

Absorption coefficients of semiconductor thin films

October 21, 2014

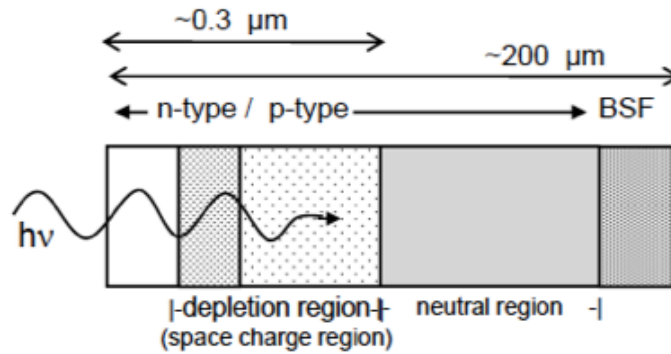
PHYS 4580, PHYS 6/7280

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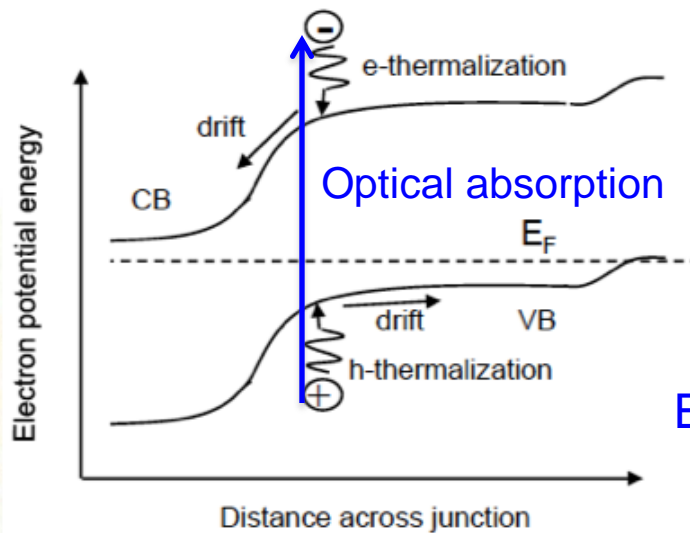
T.A. Neale O. Haugen

Typical Si PV Cell Structure



Total Device Thickness:
 $\sim 200 \mu\text{m}$

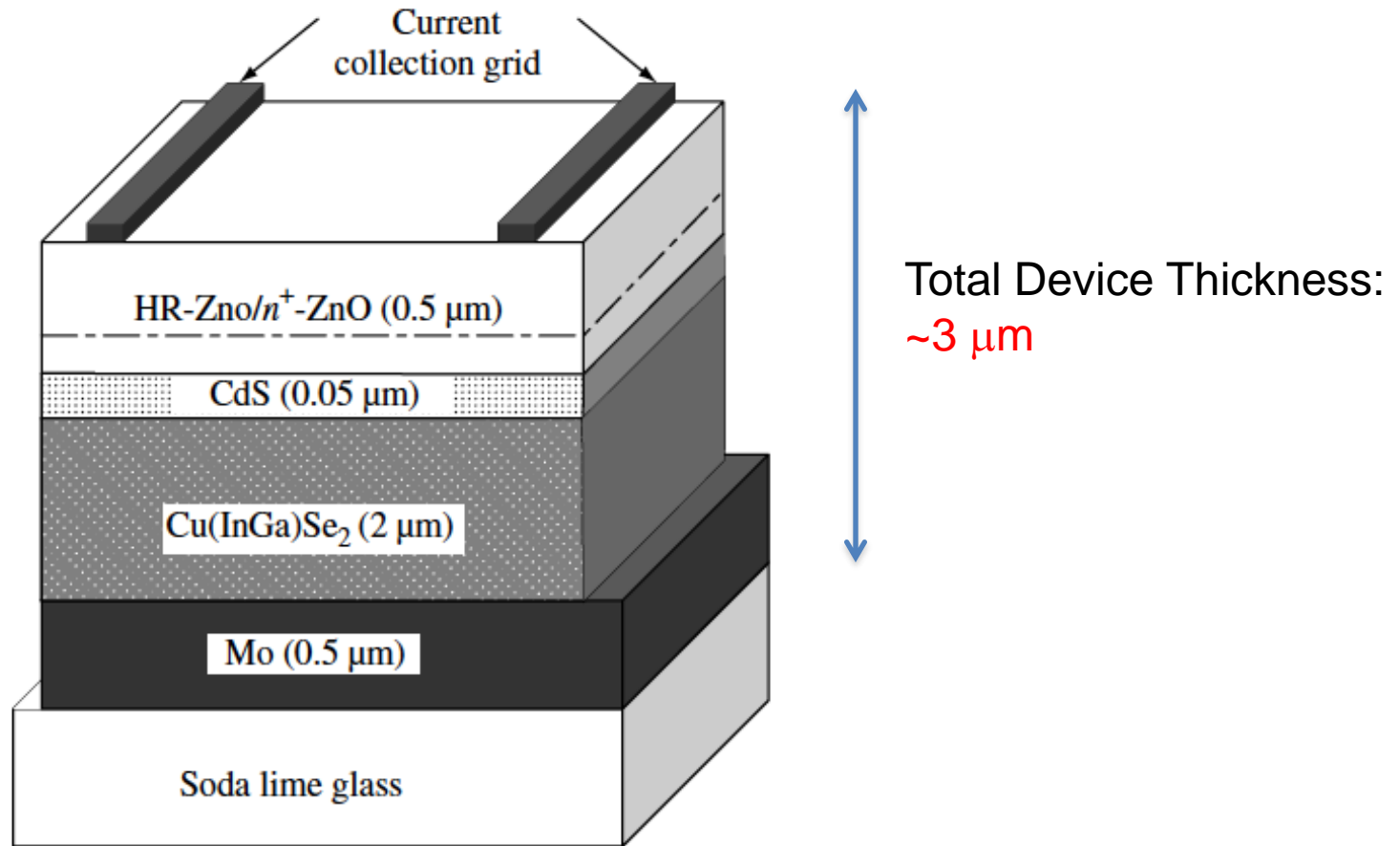
Real Space Schematic



Energy Band Diagram

Solar cell structure and energy band diagram showing valence (VB) and conduction bands (CB), Fermi level (E_F), photoabsorption, electron-hole pair generation, thermalization, and drift.
(from Compaan, APS News April, 2005)

