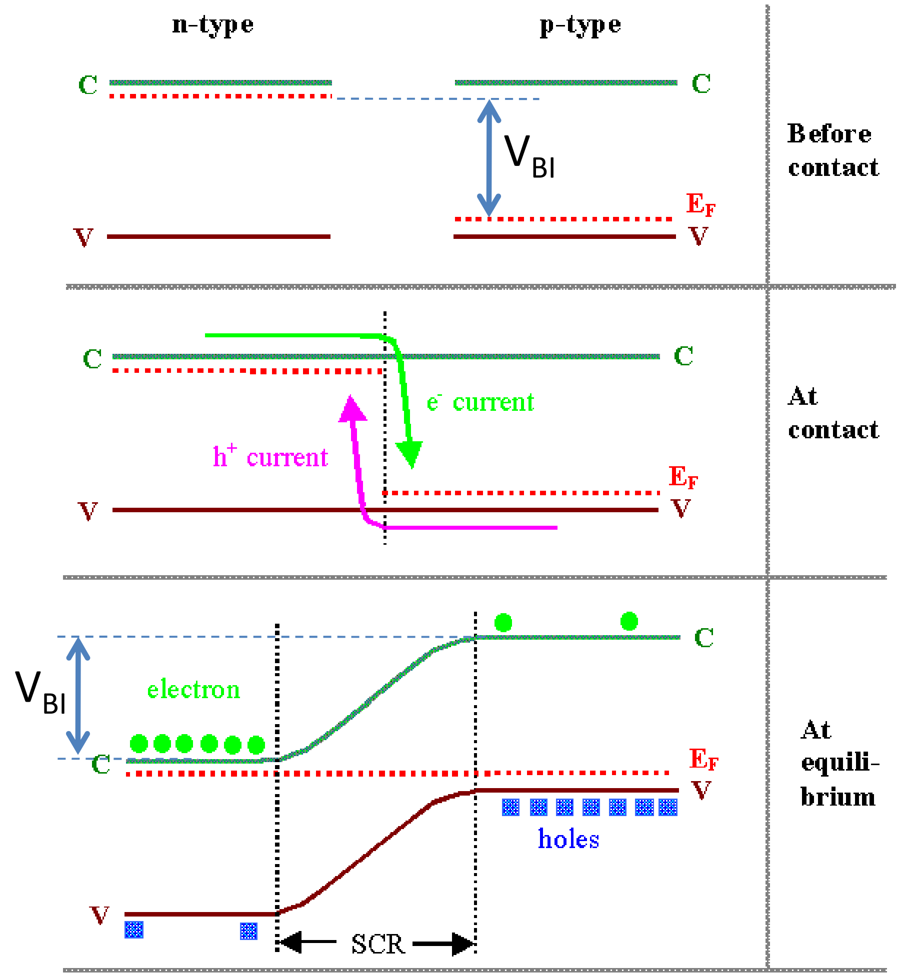
Consider the the band diagram for a *homojunction*, formed when two bits of the same type of semicondutor (e.g. Si) are doped p and n type and then brought into contact.



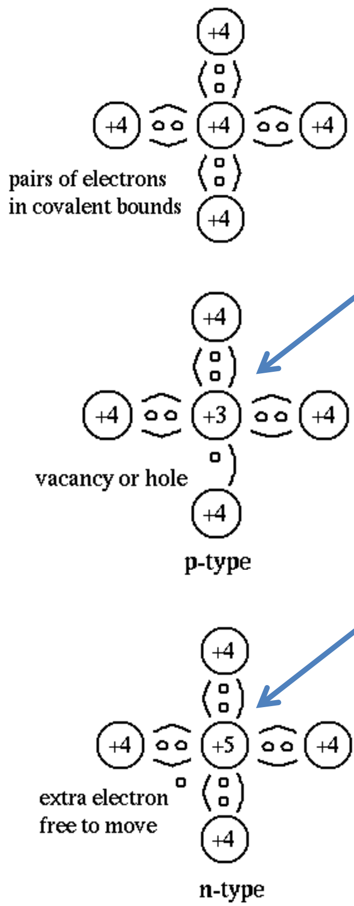
Electrons in the two bits have different electrochemical potentials (i.e. different E_f's)

