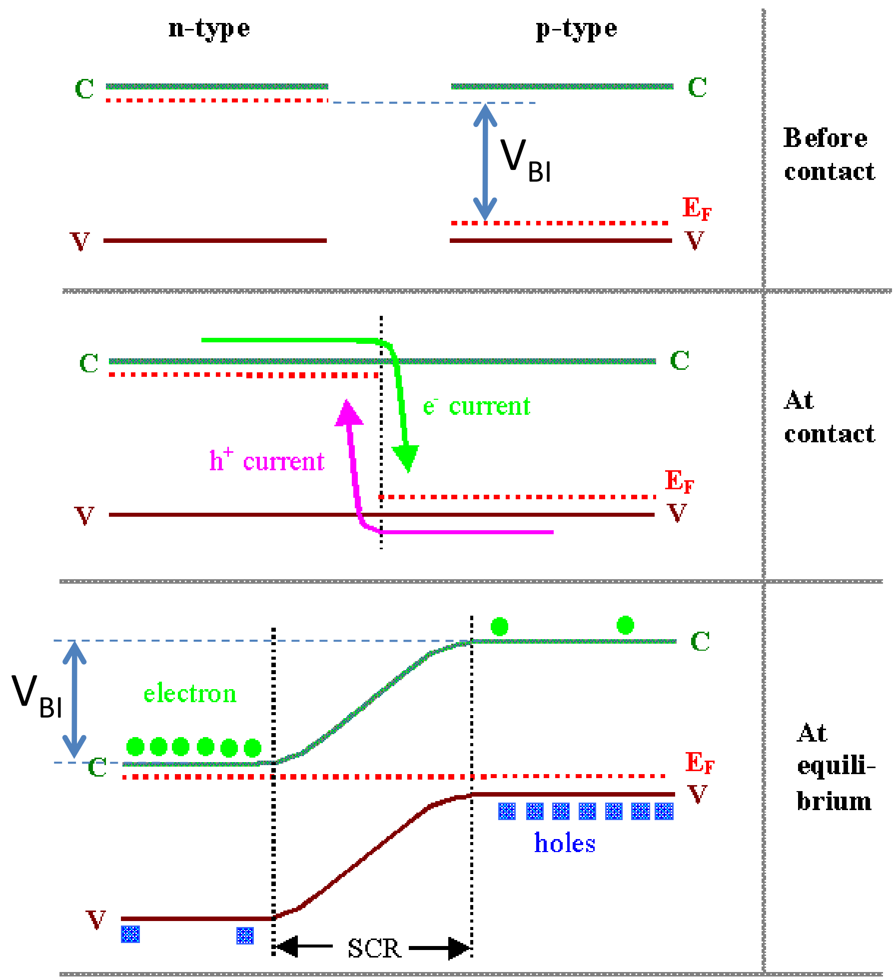
http://www.pveducation.org

V_o (aka V_{BI}) is the difference between the Fermi Levels in the two contacting "bits"

Electrons in the two bits have different electrochemical potentials (i.e. different E_f's)

Charge transfer occurs at contact (electron go down from the vacuum level, holes go "up")

At equilibrium, there is no net transport (E_f is constant throughout the device)

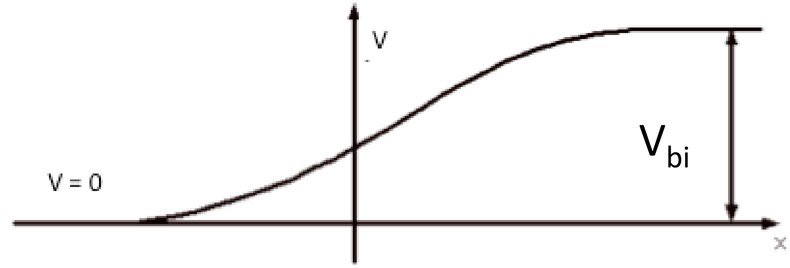


H. Föll: http://www.tf.uni-kiel.de/matwis/amat/semi_en/kap_2/backbone/r2_2_4.html

$$V_{bi} = \frac{q}{2\varepsilon}\left(N_D x_n{}^2 + N_A x_p{}^2\right)$$

Using the condition for charge neutrality:

$$N_A x_p = N_D x_n$$



We can determine the edges of the DR in terms of V_o:

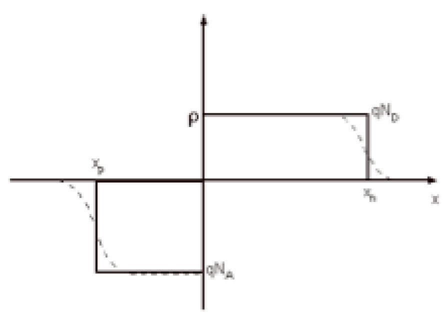
$$x_n = \left[\frac{2\varepsilon V_0}{q}\frac{N_A}{N_D(N_A+N_D)}\right]^{\frac{1}{2}} \quad \text{and} \quad x_p = \left[\frac{2\varepsilon V_0}{q}\frac{N_D}{N_A(N_A+N_D)}\right]^{\frac{1}{2}}$$

Which gives the Depletion Width, W:

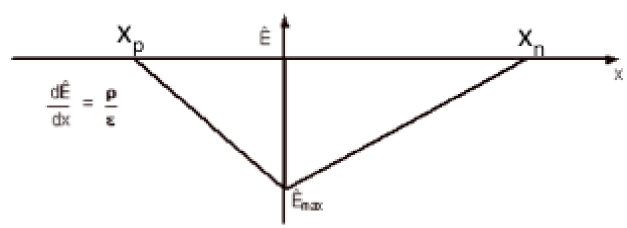
$$W = x_n + x_p = \sqrt{\frac{2\varepsilon}{q}\frac{V_0}{\left(\frac{1}{N_A}+\frac{1}{N_D}\right)}}$$

Or, x_p and x_n in terms of W:

$$x_p = W\frac{N_D}{N_A+N_D} \qquad x_n = W\frac{N_A}{N_A+N_D}$$



Plots shown for $N_A \neq N_D$



$|E|_{max}$ at x = 0

$$E_{max} = \frac{-qN_a x_p}{\varepsilon}$$

$$= \frac{-qN_D x_n}{\varepsilon}$$

http://www.pveducation.org

General Procedure using the depletion approximation:

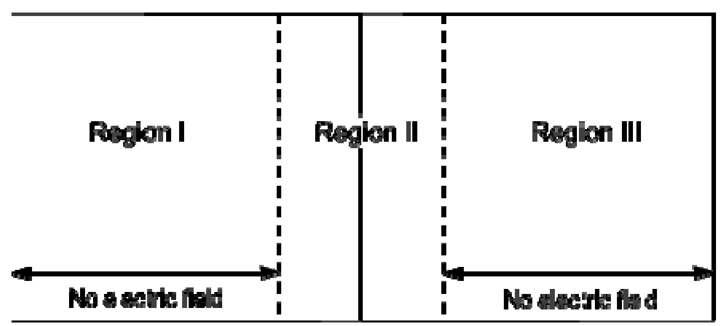
1. Divide the device into regions with an electric field and without an electric field.

2. Solve for electrostatic properties in the depletion region (Region II on the diagram). This solution depends on the doping profile assumed. Here we will restrict the calculations to constant doping profiles.

3. Solve for the carrier concentration and current in the quasi-neutral regions (Regions I and III on the diagram) under steady-state conditions.

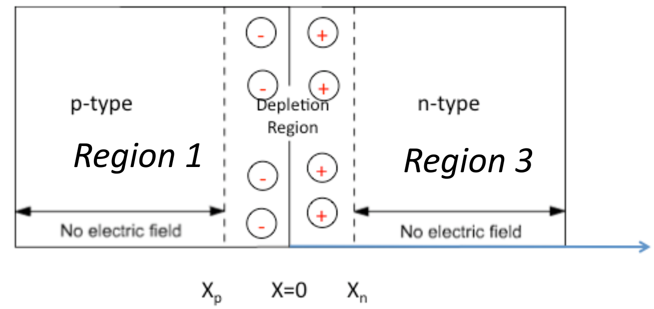
   The steps in this are:
   (a) Determine the general solution for the particular device. The general solution will depend only on the types of recombination and generation in the device.
   (b) Find the particular solution, which depends on the surfaces and the conditions at the edges of the depletion region.