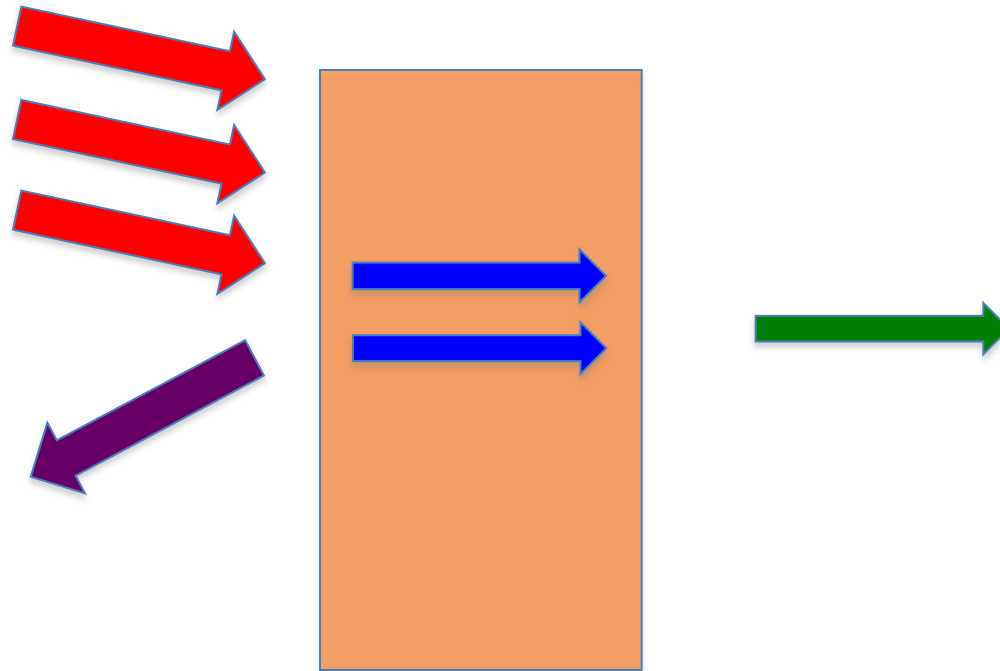
Typical Thin Film (CIGS) Cell Structure



Schematic cross section of a typical Cu(InGa)Se₂ solar cell

Conservation of Energy for each wavelength

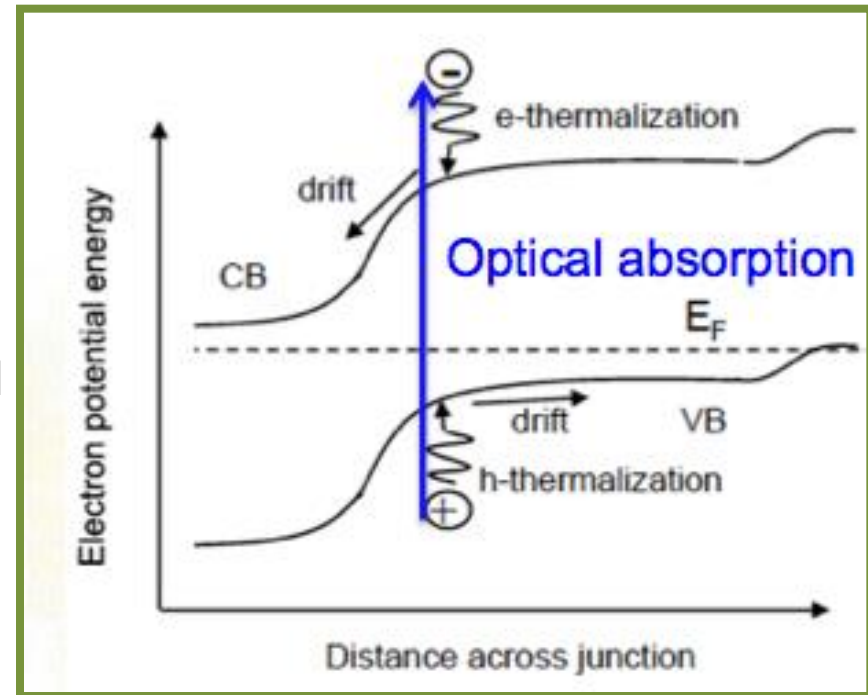
$$\text{Total Incident } (\lambda) = A(\lambda) + T(\lambda) + R(\lambda)$$



Why must some PV layers be so thick?
(want to maximize absorption)

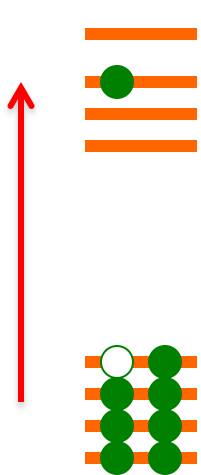
Mechanisms of Optical Absorption

- Energy is absorbed by a “system”, potential energy of system is increased.
- Consider, e.g., gravitational potential energy → energy is required to lift a book onto a high shelf.
- For semiconductors, optical absorption occurs any time a “carrier” can be excited to a higher “state”.
- Typically, the carriers we are concerned with are electrons, and the excitation is across the band gap.



