Carrier Type, Density, and Mobility Determination (Hall Effect)

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PHYS 4580, PHYS 6/7280
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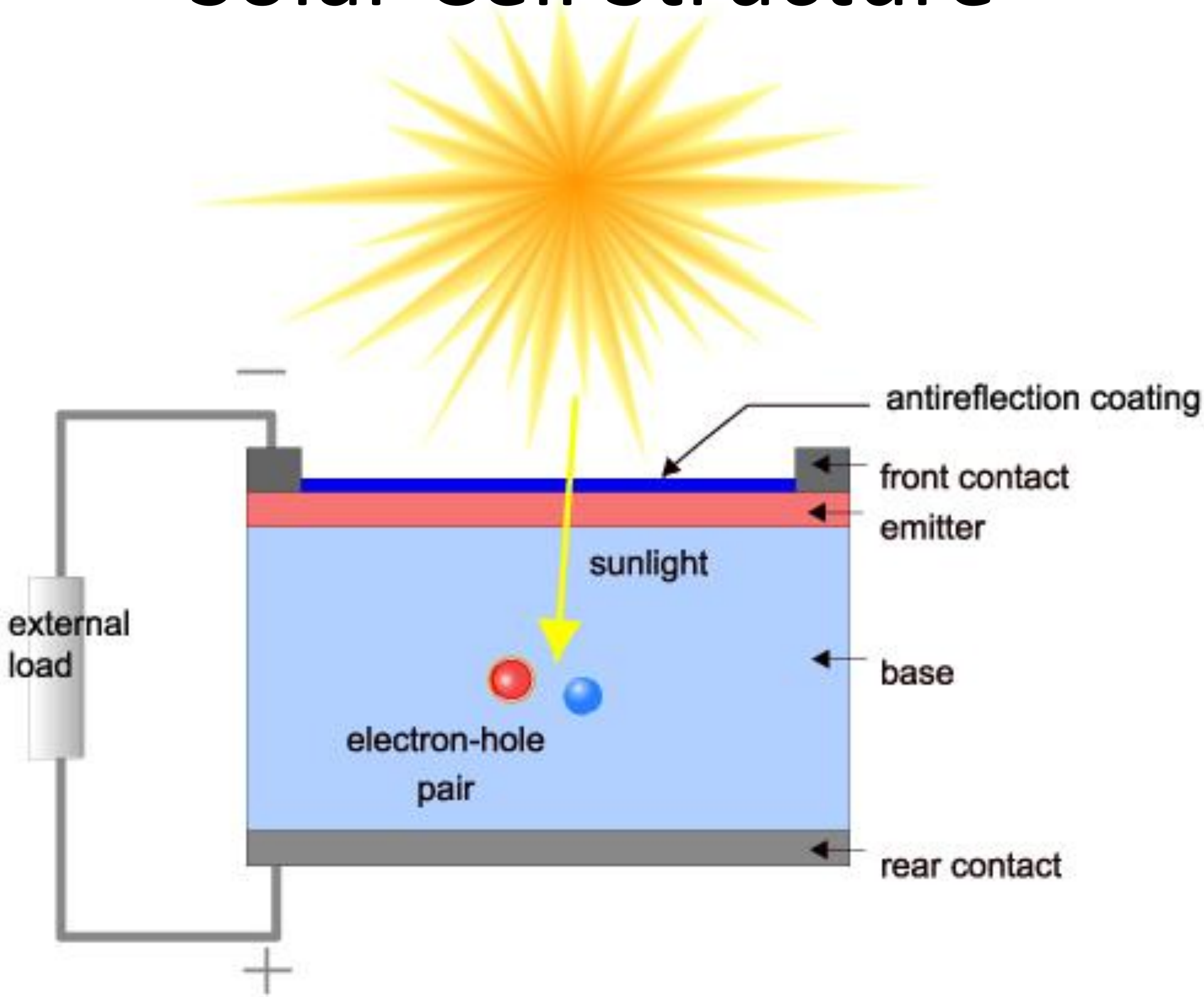
Solar Cell Structure