Charge transfer occurs at contact (electron go down from the vacuum level, holes go "up")

At equilibrium, there is no net transport (E_f is constant throughout the device)

H. Föll: http://www.tf.uni-kiel.de/matwis/amat/semi_en/kap_2/backbone/r2_2_4.html

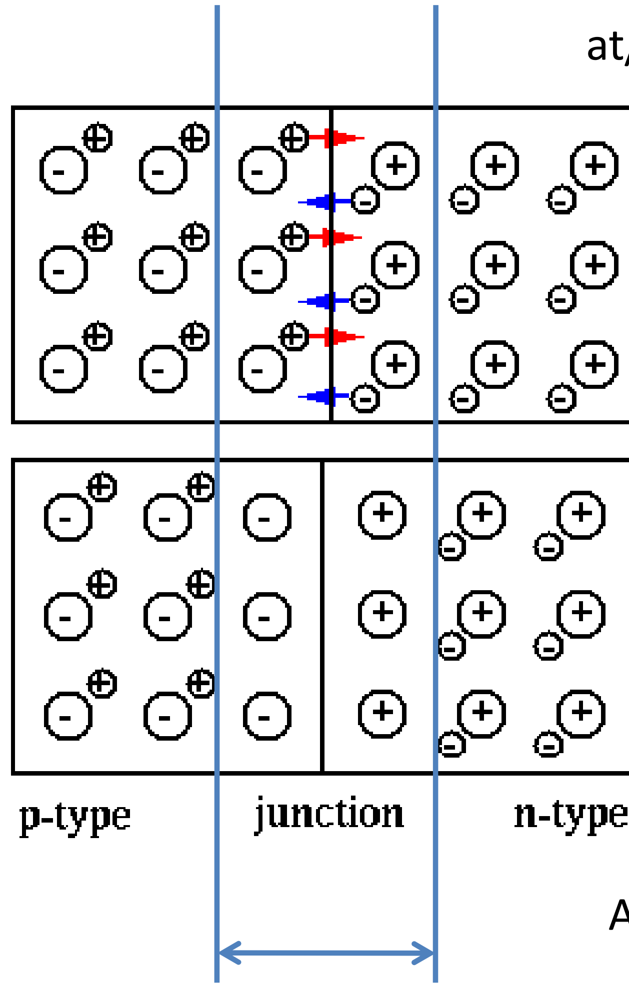
+3	+4	+5
5 B BORON	6 C CARBON	7 N NITROGEN
13 Al ALUMINUM	14 Si SILICON	15 P PHOSPHORUS
31 Ga GALLIUM	32 Ge GERMANIUM	33 As ARSENIC
49 In INDIUM	50 Sn TIN	51 Sb ANTIMONY



Charge of -1 relative to the other atoms (N_A^-)

Charge of +1 relative to the other atoms (N_D^+)