Absorption Coefficient

(connects the optical and electronic properties of materials)



$$\alpha(\lambda) \propto n_i(\lambda) P_{if}(\lambda) n_f(\lambda)$$

$\alpha(\lambda)$ - the absorption coefficient

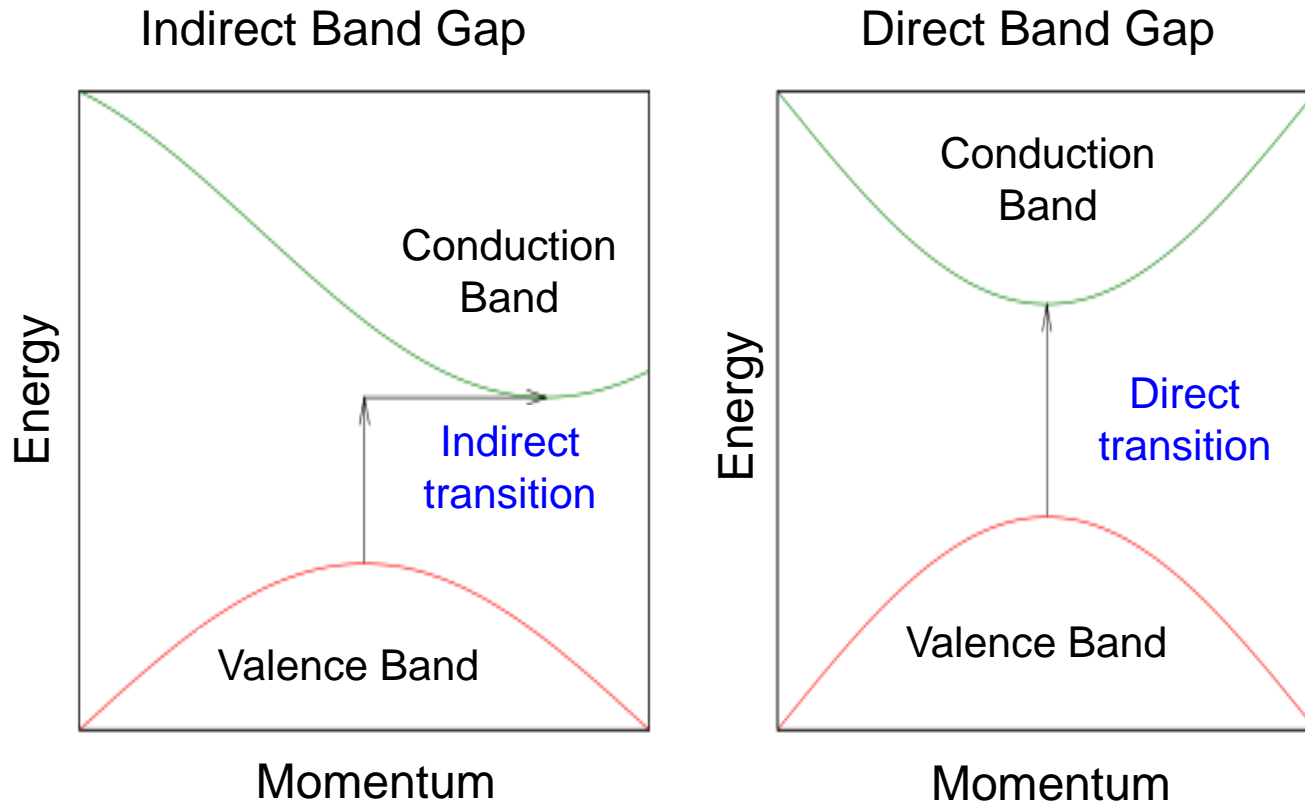
$n_i(\lambda)$ - density of carriers in an initial state

$n_f(\lambda)$ - density of unoccupied states

$P_{if}(\lambda)$ - probability that transition will take place

In addition to “band gap excitation”, another important absorption process is “free carrier absorption”, which is typically seen in metals and TCOs. Free carrier absorption refers to an *intra*band absorption in which (e.g.) electrons absorb low energy photons and are promoted to a higher energy level within the conduction band.

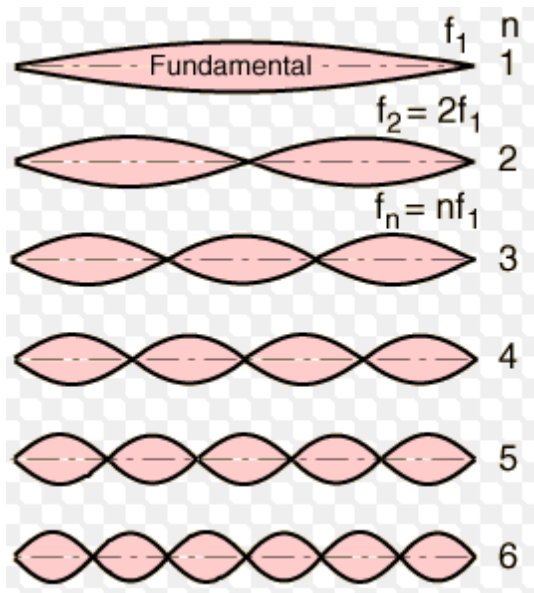
Types of Semiconductor Band Gaps



- A direct transition requires just a photon of sufficient energy
- An indirect transition requires both a photon *and* a phonon --
("two body event" is lower probability!)
- Energy *and* Momentum must be conserved.