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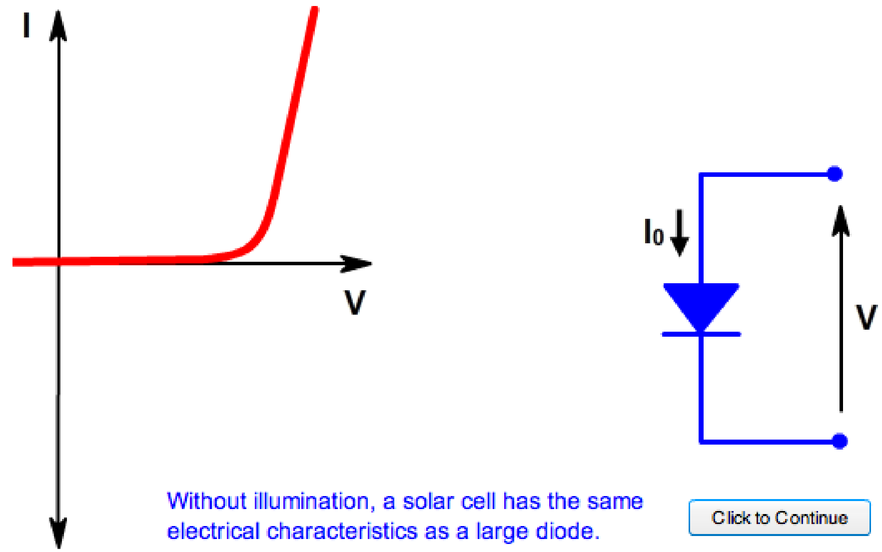
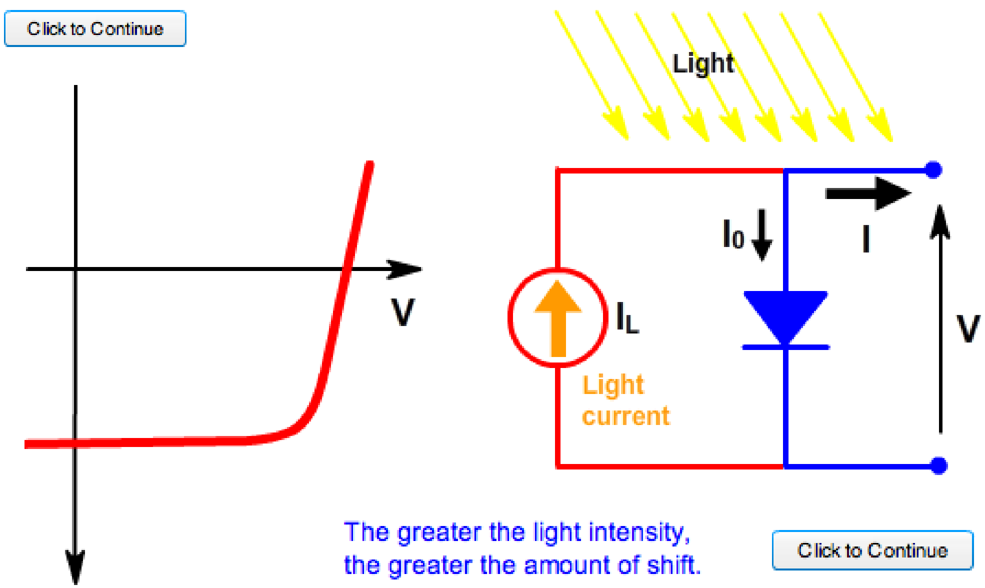
J/V Characteristics, and the Diode Eqn.



$$I = I_0 \left(e^{\frac{qV}{nkT}} - 1 \right)$$

$$I = I_0 \left[\exp\left(\frac{qV}{nkT} \right) - 1 \right] - I_L$$

where I_L = light generated current.



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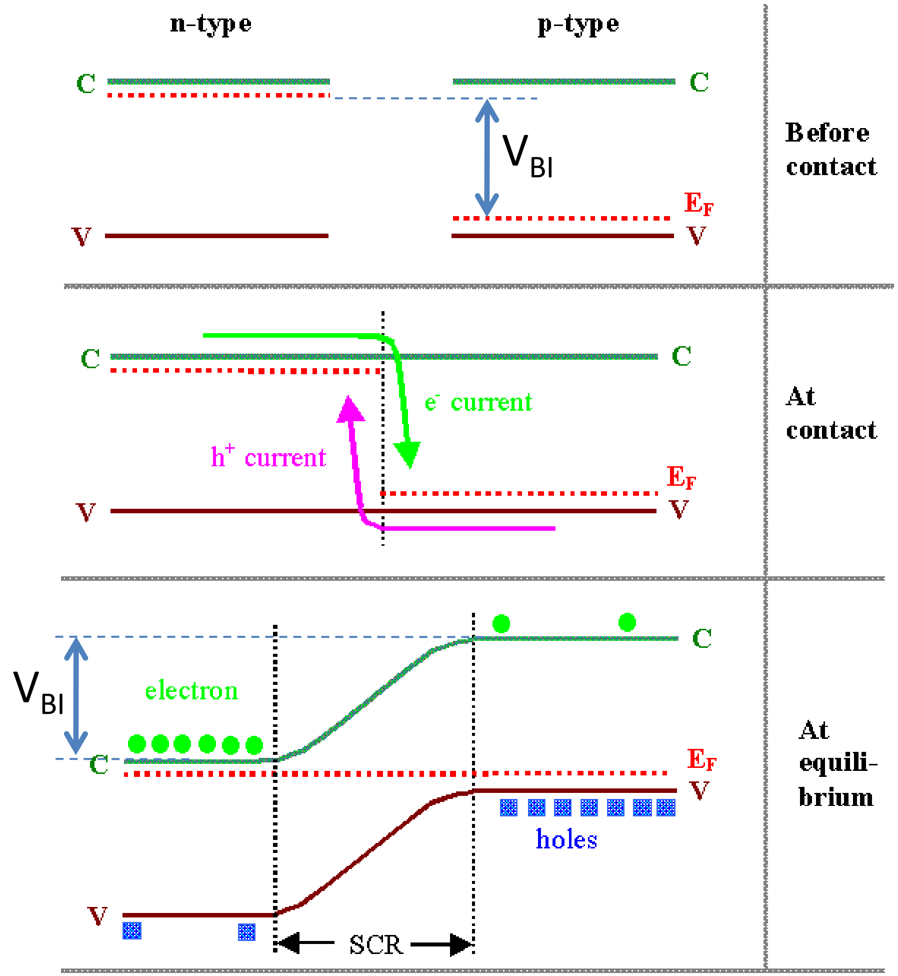
The p-n Homojunction

Consider the the band diagram for a *homojunction*, formed when two bits of the same type of semiconductor (e.g. Si) are doped p and n type and then brought into contact.