at/during contact

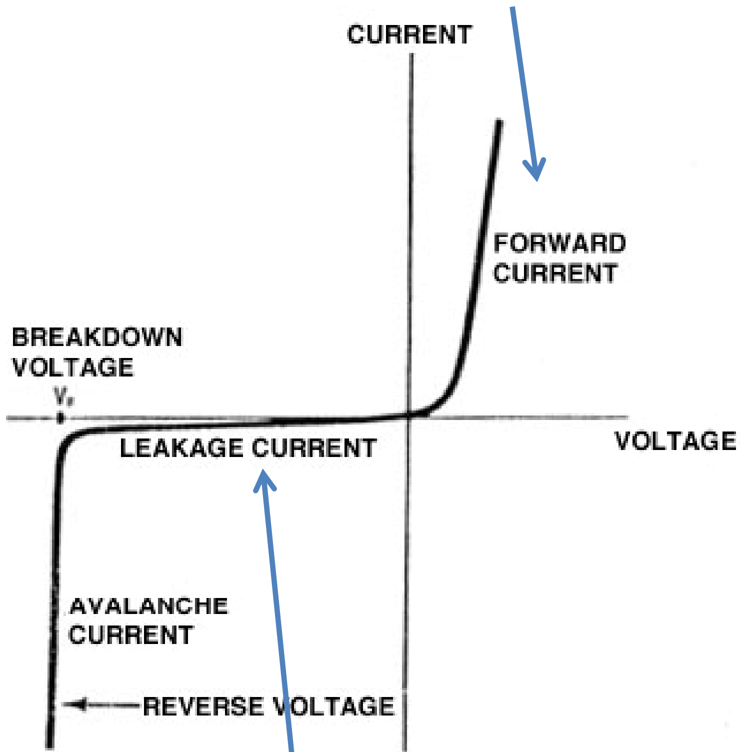


At equilibrium

Space charge region (SCR) or Depletion region

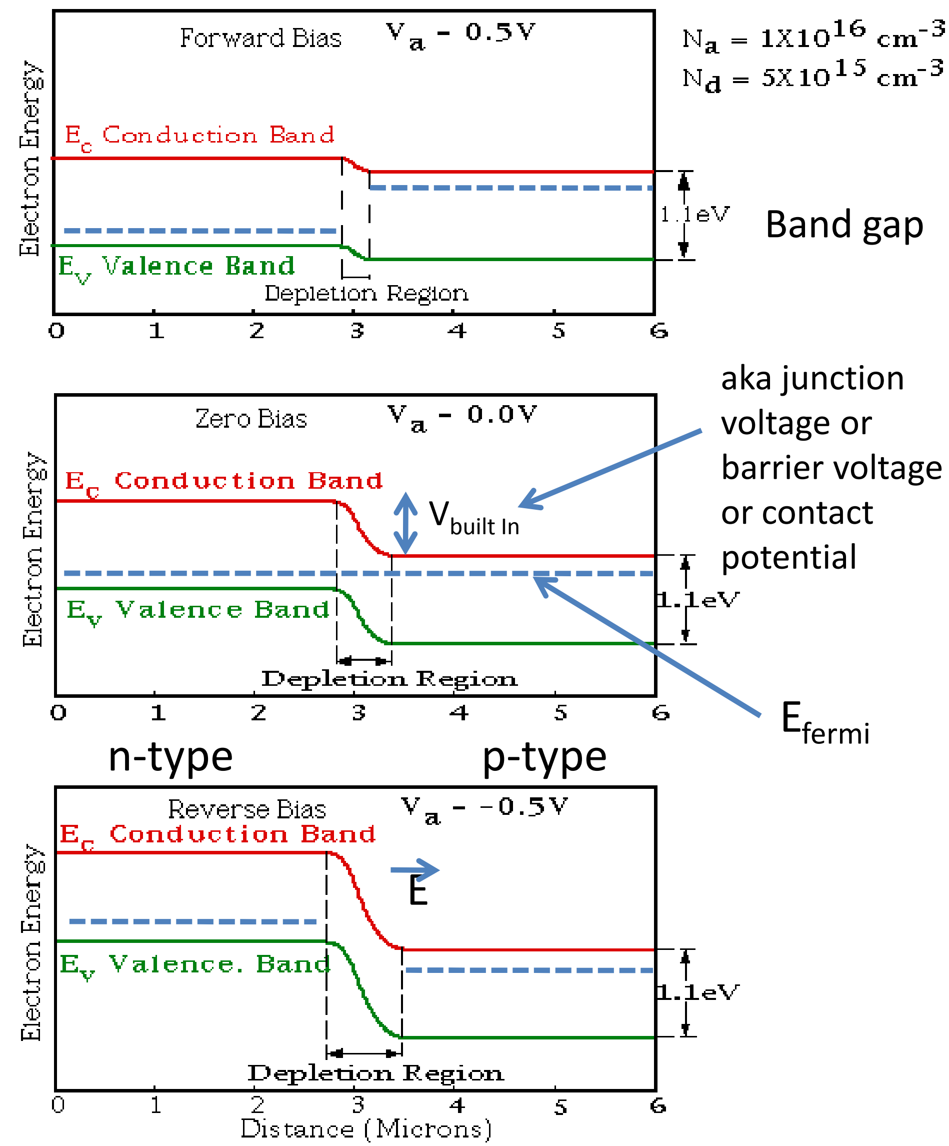
R. Wang: http://fourier.eng.hmc.edu/e84/lectures/ch4/node1.html

