Physics 6/7180:Graduate Physics Laboratory

Experiment CM3: Electrical transport and the Hall effect in metals and semiconductors

References:
1. Preston and Dietz, (Expt. 17; pp 303-315)
2. Kittel (6th or 7th edition), chaps. 6 and 8
3. L.J. van der Pauw, Philips Technical Review, 20, 220 (1959)
4. D.K. Schroder, *Semiconductor Materials and Device Characterization,* (Wiley, N.Y., 1990, p 9ff).

Goals:

- *study the Lorentz force on charge carriers moving in a static magnetic field.*
- *measure the carrier concentrations in some metals and semiconductors.*
- *observe both positive and negative signs for the charge of the carriers in various metals and semiconductors.*
- *understand the basic ideas of how the periodic lattice affects the motion of electrons, producing an "effective mass," and quasiparticles known as holes.*
- *understand some practical uses of Hall probes in the measurement of magnetic fields and in clamp-on current probes.*

Equipment:

- ♦ PC with Labview and I/O card with D/A outputs and A/D inputs including terminal strip for wiring hook-ups
- ♦ differential amplifier with gain of ~10,000 to remove dc voltage offset
- ♦ Lakeshore Gaussmeter with Hall probe
- ♦ ~0.3 T permanent magnet
- ♦ test samples; each with four contacts (evaporated Au, thin film of SnO_2:In (ITO) on glass, Ni film, Mo film, and Cu film)

Background:

The Hall Effect--

Read carefully Preston and Dietz pp. 303 - 310 (up to the section on cryogenics). We shall be performing all the measurements at room temperature. A more extensive discussion of electrical conductivity and the Hall effect is presented in Kittel pp. 168-176. Below is a figure describing the geometry for resistivity and Hall effect measurements and also Kittel's table of Hall coefficients, R_H.

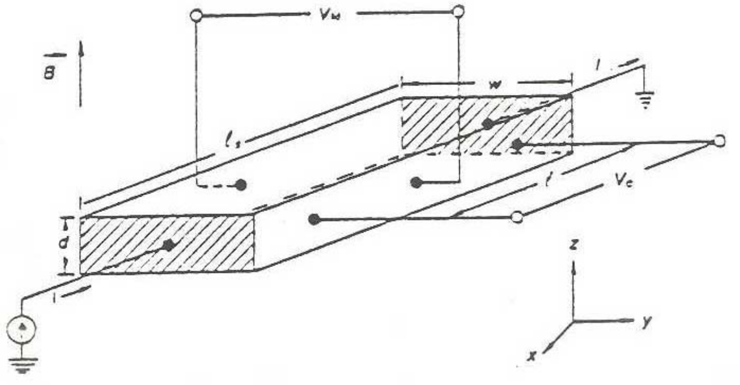
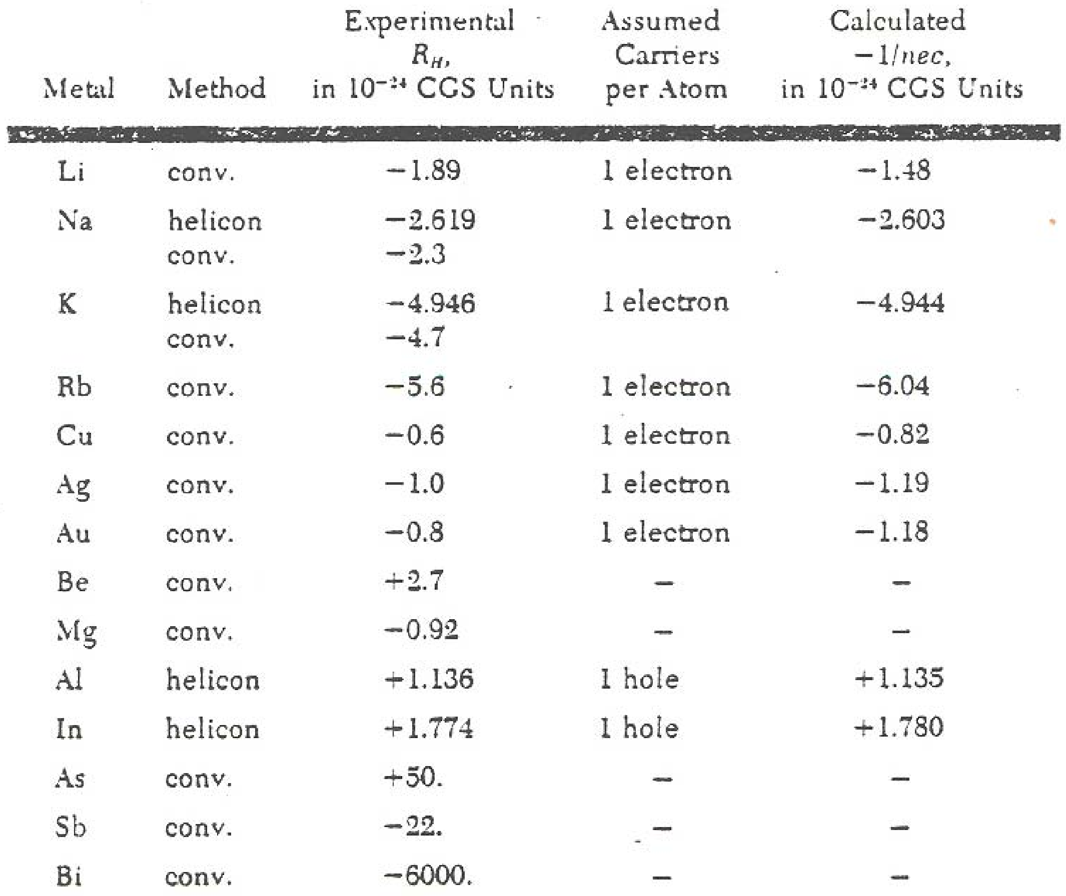