Electrons in the two bits have different electrochemical potentials (i.e. different E_f's)

Charge transfer occurs at contact (electron go down from the vacuum level, holes go "up")

At equilibrium, there is no net transport (E_f is constant throughout the device)

H. Föll: http://www.tf.uni-kiel.de/matwis/amat/semi_en/kap_2/backbone/r2_2_4.html

Basic Equations for Solving for the Electric Field, Transport, and Carrier Concentrations: see http://www.pveducation.org/pvcdrom/pn-junction/basic-equations, up through "Solving for Region With Electric Field"

1. **Poisson's equation:**

$$\frac{\partial \vec{E}}{dx} = \frac{\rho}{\varepsilon} = \frac{q}{\varepsilon}\left(p(x) - n(x) - N_A^- + N_D^+\right)$$

$$\nabla^2 \phi(x, y, z) = -\frac{\rho(x, y, z)}{\varepsilon}$$

$$\frac{d\phi(x)}{dx} = -\mathcal{E}(x)$$

2. **Transport equations:**

$$J_n = q\mu_n n(x)\vec{E} + qD_n \frac{dn(x)}{dx}$$

(first term is drift, second is diffusion)

$$J_p = q\mu_p p(x)\vec{E} - qD_p \frac{dp(x)}{dx}$$

Important material-specific properties:

- Carrier mobility (μ_p and μ_n)
- Carrier concentrations (*n* and *p*)

3. **Continuity equations:**

General conditions

$$\frac{dn}{dt} = \frac{1}{q}\frac{\partial J_n}{\partial x} - (U - G)$$

$$\frac{dp}{dt} = \frac{1}{q}\frac{\partial J_p}{\partial x} + (U - G)$$

Under thermal equilibrium and steady state conditions

$$\frac{1}{q}\frac{\partial J_n}{\partial x} = (U - G)$$

$$\frac{1}{q}\frac{\partial J_p}{\partial x} = -(U - G)$$

where U and G are the recombination and generation rates in the particular material and depend on the details of the device and may also depend on distance.

How do we measure n, p, μ_n, and μ_p ?

Through conductivity / resistivity measurements?

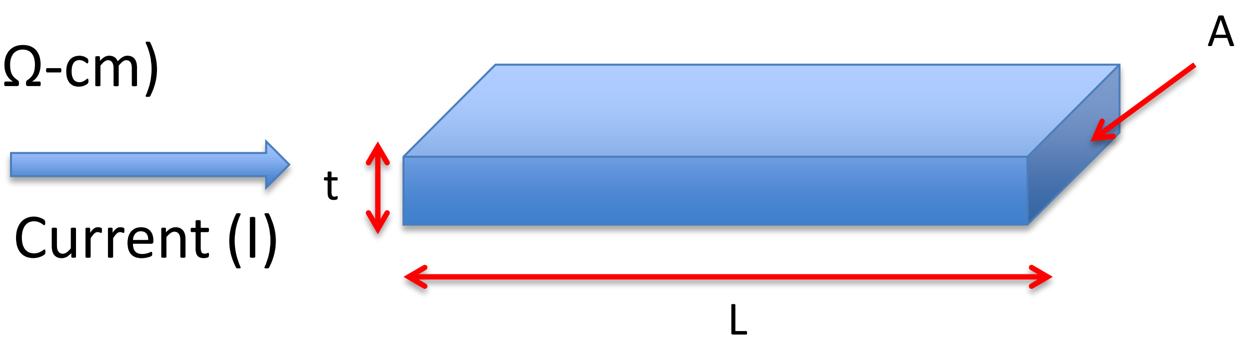
$\sigma_n = 1/\rho_n = ne\mu$ (don't confuse ρ with p)

! Generally, transport can be due to electron and holes, so;

$$\sigma_{total} = \sigma_n + \sigma_P$$

- though in most cases one deals with holes or electrons

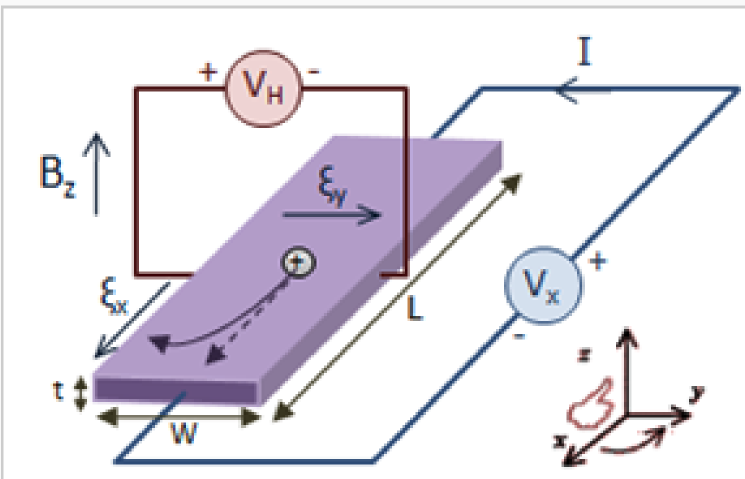
- For a chunk:
  - $R = \rho\,(L/A)$ (Ω)
- For a film:
  - $\rho = R_s \times t$ (Ω-cm)