Majority carriers diffuse across the Depletion Region (because the electric field is reduced), where they become minority carriers and recombine (aka minority carrier injection)



Aka Reverse Saturation Current- Minority carriers move across the Depletion Region with the assistance of the larger electric field, where they become minority carriers and recombine

<u>Energy Band Diagram for p-n diode</u>



http://www-tcad.stanford.edu/tcad/education/anim.html

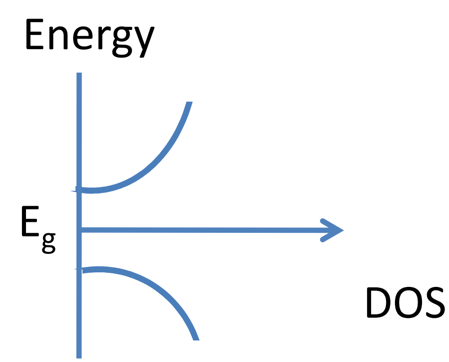
A bit about Density of States….

The DOS in a semiconductor is obtained by solving the Schrödinger equation for the particles in the semiconductor. Rather than solving for the complex potential in the semiconductor, one can use the particle-in-a box model, assuming that the particle is free to move within the material. The boundary conditions which express the fact that the particles can not leave the material, force the density of states in k-space to be constant. The DOS are approximately parabolic in energy near the conduction and valence band edges (i.e. the DOS goes with the square root of the energy distance above (below) the conduction (valence) band minimum (maximum)

Density of states as f(E):

$$g_c(E) = \frac{8\pi\sqrt{2}}{h^3} m_e^{*3/2} \sqrt{E - E_C}, \quad \text{for } E \geq E_C$$

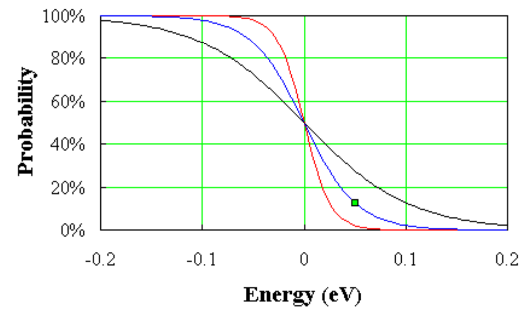
$$g_v(E) = \frac{8\pi\sqrt{2}}{h^3} m_h^{*3/2} \sqrt{E_V - E}, \quad \text{for } E \leq E_V$$



http://ecee.colorado.edu/~bart/book/carriers.htm

….The Fermi function tells us about the occupation of these states

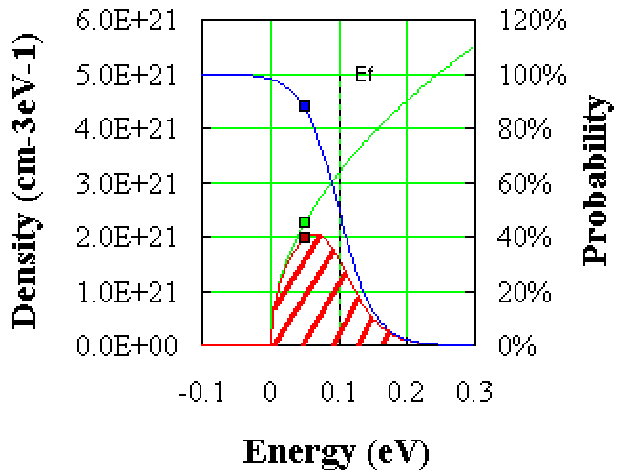
$$f(E) = \frac{1}{1 + e^{(E - E_F)/kT}}$$



Density of carriers in the band can be obtained (for, e.g., electron in the conduction band):

$$n = \int_{E_C}^{\text{top of the conduction band}} n(E)\,dE = \int_{E_C}^{\text{top of the conduction band}} g_c(E) f(E)\,dE$$

$$n = \int_{E_C}^{\infty} \frac{8\pi\sqrt{2}}{h^3} m_e^{*3/2} \sqrt{E - E_C} \frac{1}{1 + e^{\frac{E - E_F}{kT}}} dE$$



(with units of #/cm^{-3})

http://ecee.colorado.edu/~bart/book/carriers.htm

For non-degenerate semiconductors, we require that the Fermi level be at least 3kT (~ 75 meV at room temperature) away from the band edge….