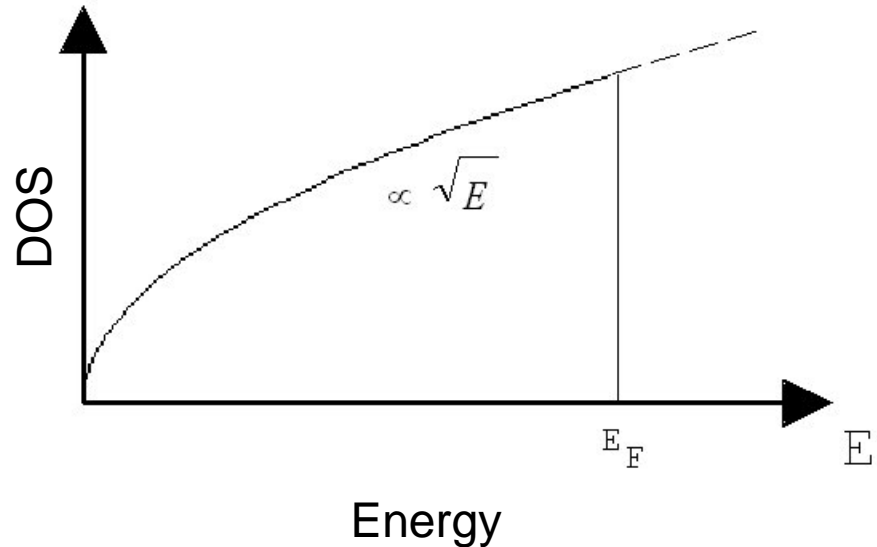
Density of States in 3D Crystal

- “States” – allowed solutions to the Shrodinger Equation
- Analogy to waves on a string, but in 3 dimensions

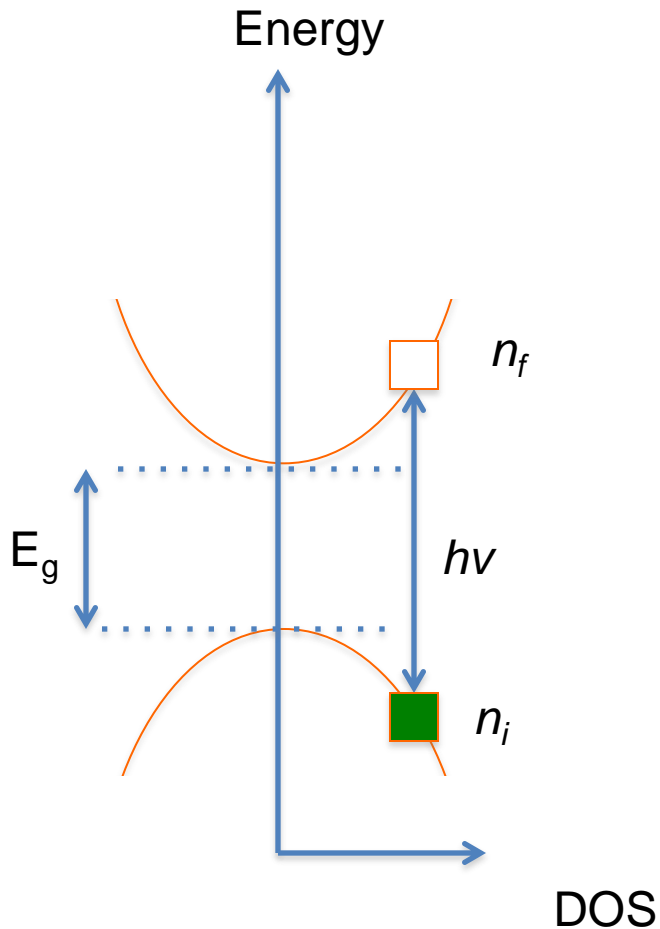


Density of States

Units of: # per energy-volume



How does α depend on photon energy?



$$\alpha(h\nu) \propto n_i(h\nu) P_{if}(h\nu) n_f(h\nu)$$

$$DOS \propto E^{1/2}$$

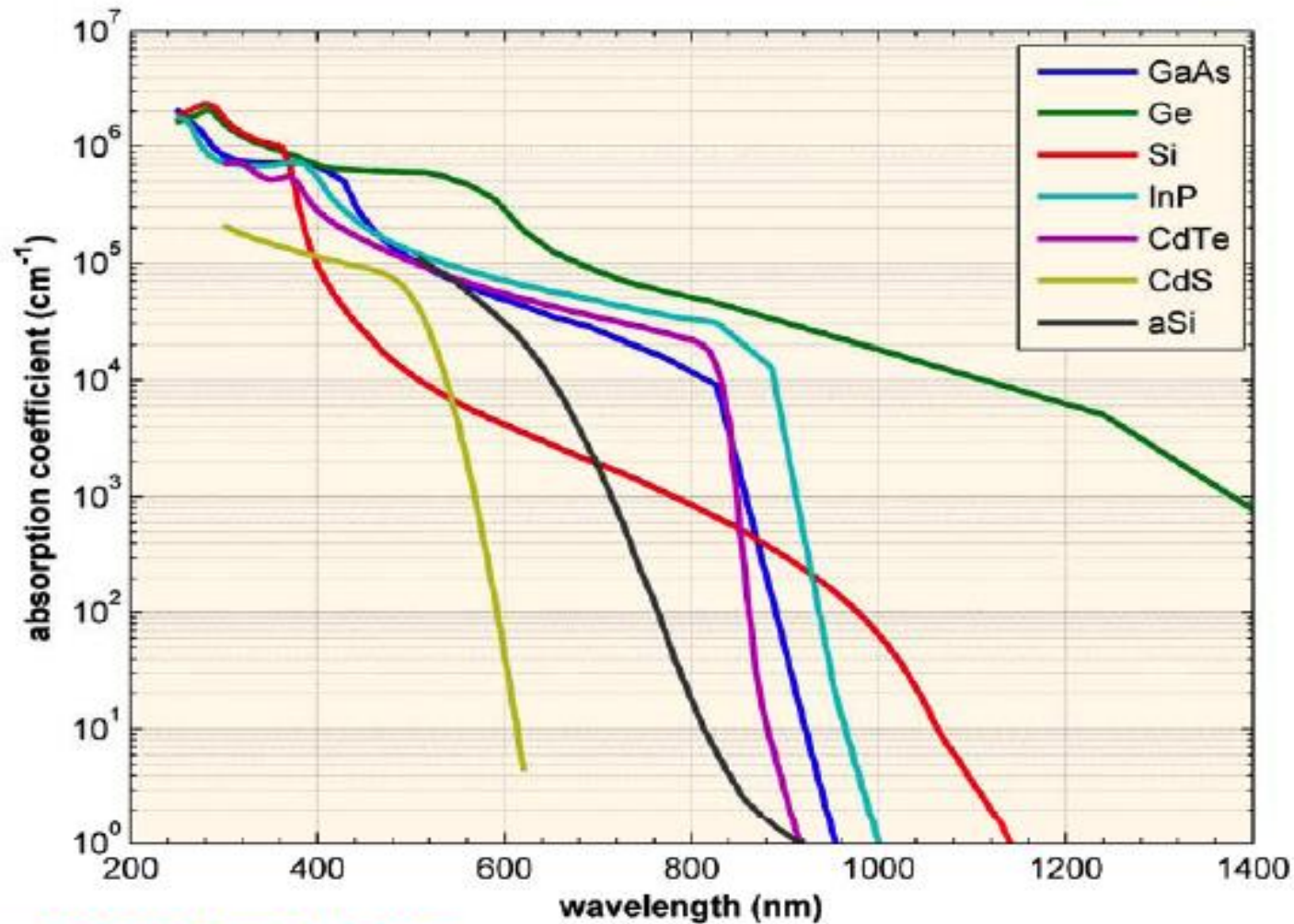
$$\alpha(h\nu) \propto E_i^{1/2} P_{if} E_f^{1/2}$$

