Carrier Type, Density, and Mobility Determination (Hall Effect)

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The University of Toledo
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Solar Cell Structure

- Antireflection coating
- Front contact
- Emitter
- Sunlight
- Electron-hole pair
- Base
- Rear contact

External load

PV Education.org
J/V Characteristics, and the Diode Eqn.

\[ I = I_0 \left( \frac{qV}{e n k T} - 1 \right) \]

Without illumination, a solar cell has the same electrical characteristics as a large diode.

\[ I = I_0 \left[ \exp \left( \frac{qV}{n k T} \right) - 1 \right] - I_L \]

where \( I_L \) = light generated current.

The greater the light intensity, the greater the amount of shift.
The p-n Homojunction

Consider the band diagram for a *homojunction*, formed when two bits of the same type of semiconductor (e.g. Si) are doped p and n type and then brought into contact.

Electrons in the two bits have different electrochemical potentials (i.e. different $E_f$’s).

Charge transfer occurs at contact (electron go down from the vacuum level, holes go “up”)

At equilibrium, there is no net transport ($E_f$ is constant throughout the device)

H. Föll: http://www.tf.uni-kiel.de/matwis/amat/semi_en/kap_2/backbone/r2_2_4.html
Basic Equations for Solving for the Electric Field, Transport, and Carrier Concentrations: see [http://www.pveducation.org/pvcdrom/pn-junction/basic-equations](http://www.pveducation.org/pvcdrom/pn-junction/basic-equations), up through “Solving for Region With Electric Field”

1. **Poisson's equation:**
   \[ \frac{\partial \phi}{\partial x} = \frac{\rho(x, y, z)}{\varepsilon} = \frac{q}{\varepsilon} \left( p(x) - n(x) - N_A^- + N_D^+ \right) \]

2. **Transport equations:**
   \[ J_n = q \mu_n n(x) \vec{E} + qD_n \frac{dn(x)}{dx} \quad \text{(first term is drift, second is diffusion)} \]
   \[ J_p = q \mu_p p(x) \vec{E} - qD_p \frac{dp(x)}{dx} \]

3. **Continuity equations:**
   - **General conditions**
     \[ \frac{dn}{dt} = \frac{1}{q} \frac{\partial J_n}{\partial x} - (U - G) \]
     \[ \frac{dp}{dt} = \frac{1}{q} \frac{\partial J_p}{\partial x} + (U - G) \]
   - **Under thermal equilibrium and steady state conditions**
     \[ \frac{1}{q} \frac{\partial J_n}{\partial x} = (U - G) \]
     \[ \frac{1}{q} \frac{\partial J_p}{\partial x} = -(U - G) \]

where $U$ and $G$ are the recombination and generation rates in the particular material and depend on the details of the device and may also depend on distance.
How do we measure $n$, $p$, $\mu_n$, and $\mu_p$?

Through conductivity / resistivity measurements?

$$\sigma_n = 1/\rho_n = ne\mu \text{ (don’t confuse } \rho \text{ with } p)$$

! Generally, transport can be due to electron and holes, so;

$$\sigma_{\text{total}} = \sigma_n + \sigma_p$$

- though in most cases one deals with holes or electrons

• For a chunk:
  - $R = \rho \ (L/A) \quad (\Omega)$

• For a film:
  - $\rho = R_s \times t \quad (\Omega\text{-cm})$
Do we have electrons or holes?

\[ \mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \]

Lorentz force

p-doped semiconductor

n-doped semiconductor
Hall effect measurements using van der Pauw sample configuration allows determination of:

• Charge carrier type (n or p)
• Charge carrier density (#/cm$^3$)
• Relevant Hall mobility (cm$^2$/V-s)
• Investigations of carrier scattering, transport phenomena as f(T) and other variables.
A METHOD OF MEASURING SPECIFIC RESISTIVITY AND HALL EFFECT OF DISCS OF ARBITRARY SHAPE

by L. J. van der PAUW

Summary
A method of measuring specific resistivity and Hall effect of flat samples of arbitrary shape is presented. The method is based upon a theorem which holds for a flat sample of arbitrary shape if the contacts are sufficiently small and located at the circumference of the sample. Furthermore, the sample must be singly connected, i.e., it should not have isolated holes.