Fig.CM3-1: Standard geometry for Hall effect and resistivity measurements (from Ref. 1).



Metal	Method	Experimental R_H, in 10^{-24} CGS Units	Assumed Carriers per Atom	Calculated $-1/nec$, in 10^{-24} CGS Units
Li	conv.	−1.89	1 electron	−1.48
Na	helicon	−2.619	1 electron	−2.603
	conv.	−2.3		
K	helicon	−4.946	1 electron	−4.944
	conv.	−4.7		
Rb	conv.	−5.6	1 electron	−6.04
Cu	conv.	−0.6	1 electron	−0.82
Ag	conv.	−1.0	1 electron	−1.19
Au	conv.	−0.8	1 electron	−1.18
Be	conv.	+2.7	–	–
Mg	conv.	−0.92	–	–
Al	helicon	+1.136	1 hole	+1.135
In	helicon	+1.774	1 hole	+1.780
As	conv.	+50.	–	–
Sb	conv.	−22.	–	–
Bi	conv.	−6000.	–	–

Fig.CM3-2: *Table of observed and calculated Hall coefficients (from Kittel, Ref. 2).*

Recall some fundamentals of electrical conductivity, recalling specifically the Lorentz force acting on a charged particle:

For a field in the x-direction (E_x) the current density is: $j_x = \sigma E_x$,
where the conductivity is: $\sigma = ne\mu = ne^2\tau/m$ [note: $1/\sigma = \rho$ (resistivity)]
where the carrier scattering rate is τ^{-1} and the mobility is: $\mu = e\tau/m = \sigma/ne$.
In the case of an applied B-field in the z-direction (B_z), you may readily show that the induced, steady-state transverse electric field will be $E_y = eB_z\tau/m$.

Referring to Figure CM3-1, the Hall coefficient is defined by:

$$R_H = E_y/(j_xB_z).$$

On the basis of a simple free-electron Fermi gas model, the Hall coefficient is equal to:

$R_H = -1/ne$ (in SI units),
where e is the electron charge and n is the electron density.

From a simple classical model of electrons all traveling at the same velocity under the influence of an applied field creating a current density, j_x, it should be intuitively clear that as the carrier density goes down the velocity must go up (to keep a constant j_x. Consequently the Lorentz force due to the magnetic field, B_z, will increase (since the Lorentz force is proportional to the velocity). This effect shows up in the second equation where R_H is inversely proportional to the carrier density. Furthermore if the effective charge of the current-carrying quasiparticle is positive, the Hall coefficient will reverse sign. For an ordinary metal with $n \sim 6 \times 10^{22}/\text{cm}^3 = 6 \times 10^{28}/\text{m}^3$, the Hall coefficient is $R_H = -1/ne \sim -1 \times 10^{10}$ m^3-coul.

Note that Kittel's discussion occurs in a chapter on the "free-electron Fermi gas" model of metals. Note also that this model, which is classical except for the inclusion of the quantum (Fermi) statistics of electrons, cannot successfully predict the existence of *positive* values of the Hall coefficient. Similarly (if less dramatically), it cannot predict or explain the existence of Hall coefficients as large as -6000 (x 10^{10}) as observed for metallic bismuth.