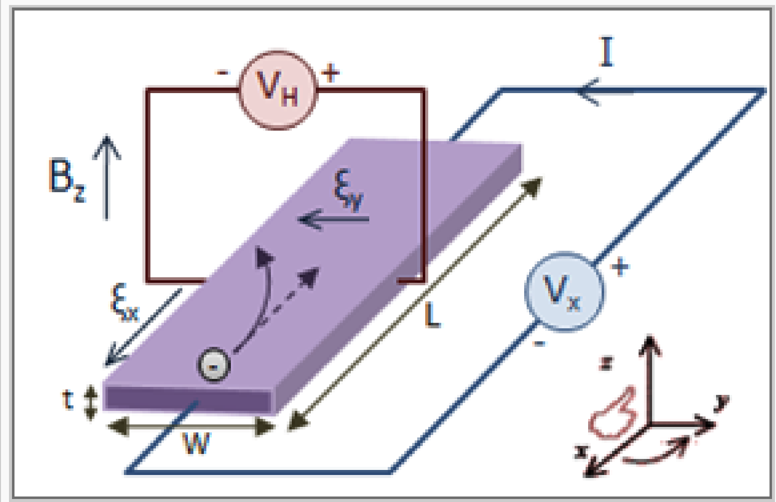
Do we have electrons or holes?

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

Lorentz force



Hall Effect measurement setup for holes



Hall Effect measurement setup for electrons

p-doped semiconductor

n-doped semiconductor

Wikipedia

Hall effect measurements

Hall effect measurements using van der Pauw sample configuration allows determination of:

- Charge carrier type (n or p)
- Charge carrier density (#/cm^3)
- Relevant Hall mobility (cm^2/V-s)
- Investigations of carrier scattering, transport phenomena as f(T) and other variables.

R 334

Philips Res. Repts 13, 1-9, 1958

A METHOD OF MEASURING SPECIFIC RESISTIVITY AND HALL EFFECT OF DISCS OF ARBITRARY SHAPE

by L. J. van der PAUW

537.723.1:53.081.7+538.632:083.9

Summary

A method of measuring specific resistivity and Hall effect of flat samples of arbitrary shape is presented. The method is based upon a theorem which holds for a flat sample of arbitrary shape if the contacts are sufficiently small and located at the circumference of the sample. Furthermore, the sample must be singly connected, i.e., it should not have isolated holes.

An ideal sample

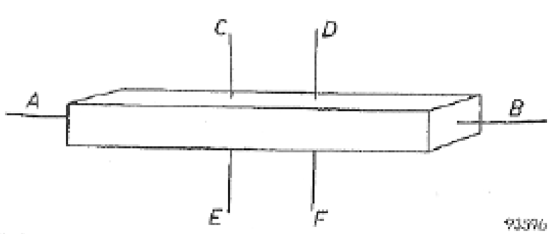


Fig. 1. The classical shape of a sample for measuring the specific resistivity and the Hall effect.

An real sample

A well-known example is the bridge-shaped sample shown in fig. 2. The

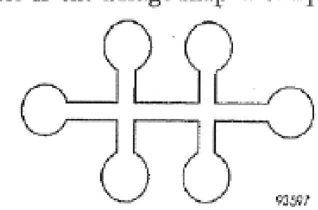


Fig. 2. The bridge-shaped sample, furnished with large areas for making low-ohmic contacts.

van der Pauw’s advance

2. A theorem which holds for a flat sample of arbitrary shape

We consider a flat sample of a conducting material of arbitrary shape with successive contacts A, B, C and D fixed on arbitrary places along the circumference such that the above-mentioned conditions (a) to (d) are fulfilled (see fig. 3). We define the resistance $R_{AB,CD}$ as the potential difference

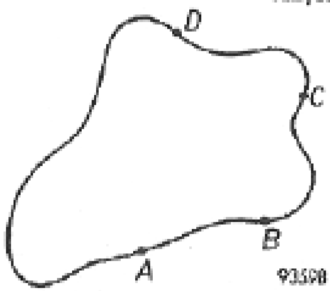


Fig. 3. A sample of arbitrary shape with four small contacts at arbitrary places along the circumference which, according to this paper, can be used to measure the specific resistivity and the Hall effect.

through the contact B. Similarly we define the resistance $R_{BC,DA}$. It will be shown that the following relation holds:

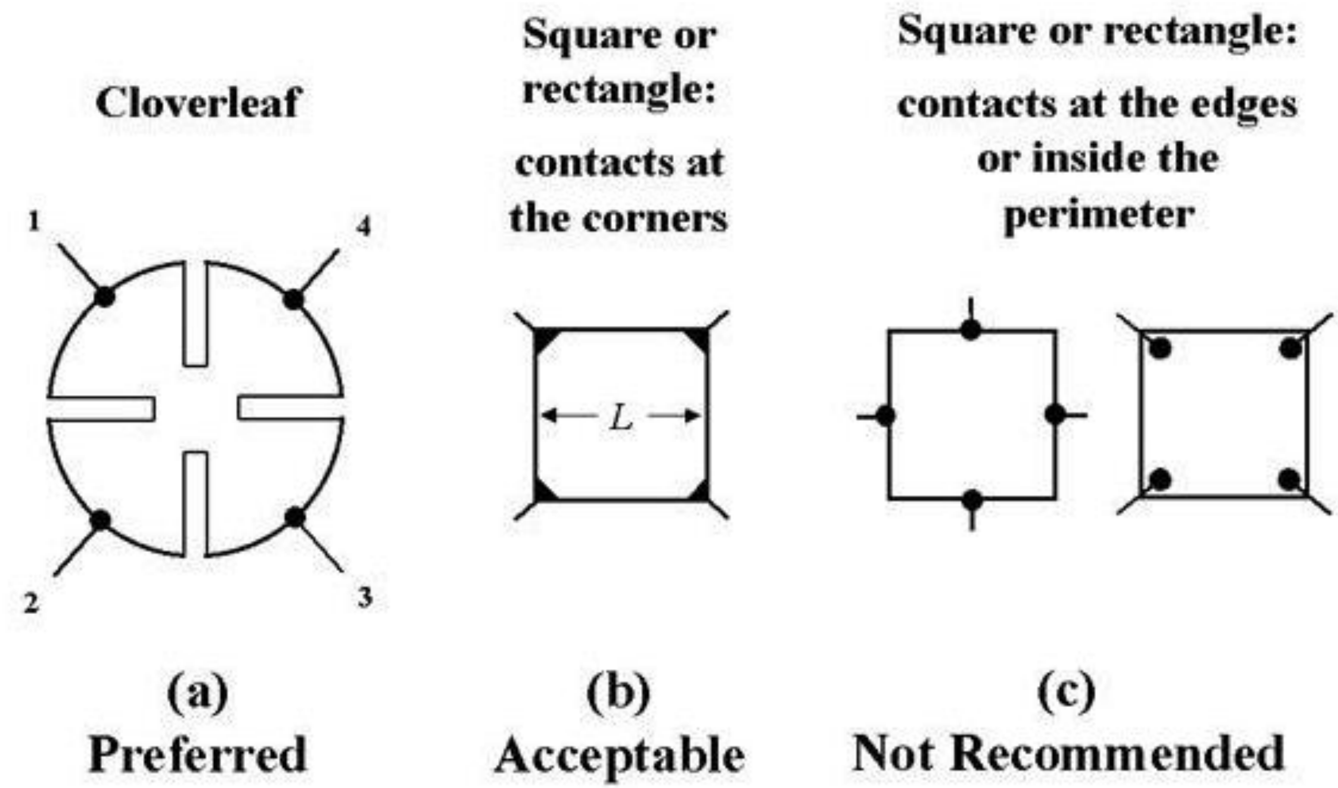
$$\exp(-\pi R_{AB,CD}\, d/\varrho) + \exp(-\pi R_{BC,DA}\, d/\varrho) = 1, \qquad (1)$$

where ϱ is the specific resistance of the material and d is the thickness of the sample.

To prove eq. (1) we shall first show that it holds for a particular shape of the sample. The second step is to prove that if it holds for a particular

Enables measurement of wafers, presents large surface area to B Field to generate larger Hall

$$V_H = \frac{IB}{qn_s}$$



http://upload.wikimedia.org/wikipedia/commons/0/03/VanderPauwContactPlacement.jpg

A few conditions for valid measurements:

1. The sample must have a flat shape of uniform thickness
2. The sample must not have any isolated holes
3. The sample must be homogeneous and isotropic
4. All four contacts must be located at the edges of the sample
5. The area of contact of any individual contact should be at least an order of magnitude smaller than the area of the entire sample.
6. The sample thickness should be << than the width and length of the sample.
7. It is preferable that the sample is symmetrical.

The measurements require that four ohmic contacts be placed on the sample. Contacts should be placed on the boundary of the sample. Contacts would ideally be infinitely small.

Practically, they must be as small as possible; any errors given by their non-zero size will be of the order D/L, where D is the average diameter of the contact and L is the distance between the contacts.

http://en.wikipedia.org/wiki/Van_der_Pauw_method

Procedures in Hall effect measurements:

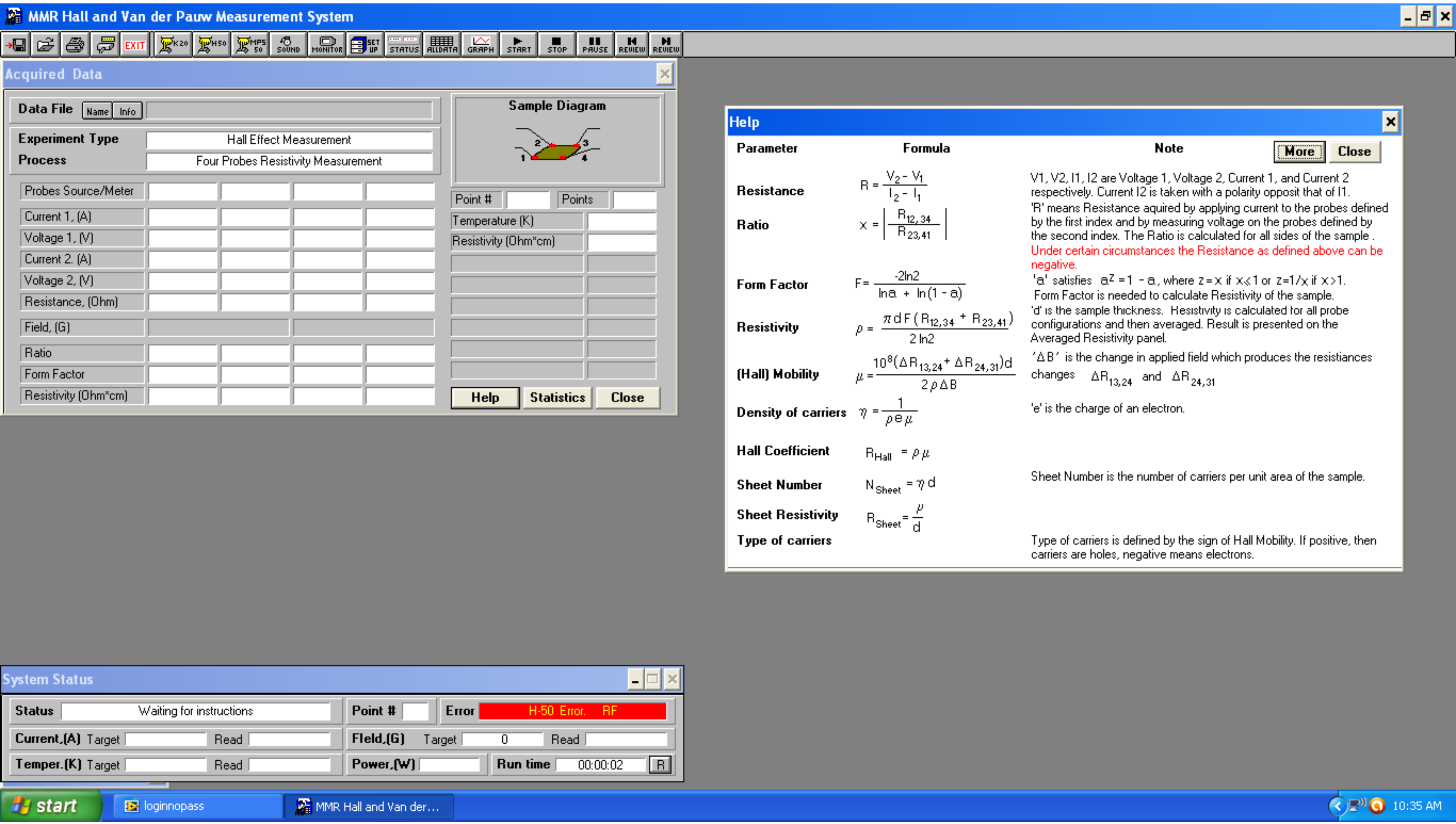
- The contacts are numbered from 1 to 4 in a counter-clockwise order, beginning at the top-left contact.

- The current I_{12} is a positive DC current injected into contact 1 and taken out of contact 2

- The voltage V_{34} is a DC voltage measured between contacts 3 and 4 with no externally applied magnetic field
- The resistivity ρ is measured in ohms·meters ($\Omega \cdot m$).

- The thickness of the sample t is measured in meters (m).
- The sheet resistance R_S is measured in ohms (Ω).