Then, the Fermi function can be replaced by a simple exponential term….

$$n \cong \int_{E_C}^{\infty} \frac{8\pi\sqrt{2}}{h^3} m_e^{*3/2} \sqrt{E - E_C}\; e^{\frac{E_F - E}{kT}} dE = N_C\, e^{\frac{E_F - E_C}{kT}}$$

$$p \cong \int_{-\infty}^{E_V} \frac{8\pi\sqrt{2}}{h^3} m_h^{*3/2} \sqrt{E_V - E}\; e^{\frac{E - E_F}{kT}} dE = N_V\, e^{\frac{E_V - E_F}{kT}}$$

Where the *effective* Density of States in the Conduction and Valence bands are:

$N_C = 2[\frac{2\pi m_e^* kT}{h^2}]^{3/2}$ and $N_V = 2[\frac{2\pi m_h^* kT}{h^2}]^{3/2}$ (with units of cm^{-3})

Interestingly,

$$np = N_V N_C e^{\frac{E_V - E_F}{kT}} e^{\frac{E_F - E_C}{kT}} = N_V N_C e^{\frac{-E_G}{kT}} = n_i^2$$

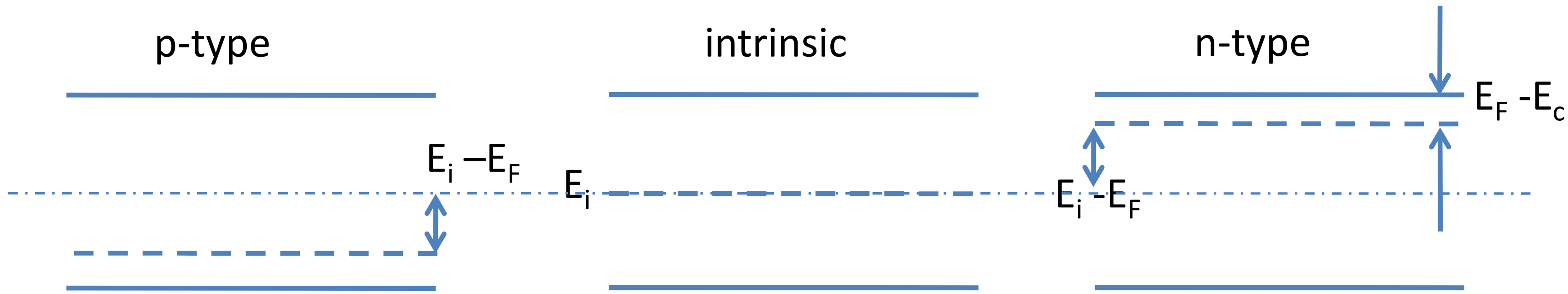
Which gives the Law of Mass action:

$$np = n_i^2$$

http://ecee.colorado.edu/~bart/book/carriers.htm

$$np = N_V N_C e^{\frac{E_V - E_F}{kT}} e^{\frac{E_F - E_C}{kT}} = N_V N_C e^{\frac{-E_G}{kT}} = n_i^2$$

$$n_i = \sqrt{N_V N_C e^{\frac{-E_G}{2kT}}}$$



A negative number   A positive number

$$\boxed{p = N_V \exp\left(\frac{E_V - E_F}{kT}\right) = n_i \exp\left(\frac{E_i - E_F}{kT}\right)\ cm^{-3}}$$

A negative number   A positive number

$$\boxed{n = N_C \exp\left(\frac{E_F - E_C}{kT}\right) = n_i \exp\left(\frac{E_F - E_i}{kT}\right)\ cm^{-3}}$$

$n_{i\ \text{ranges}}$ from 10^{13} #/cc for Ge (E_g = 0.66eV)
to 10^{6} #/cc for GaAs (E_g = 1.42 eV)

for Si: $n_i \sim 1 \times 10^{10}$ cm^{-3}
$N_C = 3.2 \times 10^{19}$ cm^{-3}
$N_V = 1.8 \times 10^{19}$ cm^{-3}