The correct understanding of Hall coefficients requires a theory which includes the effect of lattice scattering on the propagation of electrons in crystals. The "nearly free electron model" can give a qualitative understanding of such effects. (See Kittel, chap 7.) In short, it is necessary to include electron scattering from the periodic lattice potential. (The Kronig-Penney model is a simple one-dimensional example of a solution of the Schrodinger equation for the electrons in a simple square-well periodic potential.) Thus, due to the effect of the periodic lattice, the electrons may move as if their "effective mass" is very small -- as little as 0.01 m_e -- or even as if they have negative mass (and therefore a positive Hall coefficient) as in the case of aluminum!

Positive Hall coefficients in metals arise in a somewhat similar way from the peculiar shape of the Fermi surface--the locus of points in (crystal) momentum space of constant energy equal to the Fermi energy. This behavior is a little subtle and requires careful development of the various concepts. Please read Chapter 9 of Kittel, especially pages 251-260.

In the case of semiconductors, the density of free carriers will depend on the doping density. And, depending on the type of dopant, the free carriers may be either electron-like (with properties dependent on the conduction band effective mass) or hole-like (with properties dependent on the valence band effective mass). The electrons at extrema in the valence band have negative effective masses and an empty electron state can be shown to behave exactly as a positively charged particle of the same, but positive, effective mass. This is the origin of positive Hall coefficients in the case of semiconductors.

Note that for small doping densities in semiconductors it is possible to achieve large values of the Hall coefficient. That is, the induced Hall voltage is large for modest values of current density and magnetic field. This is the ideal situation for a magnetic field detector (witness our Hall probe used to measure the magnetic field in this experiment). Such a semiconductor would also be a good choice for use in a clamp-on current detector which could detect *dc* currents as well as *ac* by detecting the magnetic field surrounding a wire.

Resistivity measurements in the van der Pauw geometry: in practice, the Hall-bar geometry shown in Figure CM3-1 is very difficult to fabricate in such a way that the transverse voltage is zero when no B-field is applied. (Actually the figure does not show explicitly the current electrodes at the end or the transverse voltage electrodes on the sides.) The two contacts on the sides must be exactly opposite each other to avoid reading the IR drop which occurs along the length of the bar. However, in 1958, L. J. van der Pauw (Philips Technical Review **20**, 220 (1958) [Ref. 3]) introduced a generalized geometry and procedure for electrical resistivity measurements and Hall measurements in a sample of arbitrary shape (see Fig. CM3-3). The sample must have four leads attached, with two being current leads and two voltage leads.

We define:

$R_{ij,kl} = V_{kl}/I_{ij}$, with $V_{kl} = V_k - V_l$.

[Note, the ordering sequence is important!! ijkl should be in sequence around the periphery of the sample.]

Then the resistivity of the material can be calculated as:

$\rho = \{\pi f d/[2\ln(2)]\} \times [R_{21,34} + R_{32,14}]$,

where f is determined by the solution to the following equation:

$\cosh[\ln(2)(Q-1)/f(Q+1)] = (1/2)\exp[\ln(2)/f]$.

where

$Q = R_{21,34}/R_{32,41}$, or $R_{32,41}/R_{21,34}$ -- whichever ratio is greater than unity.

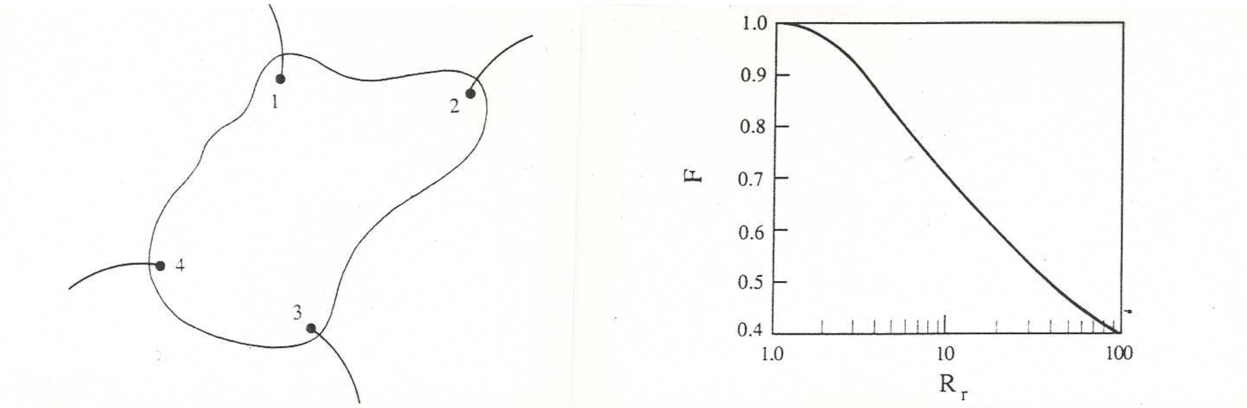


Fig. 1.6 Arbitrarily shaped sample with four contacts. **Fig. 1.7** The van der Pauw correction factor F as a function of R_r.