An ideal sample

Fig. 1. The classical shape of a sample for measuring the specific resistivity and the Hall effect.

An real sample

Fig. 2. The bridge-shaped sample, furnished with large areas for making low-ohmic contacts.

A well-known example is the bridge-shaped sample shown in fig. 2. The
van der Pauw’s advance

2. A theorem which holds for a flat sample of arbitrary shape

We consider a flat sample of a conducting material of arbitrary shape with successive contacts A, B, C and D fixed on arbitrary places along the circumference such that the above-mentioned conditions (a) to (d) are fulfilled (see fig. 3). We define the resistance $R_{AB,CD}$ as the potential difference

![Diagram](image)

Fig. 3. A sample of arbitrary shape with four small contacts at arbitrary places along the circumference which, according to this paper, can be used to measure the specific resistivity and the Hall effect.

through the contact B. Similarly we define the resistance $R_{BC,DA}$. It will be shown that the following relation holds:

$$\exp(-\pi R_{AB,CD} d/\rho) + \exp(-\pi R_{BC,DA} d/\rho) = 1,$$

(1)

where $\rho$ is the specific resistance of the material and $d$ is the thickness of the sample.

To prove eq. (1) we shall first show that it holds for a particular shape of the sample. The second step is to prove that if it holds for a particular
Enables measurement of wafers, presents large surface area to B Field to generate larger Hall

\[ V_H = \frac{IB}{q\eta_s} \]

- **Cloverleaf**
  - Preferred
- **Square or rectangle:**
  - Contacts at the corners
  - Acceptable
- **Square or rectangle:**
  - Contacts at the edges or inside the perimeter
  - Not Recommended

http://upload.wikimedia.org/wikipedia/commons/0/03/VanderPauuwContactPlacement.jpg
A few conditions for valid measurements:

1. The sample must have a flat shape of uniform thickness
2. The sample must not have any isolated holes
3. The sample must be homogeneous and isotropic
4. All four contacts must be located at the edges of the sample
5. The area of contact of any individual contact should be at least an order of magnitude smaller than the area of the entire sample.
6. The sample thickness should be $\ll$ than the width and length of the sample.
7. It is preferable that the sample is symmetrical.

The measurements require that four ohmic contacts be placed on the sample. Contacts should be placed on the boundary of the sample. Contacts would ideally be infinitely small.

Practically, they must be as small as possible; any errors given by their non-zero size will be of the order $D/L$, where $D$ is the average diameter of the contact and $L$ is the distance between the contacts.

Procedures in Hall effect measurements:

• The contacts are numbered from 1 to 4 in a counter-clockwise order, beginning at the top-left contact.

• The current $I_{12}$ is a positive DC current injected into contact 1 and taken out of contact 2.

• The voltage $V_{34}$ is a DC voltage measured between contacts 3 and 4 with no externally applied magnetic field.

• The resistivity $\rho$ is measured in ohms·meters ($\Omega \cdot m$).

• The thickness of the sample $t$ is measured in meters (m).

• The sheet resistance $R_S$ is measured in ohms ($\Omega$).

Acquired Data

Experiment Type
- Hall Effect Measurement
- Four Probe Resistivity Measurement

Process
- Probes Source/Meter
- Current 1, (A)
- Voltage 1, (V)
- Field (G)
- Resistance, (Ohm)
- Form Factor
- Resistivity (Ohm cm)

Sample Diagram

Data File

Help
- Resistance: \( R = \frac{V_2 - V_1}{I_2 - I_1} \)
- Ratio: \( X = \frac{F_{P_{24}}}{F_{P_{23}}} \)
- Form Factor: \( F = \frac{2m_0}{\ln a + \ln(1 - a)} \)
- Resistivity: \( \rho = \frac{\pi d F (R_{12,24} + R_{23,41})}{2h} \)
- Hall Mobility: \( \mu = \frac{10 \pi (\Delta R_{12,24} + \Delta R_{23,41})}{2 \rho \Delta B} \)
- Density of carriers: \( \gamma = \frac{1}{\rho \beta_{B1}} \)
- Hall Coefficient: \( R_{Hall} = \rho \mu \)
- Sheet Number: \( N_{Sheet} = \frac{n}{d} \)
- Sheet Resistivity: \( R_{Sheet} = \frac{\rho}{d} \)
- Type of carriers