Basic Equations for Solving for the Electric Field, Transport, and Carrier Concentrations

1. **Poisson's equation:**

$$\frac{\partial \vec{E}}{dx} = \frac{\rho}{\varepsilon} = \frac{q}{\varepsilon}\left(p(x) - n(x) - N_A^- + N_D^+\right)$$

$$\nabla^2 \phi(x, y, z) = -\frac{\rho(x, y, z)}{\varepsilon}$$

$$\frac{d\phi(x)}{dx} = -\mathcal{E}(x)$$

2. **Transport equations:**

$$J_n = q\mu_n n(x)\vec{E} + qD_n \frac{dn(x)}{dx}$$

(first term is drift, second is diffusion)

$$J_p = q\mu_p p(x)\vec{E} - qD_p \frac{dp(x)}{dx}$$

3. **Continuity equations:**

General conditions

$$\frac{dn}{dt} = \frac{1}{q}\frac{\partial J_n}{\partial x} - (U - G)$$

$$\frac{dp}{dt} = \frac{1}{q}\frac{\partial J_p}{\partial x} + (U - G)$$

Under thermal equilibrium and steady state conditions

$$\frac{1}{q}\frac{\partial J_n}{\partial x} = (U - G)$$

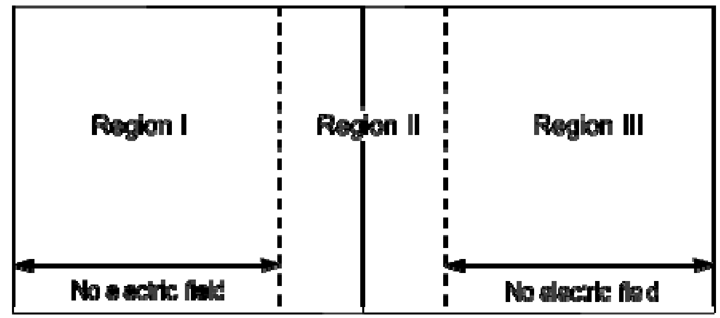
$$\frac{1}{q}\frac{\partial J_p}{\partial x} = -(U - G)$$

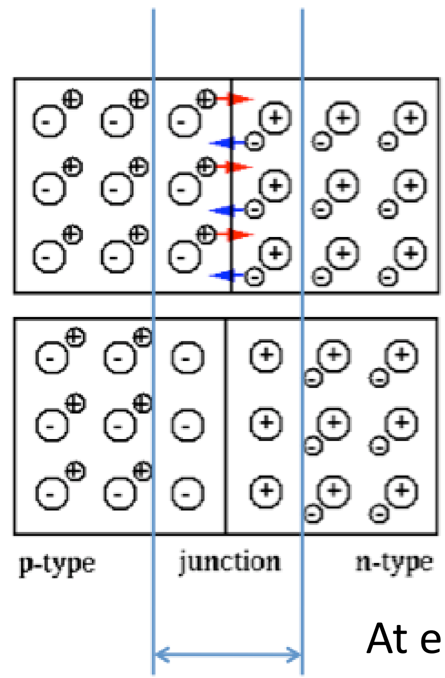
where U and G are the recombination and generation rates in the particular material and depend on the details of the device and may also depend on distance.

http://www.pveducation.org/pvcdrom/pn-junction/basic-equations

General Approach to Solving for the Electric Field, Transport, and Carrier Concentrations

For arbitrary charge distributions, band diagrams, junction types, the equations *may* be solved using numerical approaches, and many device simulators are available. In addition to the assumption of a one-dimensional device, the most valuable simplifying assumption in determining a closed form solution to the above equations is the depletion approximation. The depletion approximation assumes that the electric field in the device is confined to some region of the device. The device can then be broken up into regions that have an electric field and those that do not. This is shown below for a pn junction, where Regions I and III do not have an electric field (called quasi-neutral regions or QNR) and Region II has an electric field (which is called space-charge or depletion region).