Fig. CM3-3: *a) van der Pauw geometry for electrical resistivity and Hall measurements on samples of arbitrary shape; b)resistivity ratio function f(Q). (From Ref. 4.)*

For measuring the Hall coefficient, one measures first, IN THE ABSENCE OF ANY MAGNETIC FIELD, the value $R_{13,24}$ (Note the ordering!!) and then again, WITH THE MAGNETIC FIELD PRESENT, the change in this value $\Delta R_{13,24}$. Then the Hall coefficient is:

$R_H = (d/B)\ \Delta R_{13,24}$

provided that a) the contacts are sufficiently small, (Q)
b) the contacts are on the periphery, and
c) the film is of uniform thickness and free of holes (not holes as in positive charge carriers, but physical voids).

Additional information (possible complications):

- Recall that the relation between resistivity and conductivity is: $\rho = 1/\sigma$.

- In the general case in which there are **both** electrons and holes which carry the current, then the conductivity becomes: $\sigma = e(n\mu_n + p\mu_p)$, where p is the concentration of holes, and here are generally distinct mobilities for electrons and holes: μ_n and μ_p.

Procedure:

A. Conductivity

-Determine the resistivity (conductivity) of at least five different samples. Ideally, we'll include an *n*-type and a *p*-type semiconductor, the doped SnO_2 thin film, and two metallic thin films (Au and Al), but the sample availability may vary.

-When attaching leads to the sample or connecting the leads to the instruments, pay attention to the order of the contacts and note that the order is different for conductivity measurements than it will be for the later Hall measurements.

-Measure the resistivity for both "forward" and "reverse" current directions in order to determine whether there are any non-ohmic contacts.

-Compute the resistivity and conductivity for each sample. (Use the film thicknesses provided for the thin-film samples.

B. Hall effect (carrier type, carrier density, and mobility)

-Measure the same five samples as in part A.

-Use at least two different values of magnetic field. B should be measured with the Lakeshore gaussmeter. Note that the probe is a x10 probe so that the indicated field must be multiplied by 10.

-Measure the "Hall voltage" (transverse voltage) with the sample outside of the field and then, without disconnecting the leads, measure the change in voltage as the sample is placed in the center of the B-field.

-Note that (although we're not using the electromagnet in Spring 2010) when using the electromagnet, you cannot assume that B = 0 when the current is 0 -- due to to hysteresis effects (remanence).

-For the Hall measurements, as for the conductivity measurements, you should measure the Hall voltage for current flowing in both directions through the sample and for both directions of the B field. (This may be done, of course, by inverting the sample.)

C. Analysis

-For each sample calculate the conductivity, carrier type, carrier density, and carrier mobility.

-Discuss the results, provide error estimates, and explain any discrepancies.

Nota Bene: If the differential voltage amplifier is used, please account for the amplification factor of 10^4. The differential amplifier requires a bias supply of 12 V.