$$\alpha(h\nu) \propto \Delta E^{1/2}$$

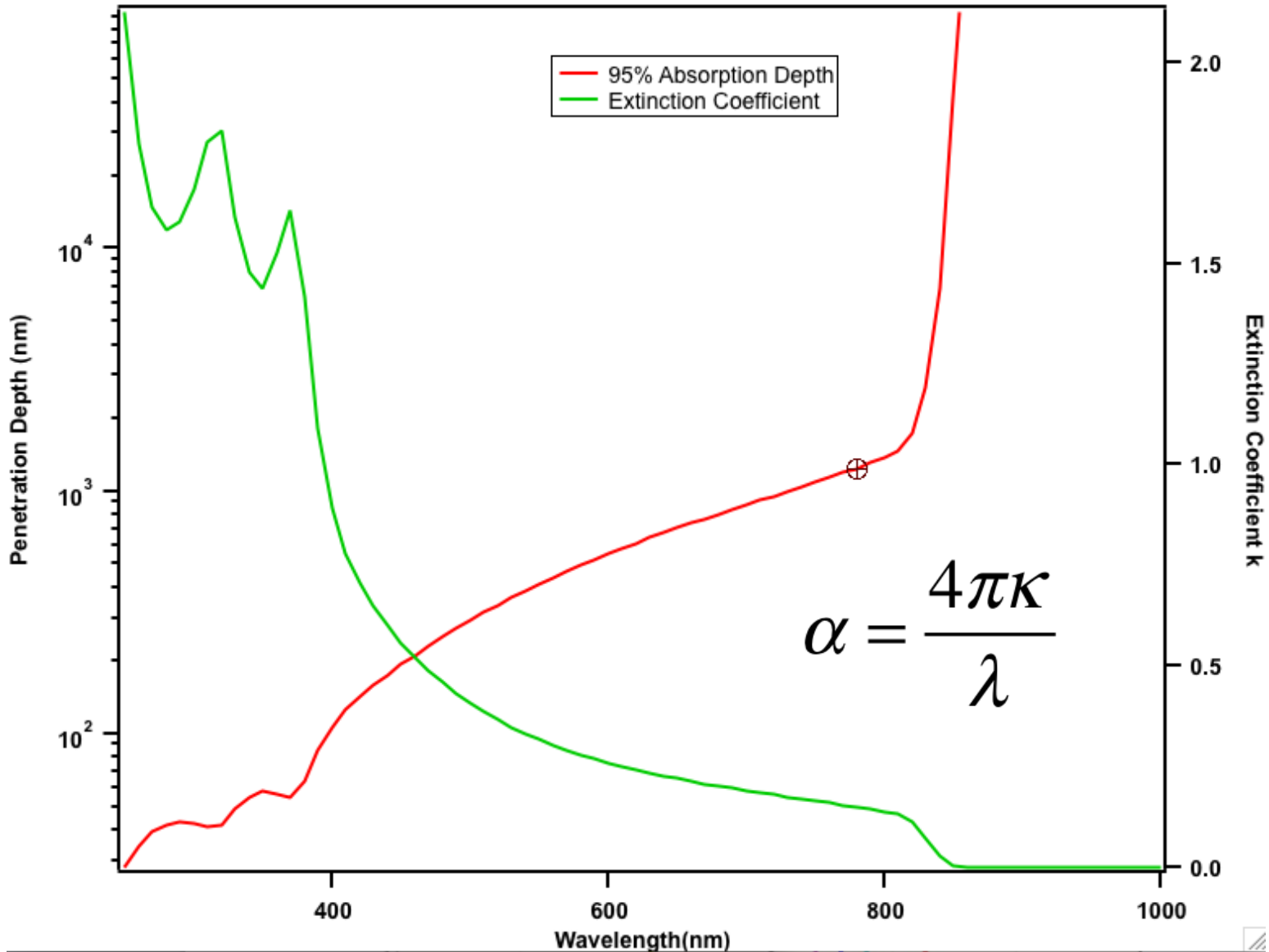
$$\Delta E = h\nu - E_g$$

$$\alpha(h\nu) \propto (h\nu - E_g)^{1/2}$$

Absorption Coefficients for some Semiconductors

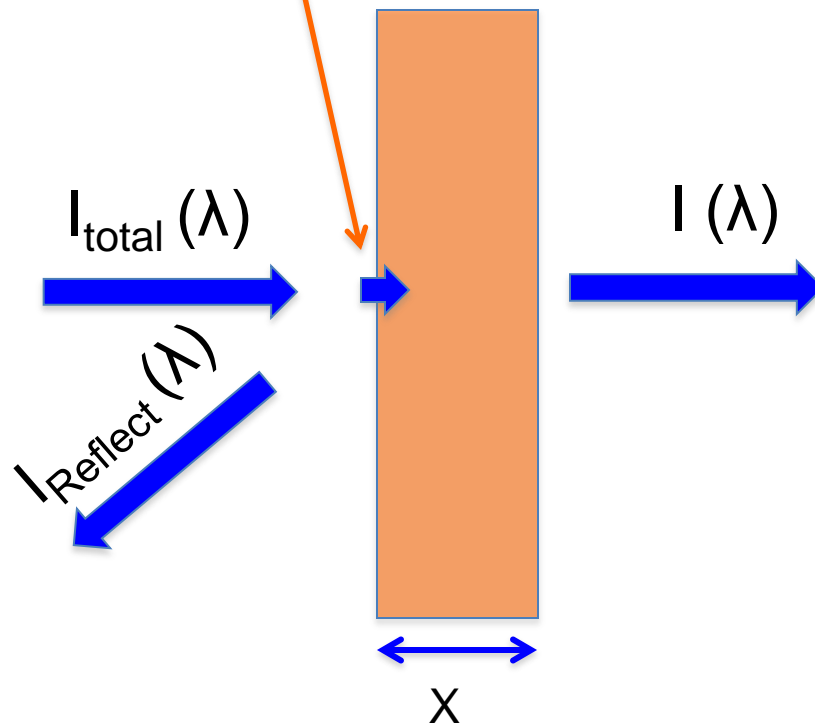


95% Absorption Depth (CdTe)



Beer-Lambert Law

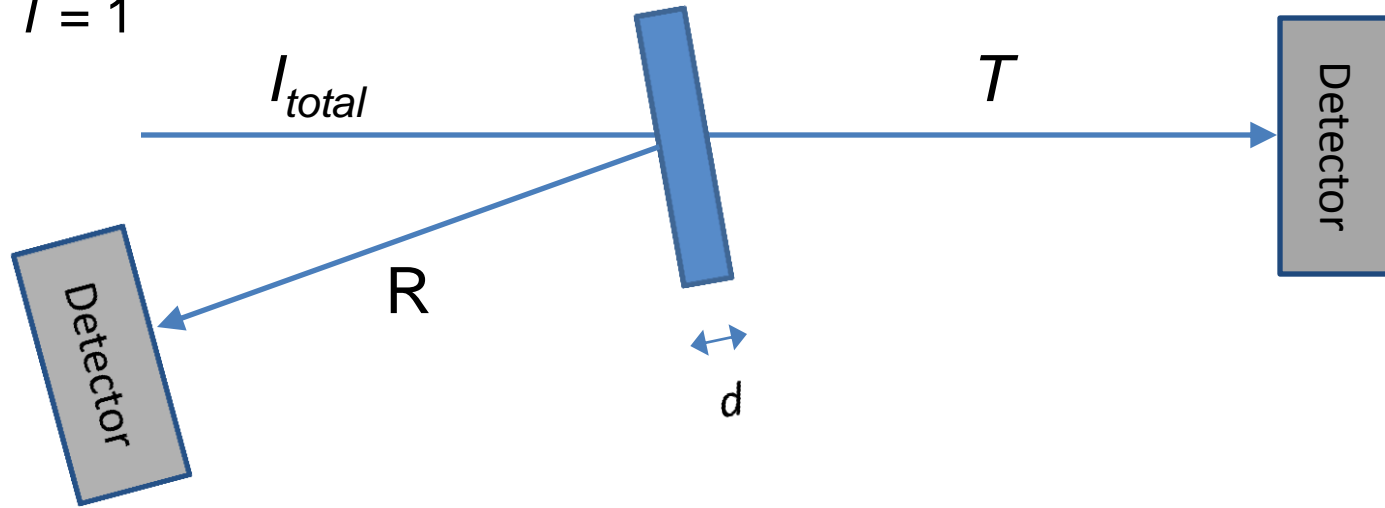
$$I(\lambda) = I_0(\lambda)e^{-\alpha(\lambda)x} \leftrightarrow I(E) = I_0(E)e^{-\alpha(E)x}$$



$$\alpha = -\frac{1}{x} \ln \left(\frac{I}{I_0} \right)$$

Calculation of the absorption coefficient, $\alpha(\lambda)$, for a thin film

$$A + R + T = 1$$



$$T = \frac{I_t}{I_{total}}$$

$$I_t = I_0 e^{-\alpha(\lambda)d}$$

$$I_0 = I_{total}(1 - R)$$

$$I_t = I_{total}(1 - R)e^{-\alpha(\lambda)d}$$

$$\text{Since } I_t = (T)(I_{total}) \rightarrow (T)(I_{total}) = (1 - R)(I_{total})e^{-\alpha(\lambda)d}$$

$$e^{-\alpha(\lambda)d} = \frac{T}{1 - R} \Rightarrow \alpha(\lambda)d = -\ln\left(\frac{T}{1 - R}\right)$$