SCR

At equilibrium

http://ecee.colorado.edu/~bart/book/carriers.htm

General Procedure using the depletion approximation:

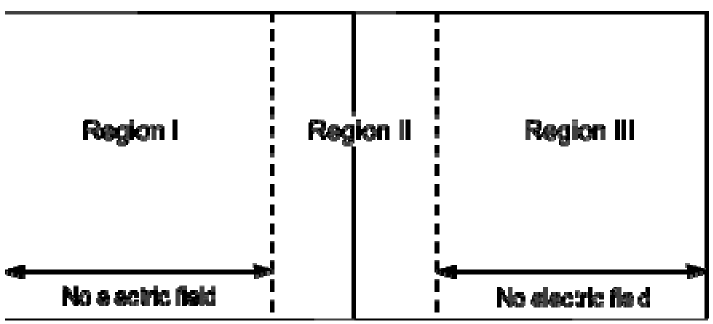
1. Divide the device into regions with an electric field and without an electric field.

2. Solve for electrostatic properties in the depletion region (Region II on the diagram). This solution depends on the doping profile assumed. Here we will restrict the calculations to constant doping profiles.

3. Solve for the carrier concentration and current in the quasi-neutral regions (Regions I and III on the diagram) under steady-state conditions.

   The steps in this are:
   (a) Determine the general solution for the particular device. The general solution will depend only on the types of recombination and generation in the device.
   (b) Find the particular solution, which depends on the surfaces and the conditions at the edges of the depletion region.

4. Find the relationship between the currents on one side of the depletion region and the currents on the other side. This depends on the recombination/generation mechanisms in the depletion region.



http://www.pveducation.org/pvcdrom/pn-junction/basic-equations

Basic Equations for Solving for the Electric Field, Transport, and Carrier Concentrations

1. **Poisson's equation:**

$$\frac{\partial \vec{E}}{dx} = \frac{\rho}{\varepsilon} = \frac{q}{\varepsilon}\left(p(x) - n(x) - N_A^- + N_D^+\right)$$

$$\boxed{\begin{gathered}\nabla^2 \phi(x,y,z) = -\frac{\rho(x,y,z)}{\varepsilon} \\ \frac{d\phi(x)}{dx} = -\mathcal{E}(x)\end{gathered}}$$

2. **Transport equations:**

$$J_n = q\mu_n n(x)\vec{E} + qD_n \frac{dn(x)}{dx}$$

(first term is drift, second is diffusion)

$$J_p = q\mu_p p(x)\vec{E} - qD_p \frac{dp(x)}{dx}$$

3. **Continuity equations:**

General conditions

$$\frac{dn}{dt} = \frac{1}{q}\frac{\partial J_n}{\partial x} - (U - G)$$

$$\frac{dp}{dt} = \frac{1}{q}\frac{\partial J_p}{\partial x} + (U - G)$$

Under thermal equilibrium and steady state conditions

$$\frac{1}{q}\frac{\partial J_n}{\partial x} = (U - G)$$

$$\frac{1}{q}\frac{\partial J_p}{\partial x} = -(U - G)$$

where U and G are the recombination and generation rates in the particular material and depend on the details of the device and may also depend on distance.

http://www.pveducation.org/pvcdrom/pn-junction/basic-equations

Solving for Region With Electric Field

1. Depletion approximation: the electric field is confined to a particular region.
2. No free carriers (*$n(x)$, $p(x) = 0$) in depletion region.*
3. *We can assume no free carriers since the electric field sweeps them out of the depletion region quickly. No free carriers means (1) transport equations drop out and (2) no recombination or generation, so the continuity equation becomes:*

$$\frac{1}{q}\frac{\partial J_n}{\partial x} = (U - G) = 0$$

This means that J_n is constant across the depletion region. Similarly, J_p is also constant across the depletion region.