4. Find the relationship between the currents on one side of the depletion region and the currents on the other side. This depends on the recombination/generation mechanisms in the depletion region.



http://www.pveducation.org/pvcdrom/pn-junction/basic-equations

What's happening in the QNRs (regions 1 and 3) …



According to the Depletion Approximation:

-no E Field $\frac{\partial \hat{E}}{\partial x} = 0$

-Transport equations also simplify

$$J_p = q\mu_p p(x)\bar{E} - qD_p \frac{dp(x)}{dx}$$

$$J_n = q\mu_n n(x)\bar{E} + qD_n \frac{dn(x)}{dx}$$

0

$$J_n = qD_n \frac{dn(x)}{dx} \qquad J_p = -qD_p \frac{dp(x)}{dx}$$

http://www.pveducation.org

A bit about the continuity equations

one for electrons, one for holes….

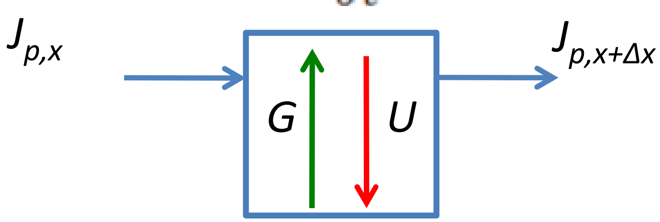
Result of "mass balance"

$$\frac{\partial n}{\partial t} = \frac{1}{q}\frac{\partial J_n}{\partial x} + G_n - U_n$$

$$\frac{\partial p}{\partial t} = -\frac{1}{q}\frac{\partial J_p}{\partial x} + G_p - U_p$$

Consider a box, and the rate of change of particles (electrons or holes in that box)

$\frac{\partial n}{\partial t}$ *Units of #/cc-s*
i.e. change in concentration over time



Under thermal equilibrium and *steady* state $\frac{\partial n}{\partial t}$ *and* $\frac{\partial p}{\partial t}$ *= 0*

$$\frac{1}{q}\frac{dJ_p}{dx} = -(U - G) \qquad \frac{1}{q}\frac{dJ_n}{dx} = U - G$$

Back to the Solution for Carrier Concentration and Currents in QNRs….

transport $J_n = qD_n \frac{dn(x)}{dx}$ $J_p = -qD_p \frac{dp(x)}{dx}$

continuity $\frac{1}{q}\frac{dJ_n}{dx} = U - G$ $\frac{1}{q}\frac{dJ_p}{dx} = -(U - G)$

Differentiating the transport Eqns. and substituting into the continuity Eqns. Gives:

$$\frac{dJ_n}{dx} = qD_n \frac{d^2n(x)}{dx^2} = q[U(x) - G(x)] \Rightarrow \frac{d^2n(x)}{dx^2} = \frac{U(x) - G(x)}{D_n}$$

and

$$\frac{dJ_p}{dx} = -qD_p \frac{d^2p(x)}{dx^2} = -q[U(x) - G(x)] \Rightarrow \frac{d^2p(x)}{dx^2} = \frac{U(x) - G(x)}{D_p}$$

$$\frac{d^2n(x)}{dx^2} = \frac{U(x) - G(x)}{D_n} \qquad \frac{d^2p(x)}{dx^2} = \frac{U(x) - G(x)}{D_p}$$

•Two second-order differential equations which are general valid under steady state conditions as long as the depletion approximation holds and as long as drift and diffusion are the only transport mechanisms.

•Also requires “low level injection”, i.e. photogenerated carrier density is small compared the background doping density

•For steady state and low injection, these are the equations that must be solved to determine the electrical characteristics, of the IV equation, of a device.

Importance of the minority carrier currents:
In Regions I and III, the total current consists of two component;, J_n *(the current composed of electrons) and* J_p *(the current composed of holes).*

However, we need only to consider the minority carrier currents since the recombination of minority carriers control the current flow. Later, we will also calculate the majority carrier currents based on the fact that the total current must be constant.

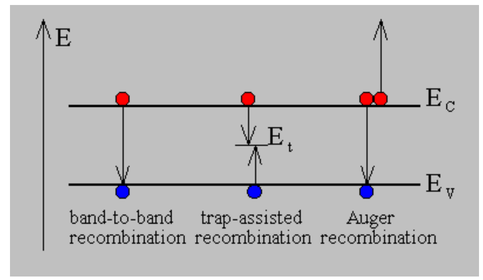
http://www.pveducation.org

The solution to the equations involves first finding relations for U and G to determine the general solution to the differential equation, and secondly determining boundary conditions to find the particular solution.