$$\alpha(\lambda) = \frac{-\ln\left(\frac{T(\lambda)}{1 - R(\lambda)}\right)}{d}$$

Measuring the bandgap energy (optical absorption)

$$I(\lambda) = I_0(\lambda)e^{-\alpha(\lambda)x} \longrightarrow I(E) = I_0(E)e^{-\alpha(E)x}$$

Direct-gap semiconductor

$$\alpha(E) = \alpha_0(E - E_g)^{\frac{1}{2}}$$

Indirect-gap semiconductor

$$\alpha(E) \propto (E - E_g)^2$$

Crystal	Gap	E_{gn} eV		Crystal	Gap	E_{gn} eV	
		0 K	300 K			0 K	300 K
Diamond	i	5.4	-	HgTe ^a	d	-0.30	-
Si	i	1.17	1.14	PbS	d	0.286	0.34-0.37
Ge	i	0.744	0.67	PbSe	d	0.165	0.27
α Sn	d	0.00	0.00	PbTe	d	0.190	0.30
InSb	d	0.24	0.18	CdS	d	2.582	2.42
InAs	d	0.43	0.35	CdSe	d	1.840	1.74
InP	d	1.42	1.35	CdTe	d	1.607	1.45
GaP	i	2.32	2.26	ZnO		3.436	3.2
GaAs	d	1.52	1.43	ZnS		3.91	3.6
GaSb	d	0.81	0.78	SnTe	d	0.3	0.18
AlSb	i	1.65	1.52	AgCl		-	3.2
SiC(hex)		3.0	-	AgI		-	2.8
Te	d	0.33	-	Cu ₂ O		2.172	-
ZnSb		0.56	0.56	TiO ₂		3.03	-

Indirect gap: plotting α vs E shows an E^2 dependence, so plotting $\alpha^{1/2}$ shows a linear dependence.