4. *Abrupt or step doping profile (N_A-, N_D^+ are constant).*
5. *All dopants are ionized $N_A^- = N_A$, $N_D^+ = N_D$).*
6. *One-dimensional device.*

http://www.pveducation.org

Solution

The only equation left to solve is Poisson's Equation, with *n(x) and p(x) =0, abrupt doping profile and ionized dopant atoms. Poisson's equation then becomes:*

$$\frac{\partial \hat{E}}{\partial x} = \frac{\rho}{\varepsilon} = \frac{q}{\varepsilon}(-N_A + N_D)$$

$$\rho = \begin{cases} -qN_A, & when - x_p \leq x \leq 0 \\ qN_D, & when\ 0 \leq x \leq x_n \end{cases}$$

Where $\varepsilon = \varepsilon_0 \varepsilon_s$, ε_0 is the permittivity in free space, and ε_s is the permittivity in the semiconductor and x_p and x_n are the edges of the depletion region in the p- and n-type side respectively, measured from the physical junction between the two materials. The electric field then becomes

$$E = \begin{cases} \int -\frac{qN_A}{\varepsilon} dx = -\frac{qN_A}{\varepsilon} x + C_1, & for - x_p \leq x < 0 \\ \int \frac{qN_D}{\varepsilon} dx = \frac{qN_D}{\varepsilon} x + C_2, & for\ 0 \leq x < x_n \end{cases}$$

http://www.pveducation.org

Solution (continued…)

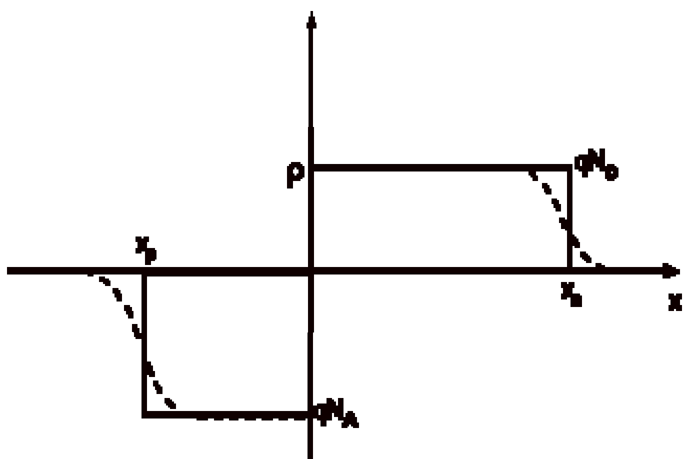
The integration constants C_1 *and* C_2 *can be determined by using the depletion approximation, which states that the electric field must go to zero at the boundary of the depletion regions. This gives:*

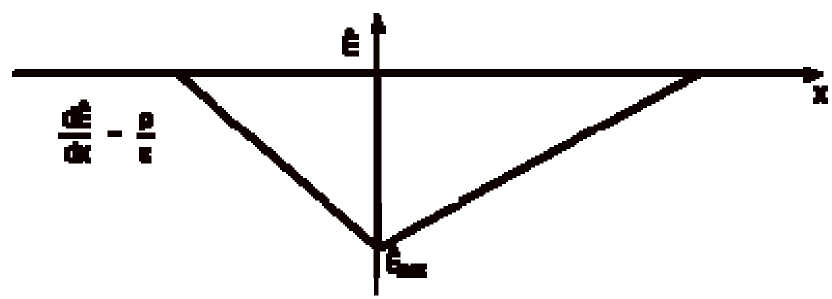
$$E(x = -x_p) = 0 \quad \Rightarrow \quad C_1 = \frac{-qN_A}{\varepsilon} x_p$$

$$E(x = x_n) = 0 \;\Rightarrow\; C_2 = -\frac{qN_D}{\varepsilon} x_n$$

Solution (Electric field distribution in the space charge)

The maximum electric field occurs at the junction between the p- and n-type material. Further, we know that the electric field lines must be continuous across the interface, such that the electric field in the p-type side and the n-type side must equal each other at the interface or when *$x = 0$. Putting $x = 0$ in the above equation for electric field and setting the two values of E equal to each other gives: $N_A x_p = N_D x_n$ .This equation makes physical sense since it states that the total charge on one side of the junction must be the same as the total charge on the other. In other words, if the electric field is confined to the depletion region, then the net charge in Region II must be zero, and hence the negative charge and the positive charge must be equal. $N_A x_p A$ is the total negative charge, since N_A is the charge density and $x_p A$ is the volume of the depletion region (A is the cross-sectional area and xp is the depth). Similarly, $N_D x_n A$ is the positive charge. The cross sectional area (A) is the same and cancels out.*





http://www.pveducation.org