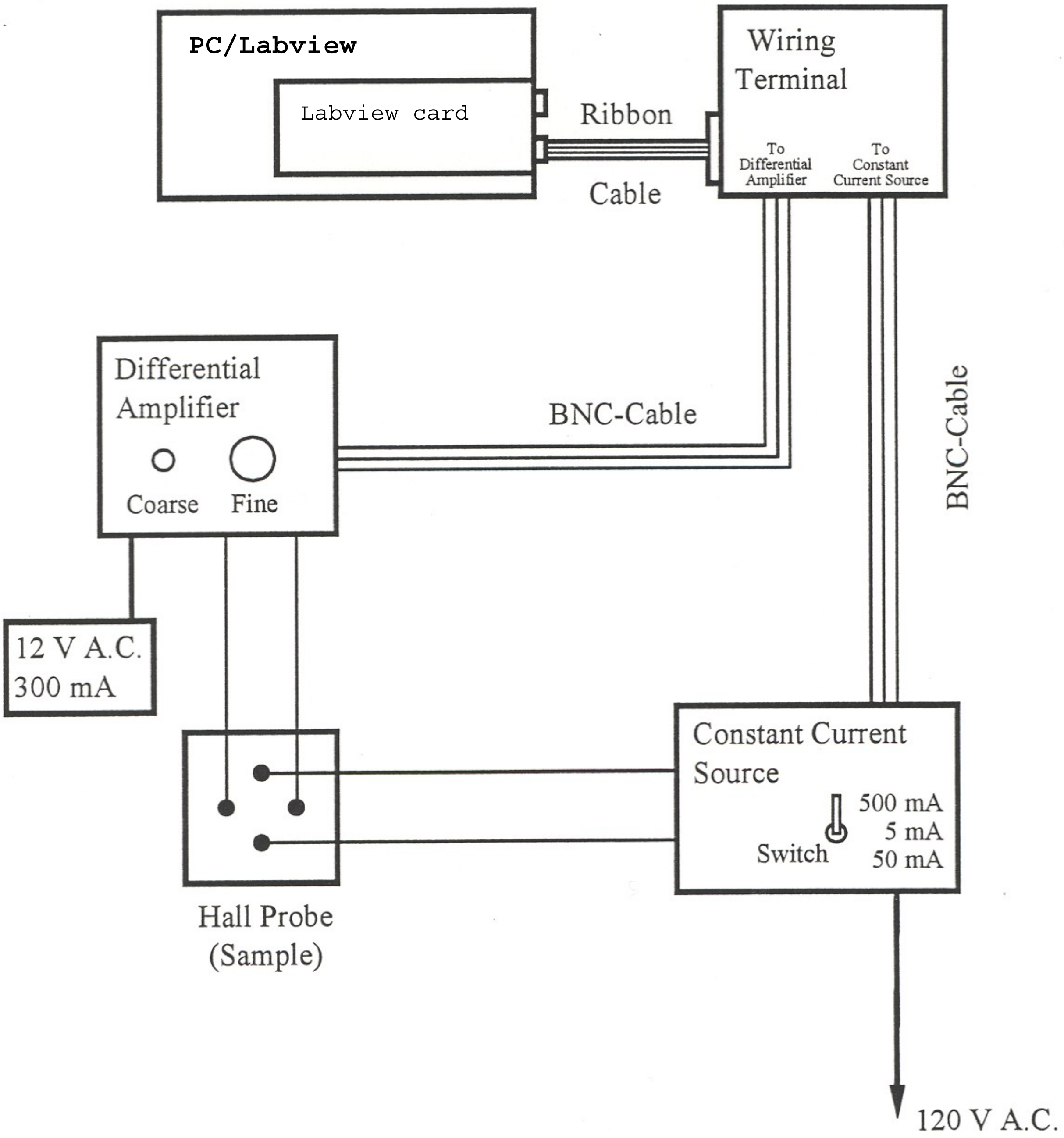


Fig. CM3-4. Hall effect measurement system wiring plan

Information for the Hall Effect Measurements write-up (Experiment CM3)

1. Details on the geometry and materials for the commercial Hall probe: they are (as reported by the manufacturer, typically):
    - -material InAs
    - -carrier concentration (electrons) n= 0.82 - 1.4 x 10^{18} cm^{-3}
    - -mobility μ_e = 12100 - 23900 cm^2/V-s
    - -Hall constant R_H = 4.6 - 302 !! (take your pick!)
    - -I estimate the thickness to be 1.5 μm

2. Thickness of the SnO_2:F (fluorine-doped tin oxide) estimated from the optical measurement
    - -0.7 μm (This assumes that the index of refraction is n = 2.)

3. Thickness of the gold film deposited in a thermal evaporator:
    - = 40 nm (This based on a measurement of Xiangxin Liu using the Dektak profilometer in MH 3023.)

4. Thickness of the Si sample—measure an equivalent piece of Si with a vernier calipers or micrometer—should be a little less than 0.5 mm

This experiment report will be a synopsis (short report).

1. a diagram of the experimental set up including a sketch and definition of the van der Pauw geometry and how it is used for samples of arbitrary geometry—for both resistivity measurements and Hall effect measurements
2. a) a short section on theory including definition of resistivity and conductivity and how they are determined from the raw measurements; and

    b) definition of the Hall constant and how it is determined from the raw measurements
3. Discussion of how the resistivity and Hall measurements can be analyzed to yield:

    carrier concentration, carrier type, mobility
4. measurement of the magnetic field of the permanent magnet. (In the spring of 04 we will have to finesse this one, at least for the time being, since the Bell Gaussmeter is malfunctioning.)
5. the results for all of these parameters for the four samples (a table would be appropriate):

    [*gold film (metal), SnO_2 film doped with F, doped Si wafer, Ni film, Mo film, Cu film, commercial InAs probe*]
6. analysis of errors and comparison with known values where possible