- http://en.wikipedia.org/wiki/Van_der_Pauw_method



$$R = \frac{V_2 - V_1}{I_2 - I_1}$$

$$x = \left| \frac{R_{12,34}}{R_{23,41}} \right|$$

$$F = \frac{-2\ln 2}{\ln a + \ln(1-a)}$$

$$\rho = \frac{\pi d F (R_{12,34} + R_{23,41})}{2 \ln 2}$$

$$\mu = \frac{10^8(\Delta R_{13,24} + \Delta R_{24,31}) d}{2 \rho \Delta B}$$

$$\eta = \frac{1}{\rho e \mu}$$

$$R_{Hall} = \rho \mu$$

$$N_{Sheet} = \eta d$$

$$R_{Sheet} = \frac{\rho}{d}$$

Help

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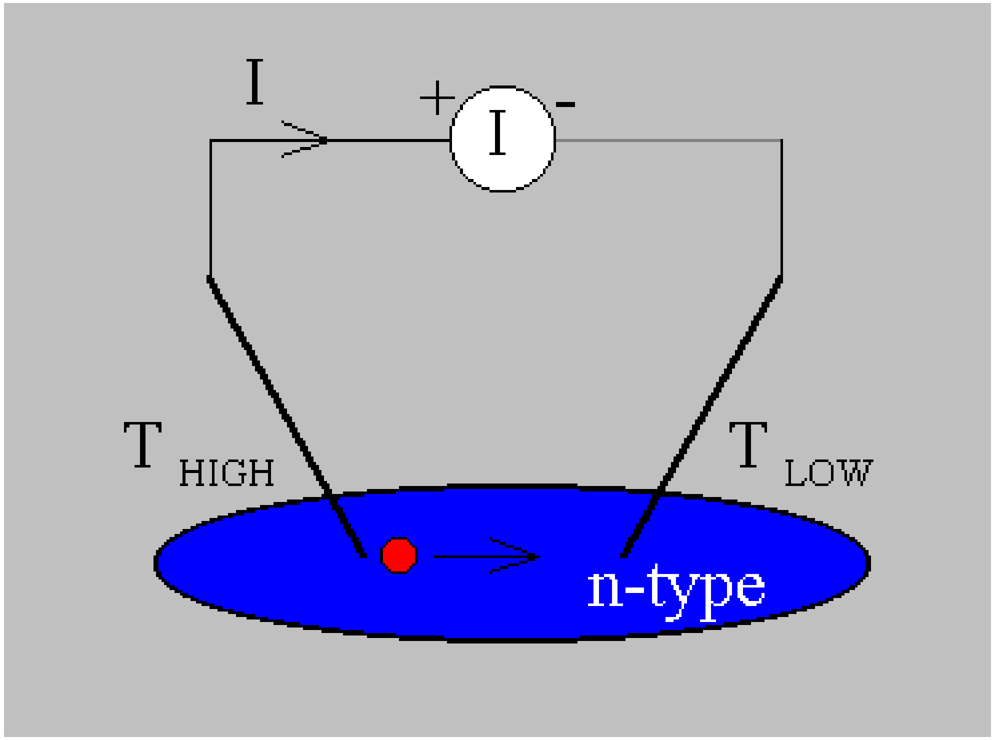
Parameter	Formula	Note
Resistance	$R = \dfrac{V_2 - V_1}{I_2 - I_1}$	V1, V2, I1, I2 are Voltage 1, Voltage 2, Current 1, and Current 2 respectively. Current I2 is taken with a polarity opposit that of I1.
Ratio	$x = \left\lvert \dfrac{R_{12,34}}{R_{23,41}} \right\rvert$	'R' means Resistance aquired by applying current to the probes defined by the first index and by measuring voltage on the probes defined by the second index. The Ratio is calculated for all sides of the sample . Under certain circumstances the Resistance as defined above can be negative.
Form Factor	$F = \dfrac{-2\ln 2}{\ln a + \ln(1-a)}$	'a' satisfies $a^z = 1 - a$, where $z = x$ if $x < 1$ or $z = 1/x$ if $x > 1$. Form Factor is needed to calculate Resistivity of the sample.
Resistivity	$\rho = \dfrac{\pi d F (R_{12,34} + R_{23,41})}{2 \ln 2}$	'd' is the sample thickness. Resistivity is calculated for all probe configurations and then averaged. Result is presented on the Averaged Resistivity panel.
(Hall) Mobility	$\mu = \dfrac{10^8(\Delta R_{13,24} + \Delta R_{24,31})d}{2 \rho \Delta B}$	'ΔB' is the change in applied field which produces the resistiances changes $\Delta R_{13,24}$ and $\Delta R_{24,31}$
Density of carriers	$\eta = \dfrac{1}{\rho e \mu}$	'e' is the charge of an electron.
Hall Coefficient	$R_{Hall} = \rho \mu$	
Sheet Number	$N_{Sheet} = \eta d$	Sheet Number is the number of carriers per unit area of the sample.
Sheet Resistivity	$R_{Sheet} = \dfrac{\rho}{d}$	
Type of carriers		Type of carriers is defined by the sign of Hall Mobility. If positive, then carriers are holes, negative means electrons.

Hot Probe Test to Determine Carrier Type

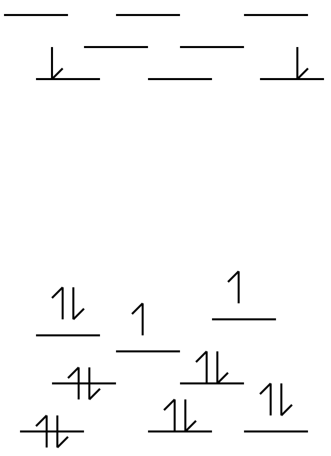
Seebeck effect: All you need is a soldering iron, and an ammeter!



http://ecee.colorado.edu/~bart/book/hotprobe.htm

Hot Probe Test to determine Carrier Type

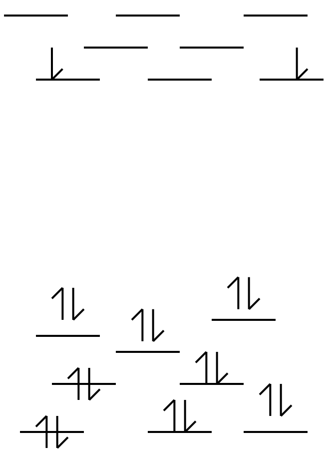
Intrinsic



$p = n = n_i$

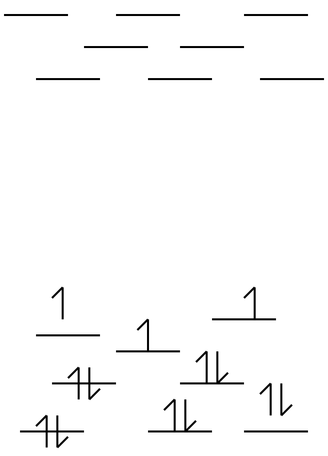
Number of thermally generated Holes equals number thermally generated free electrons