Finding The Recombination Rate:

Assumption of Low Injection, the general form of the recombination rate for SRH recombination is:

$$U = \frac{(np - n_i^2)}{\tau_{p0}(n + n_1) + \tau_{n0}(p + p_1)}$$



http://ecee.colorado.edu/~bart/book/recomb.htm

where n_1, p_1 *are the numbers of carriers in recombination sites,* n_0, p_0 *are the electron and hole concentrations in equilibrium and* τ_{p0} *and* τ_{n0} *are the minority carrier lifetimes for holes and electrons.*

In p-type material and under low injection (low bias) conditions, *p >> n also* $p \approx p_0$. *Further, we will assume that the number of carriers is low, i.e.* $n >> n_1$ *and* $p >> p_1$, *and that the lifetimes do not vary dramatically. The above equation then reduces to the recombination rate for electrons in p-type material as:*

And for holes in n-type material we have

$$U = \frac{(np - n_i^2)}{\tau_{n0}p} = \frac{(np - n_0 p_0)}{\tau_{n0}p} = \frac{(np_0 - n_0 p_0)}{\tau_{n0}p} = \frac{(n - n_0)}{\tau_{n0}} = \frac{\Delta n}{\tau_n}$$

$$U = \frac{\Delta p}{\tau_p}$$

http://www.pveducation.org

Finding the Generation Rate:

In general, *G is given by:*

$$G = \alpha N_S e^{-\alpha x}$$

- N_S *is the photon flux at the surface and may vary with time and wavelength;* $N_S(\lambda,t)$
- *Thus, G is also a function of wavelength and time.*
- *However, the time dependence can be ignored since we are looking for steady state solutions.*
- *Wavelength dependence is often included by summing the result obtained at one wavelength over all wavelengths of interest.*
- *The exponential form of the generation rate generally makes the differential equation a nonhomogeneous second order differential equation.*
- *Therefore, approximation to the generation rate is often needed*
- *A common approximation is that the generation is constant*

  Which is valid when the dimensions of interest are small compared to a^{-1}*, when there is an impulse generation at the surface, or when the generation is zero.*

http://www.pveducation.org

$$\frac{d^2n(x)}{dx^2} = \frac{U(x) - G(x)}{D_n} \qquad \frac{d^2p(x)}{dx^2} = \frac{U(x) - G(x)}{D_p}$$

Finding the general solution

The general solution to the differential equation in (3a) and (3b) will depend on the equation for *U* and *G*. There are several common general solutions. These are shown below, where is any function dependent only on *x*, and for the pn junction equations $\zeta(x)$ will usually correspond to one of *n(x), p(x), Dn(x)* or*Dp(x)*. *A* and *B* are constants that need to be determined by the boundary conditions. *C* and *K* are semiconductor or device constants. *C* for pn junction equations is usually L_n or L_p (the minority carrier diffusions length) and *K* is often a constant generation term, *G*.

Differential equation of the form	General solution	When used
$\frac{d^2\zeta(x)}{dx^2} = \frac{\zeta(x)}{C^2}$	$\zeta(x) = Ae^{-\frac{x}{C}} + Ae^{\frac{x}{C}}$	Bulk recombination, no generation
$\frac{d^2\zeta(x)}{dx^2} = \frac{\zeta(x)}{C^2} + \frac{K}{C^2}$	$\zeta(x) = Ae^{-\frac{x}{C}} + Ae^{\frac{x}{C}} + K$	Bulk recombination, constant geneneration
$\frac{d^2\zeta(x)}{dx^2} = 0$	$\zeta(x) = Ax + B$	Zero recombination and generation
$\frac{d^2\zeta(x)}{dx^2} = K$	$\zeta(x) = \frac{K}{2}x^2 + Ax + B$	Zero recombination, constant generation

http://www.pveducation.org

Finding diffusion currents in Regions I and III

Working towards finding expressions for *n(x) and p(x) so that we can find the minority carrier currents by using the general equations:*

$$J_n = qD_n \frac{dn(x)}{dx} \qquad\qquad J_p = -qD_p \frac{dp(x)}{dx}$$

Finding Total Current

To find the total current, we note that the TOTAL current in the device must be constant, independent of distance as long as there is not a contact that can extract or inject carriers and as long as the device is under steady state conditions. This can be shown by:

$$\frac{dJ_T}{dx} = \frac{d(J_p + J_n)}{dx} = \frac{dJ_p}{dx} + \frac{dJ_n}{dx} = -q\left(U_p + G_p\right) + q\left(U_n + G_n\right) = q\left(G_n - G_p\right) - q\left(U_n - U_p\right)$$