Direct gap: Plotting α vs E shows an $E^{1/2}$ dependence, so plotting α^2 shows a linear dependence.

^aHgTe is a semimetal; the bands overlap.

General references: D. Long, Energy bands in semiconductors. Interscience, 1968; also the A.I.P. Handbook, 3rd ed., Sec. 9.

Measuring the bandgap energy (optical absorption)

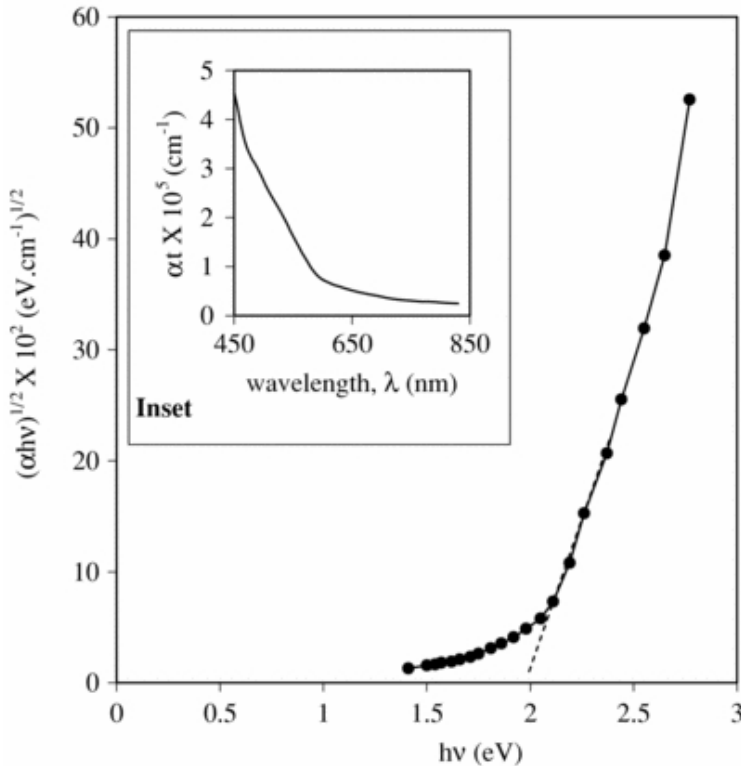
$$I(\lambda) = I_0(\lambda)e^{-\alpha(\lambda)x} \quad \longrightarrow \quad I(E) = I_0(E)e^{-\alpha(E)x}$$

Direct-gap semiconductor

$$\alpha(E) = \alpha_0(E - E_g)^{\frac{1}{2}}$$

Indirect-gap semiconductor

$$\alpha(E) \propto (E - E_g)^2$$



Fe_2O_3 , (haematite) –
direct or indirect gap?

if a plot of $h\nu$ versus $\alpha^{1/2}$ forms
a straight line, \rightarrow indirect gap
material

Semicond. Sci. Technol. **20** No 8 (August 2005) 705-709

doi:10.1088/0268-1242/20/8/009

**Nanocrystalline haematite thin films by chemical solution
spray**

J D Desai, H M Pathan, Sun-Ki Min, Kwang-Deog Jung
and Oh-Shim Joo

Measuring the bandgap of a thin film (optically)

<http://www.chalcogen.infim.ro/Patidar-CdSe-Final.pdf>

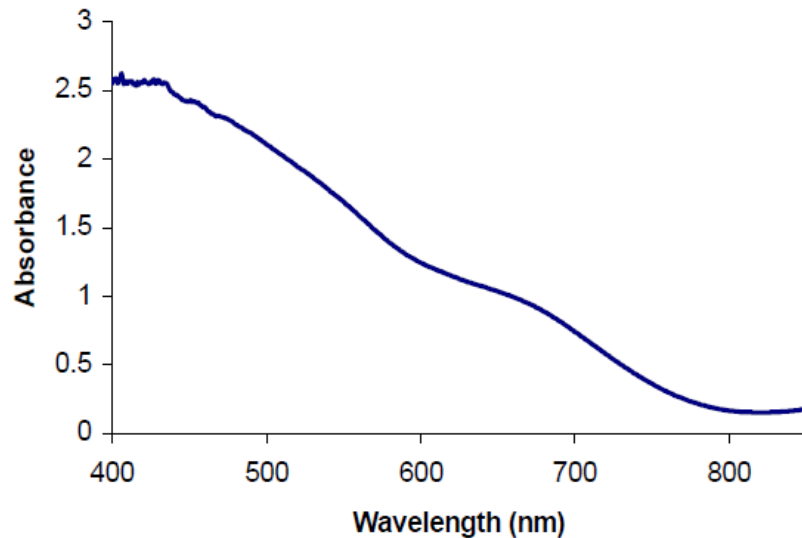


Fig 2. The absorption spectra of CdSe thin film.