n-type



Number of free electrons equals number of positively charged donor ions

p-type

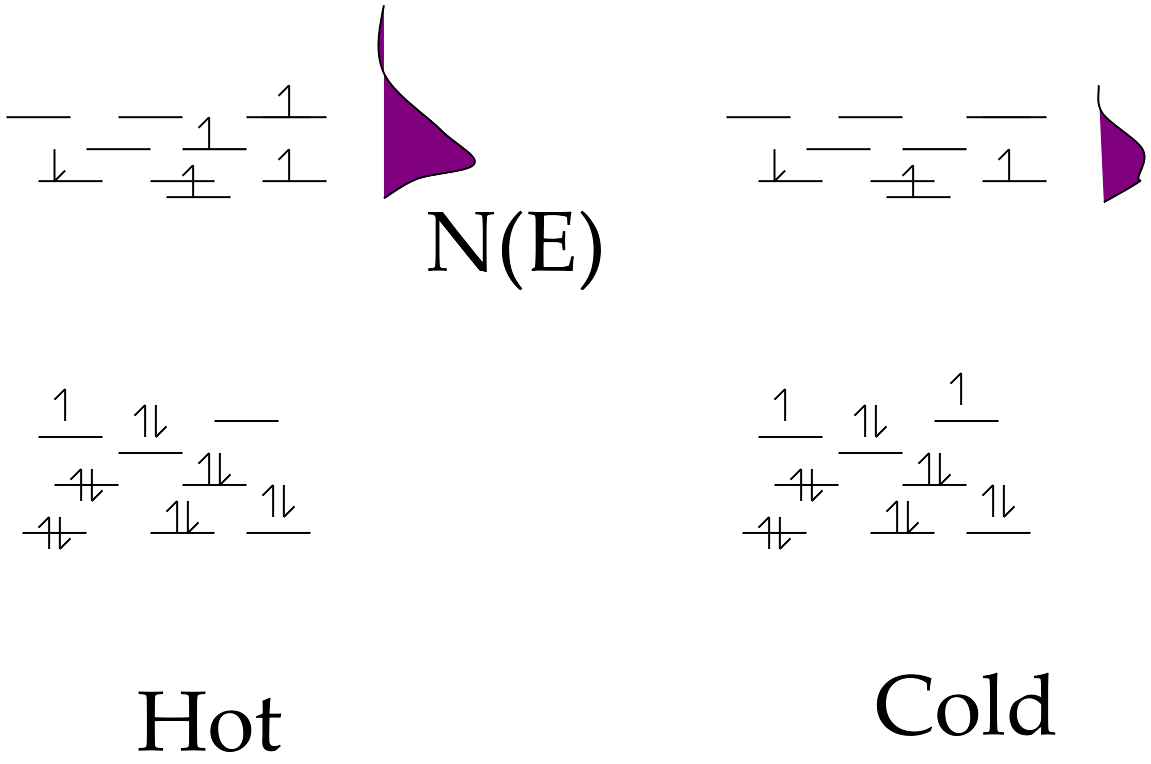


Number of free holes equals number of Negatively charged acceptor cores

After Hamers

Hot Probe Test to determine Carrier Type

Distribution of OCCUPIED C.B. levels:

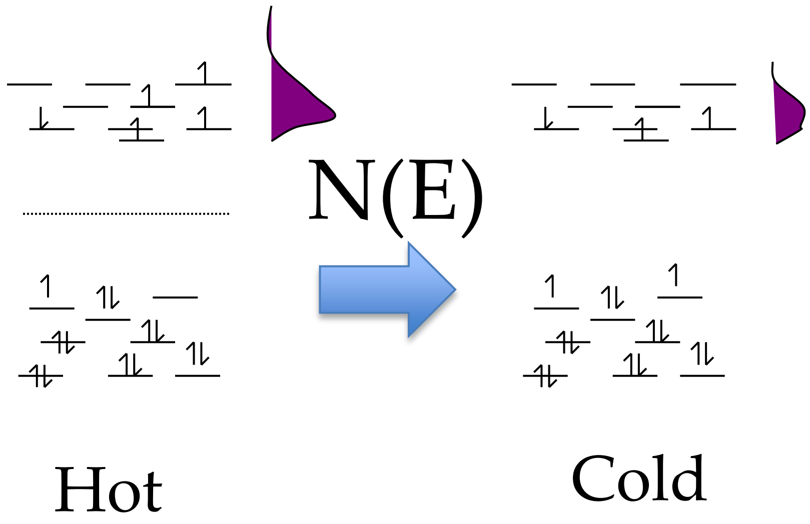


These are *not* in equilibrium!

After Hamers

Hot Probe Test to determine Carrier Type

Seebeck effect, n-type semiconductor



Fick's Law of Diffusion:

$$J = -D\frac{\partial c}{\partial x}$$

Electrons diffuse from region of high
Concentration to region of
lower concentration

"Cold" side becomes slightly
negatively charged
Hot side becomes positively charged

After Hamers