System Status
- Status: Waiting for instructions
- Point: Error
- Error: FF
- Current (A): Target / Read
- Field (G): Target / Read
- Temp (K): Target / Read
- Power (W): Target / Read
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Formula</th>
<th>Note</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resistance</td>
<td>[ R = \frac{V_2 - V_1}{I_2 - I_1} ]</td>
<td>( V_1, V_2, I_1, I_2 ) are Voltage 1, Voltage 2, Current 1, and Current 2 respectively. Current ( I_2 ) is taken with a polarity opposite that of ( I_1 ). 'R' means Resistance acquired by applying current to the probes defined by the first index and by measuring voltage on the probes defined by the second index. The Ratio is calculated for all sides of the sample. Under certain circumstances the Resistance as defined above can be negative.</td>
</tr>
<tr>
<td>Ratio</td>
<td>[ X = \left</td>
<td>\frac{R_{12,34}}{R_{23,41}} \right</td>
</tr>
<tr>
<td>Form Factor</td>
<td>[ F = \frac{-2\ln2}{\ln a + \ln(1 - a)} ]</td>
<td>( '\Delta B' ) is the change in applied field which produces the resistances changes ( \Delta R_{13,24} ) and ( \Delta R_{24,31} ).</td>
</tr>
<tr>
<td>Resistivity</td>
<td>[ \rho = \frac{\pi d F (R_{12,34} + R_{23,41})}{2 \ln2} ]</td>
<td>( 'e' ) is the charge of an electron.</td>
</tr>
<tr>
<td>(Hall) Mobility</td>
<td>[ \mu = \frac{10^8(\Delta R_{13,24} + \Delta R_{24,31})d}{2 \rho \Delta B} ]</td>
<td></td>
</tr>
<tr>
<td>Density of carriers</td>
<td>[ \eta = \frac{1}{\rho e \mu} ]</td>
<td>Sheet Number is the number of carriers per unit area of the sample.</td>
</tr>
<tr>
<td>Hall Coefficient</td>
<td>[ R_{Hall} = \rho \mu ]</td>
<td></td>
</tr>
<tr>
<td>Sheet Number</td>
<td>[ N_{Sheet} = \gamma d ]</td>
<td></td>
</tr>
<tr>
<td>Sheet Resistivity</td>
<td>[ R_{Sheet} = \frac{\mu}{d} ]</td>
<td>Type of carriers is defined by the sign of Hall Mobility. If positive, then carriers are holes, negative means electrons.</td>
</tr>
<tr>
<td>Type of carriers</td>
<td></td>
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</tbody>
</table>
Hot Probe Test to Determine Carrier Type

Seebeck effect: All you need is a soldering iron, and an ammeter!

http://ecee.colorado.edu/~bart/book/hotprobe.htm
### Hot Probe Test to determine Carrier Type

#### Intrinsic

| Holes | | | |
|-------|-------|-------|
|       |       |       |

#### n-type

<table>
<thead>
<tr>
<th>Holes</th>
<th>Free Electrons</th>
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<td></td>
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</table>

#### p-type

| Free Holes | | | |
|------------|-------|-------|
|            |       |       |

\[ p = n = n_i \]

Number of thermally generated holes equals number thermally generated free electrons

Number of free electrons equals number of positively charged donor ions

Number of free holes equals number of negatively charged acceptor cores

*After Hamers*
Hot Probe Test to determine Carrier Type

Distribution of OCCUPIED C.B. levels:

\[ \text{N(E)} \]

Hot

Cold

These are *not* in equilibrium!

*After Hamers*
Seebeck effect, n-type semiconductor

Fick’s Law of Diffusion:

\[ J = -D \frac{\partial c}{\partial x} \]

Electrons diffuse from region of high Concentration to region of lower concentration

“Cold” side becomes slightly negatively charged
Hot side becomes positively charged

After Hamers