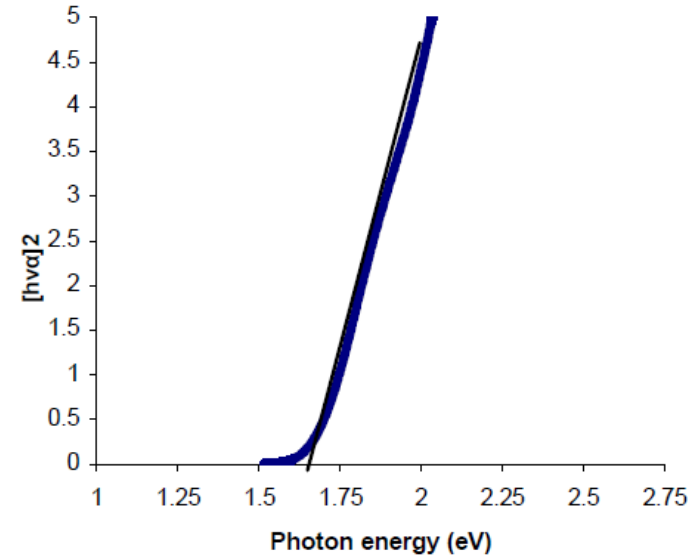


Fig.3. Energy band gap determination of CdSe thin film.

Therefore, if a plot of $h\nu$ versus α^2 forms a straight line, it can normally be inferred that there is a direct band gap, measurable by extrapolating the straight line to the $\alpha = 0$ axis. On the other hand, if a plot of $h\nu$ versus $\alpha^{1/2}$ forms a straight line, it can normally be inferred that there is an indirect band gap, measurable by extrapolating the straight line to the $\alpha = 0$ axis.

From http://en.wikipedia.org/wiki/Direct_and_indirect_band_gaps

	Energy gap		Lowest conduction-band minimum, direct or indirect	$\left(\frac{dE_g}{dT}\right) \times 10^4$ (300°K) eV/°K	$\left(\frac{dE_g}{dP}\right)_T \times 10^6$ (300°K) eV/bar	Effective mass		Refractive index n	Static dielectric constant ϵ	Lattice constant a Å	Mobility			
	E_g (0°K) eV	E_g (300°K) eV				m_l^*	m_h^*				μ_e cm ² / V·sec	μ_h cm ² / V·sec		
IV	Si	1.166	1.11	ind 100	-2.3	-1.5	m_l 0.98 m_t 0.19	0.52	3.44	11.7	5.43	1,350	480	
	Ge	0.74	0.67	ind 111	-3.7	5.0	m_l 1.58 m_t 0.08	0.3	4.00	16.3	5.66	3,900	1,900	
IV-IV	SiC	-0.2†		dir 000		5.0					6.489	2,000	1,000	
		3.0 (6H)	2.8-3.2†	ind	-3.3				2.69 // c 2.65 ⊥ c	10.2	a 3.0817 c 15.1123	400		
VI	Se	1.95	1.74	dir 0001	-14	-20		0.12	5.56 // c 3.72 ⊥ c	8.5			1	
		Te	0.334	0.32	dir 0001	-0.3	-19		0.038 ⊥	0.26 // 0.10 ⊥	3.07 // c 2.68 ⊥ c	5.0 // c 2.2 ⊥ c		1,100
III-V	BP		2	ind					2.6	6.9	4.538			
	AlP	2.5	2.43	ind 100	-3.5			0.13†		3.0	9.8	5.462	80	
	AlAs	2.24	2.16	ind				0.5	m_l 1.06 m_t 0.49		12	5.66	1,000	~100
	AlSb	1.6	1.6	ind 100	-4	-1.6		0.11	0.39	3.4	11	6.135	50	400
	GaN	3.5	3.4	dir 0000	-4.8	4.2		0.2	0.8	2.4	12	a 3.18 c 5.16	300	
	GaP	2.4	2.25	ind 100	-5.4	-1.7		0.13	0.678	3.37	10	5.450	120	120
	GaAs	1.520	1.43	dir 000	-5.0	11		0.07	0.5	3.4	12	5.653	8,600	400
	GaSb	0.81	0.69	dir 000	-4.1	12		0.045	0.39	3.9	15	6.095	4,000	650
	InP	1.42	1.28	dir 000	-4.6	4.6		0.07	0.40	3.37	12.1	5.8687	4,000	650
	InAs	0.43	0.36	dir 000	-3.3	3		0.028	0.33	3.42	12.5	6.058	30,000	240
	InSb	0.235	0.17	dir 000	-2.9	15		0.0133	0.18	3.75	18	6.4787	76,000	5,000 (78°K)
	ZnO		3.2	dir 0000	-9.5	0.6		0.32	0.27	2.02	7.9	a 3.2496 c 5.2065	180	
	ZnS		3.8	dir 0000	-3.8	9		0.28	>1 // 0.5 ⊥	2.4	8.3	a 3.814 c 6.257		
	ZnSe	2.80	2.58	dir 000	-5.3	5.7		0.39		2.4	8.3	5.406		
ZnTe	2.39	2.28	dir 000	-7.2	6		0.17		2.89	8.1	5.667	100		
II-VI	CdS	2.58	2.53	dir 0000	-5	3.3		0.15		3.56	9.7	6.101		7
	CdSe	1.85	1.74	dir 0000	-4.6			0.20	0.7 ⊥ c 5 // c	2.5	8.9	a 4.136 c 6.713	210	
	CdTe	1.60	1.50	dir 000	-4.1	1.5		0.11	2.5 // † 0.4 ⊥		10.6	a 4.299 c 7.010	500	
	HgS		2.5										600	
IV-VI	HgSe	-0.24	-0.15	dir 000				0.045		25	6.085	5,500		
	HgTe	-0.28	-0.15 0.14	dir 000	+5.6			0.029	~0.3	3.7	20	6.42	22,000 (20°K)	100
	PbS	0.29	0.37	dir 111	+4	-7		0.1	0.1	3.7	170	5.936	550	600
	PbSe	0.15	0.26	dir 111	+4	-8		m_l 0.07 m_t 0.039	m_l 0.06 m_t 0.03		250	6.124	1,020	930
IV-VI	PbTe	0.19	0.29	dir 111	+4	-9		m_l 0.24 m_t 0.02	m_l 0.3 m_t 0.02	3.8	412	6.460	1,620	750
	SnTe	0.3	0.18	dir 111							6.328			