

## Lab #6

# Measuring the Spectral Response of a PV Cell

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*November 5, 2013*

*PHYS 4580, 6/7280*

# Pop Quiz

Note: quiz does not count toward grade...

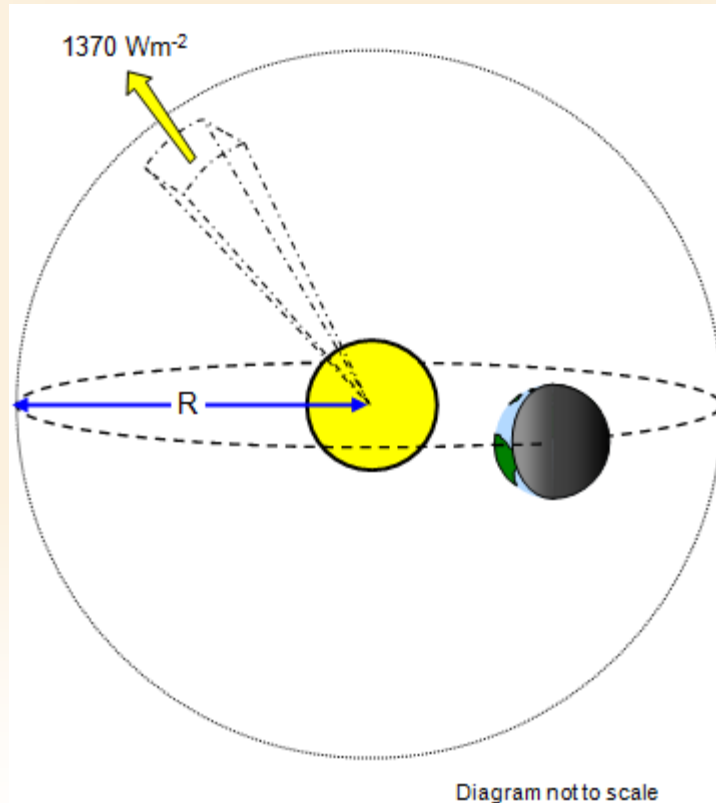
In which spectral region do *most* solar cells work best, and why?

1. Visible
2. Ultraviolet
3. Near-infrared
4. Infrared

# Solar Constant

Luminosity (power) of the Sun:  $3.839 \times 10^{26}$  W

Energy radiates outward and at  $r = 1$  A.U. is spread evenly over an area  $4\pi r^2$ . In this way, one can calculate the intensity of sunlight at Earth.



# What is AM1.5?

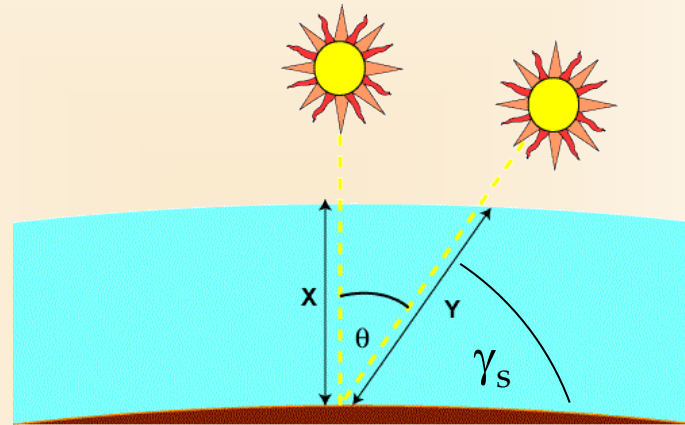
## Air Mass – example (Toledo, Ohio)

Today (January 24):

From sunposition.info, at 12:15 pm today, the sun was at a zenith angle of  $\gamma_s = 33^\circ$  above the horizon.

$$AM = \frac{1}{\cos \theta}$$

$$AM = \csc(\gamma_s) = \frac{1}{\sin(\gamma_s)}$$

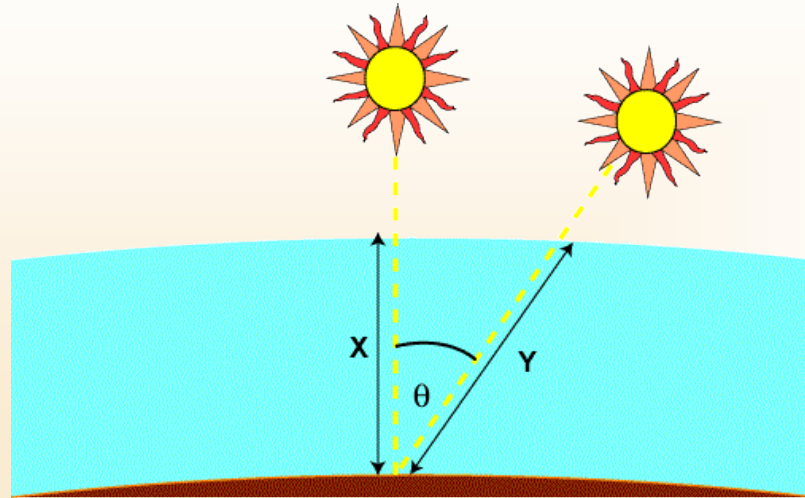


Therefore,  $AM \approx 1.84$  (at 12:15 pm, assuming a clear sky). How about June 21, at noon? In that case,  $\gamma_s = 63^\circ$ , so that  $AM \approx 1.12$ .

## Air Mass (continued)

$$AM = \frac{1}{\cos \theta}$$

However,  $AM \neq \infty$  when  $\theta = 90^\circ$



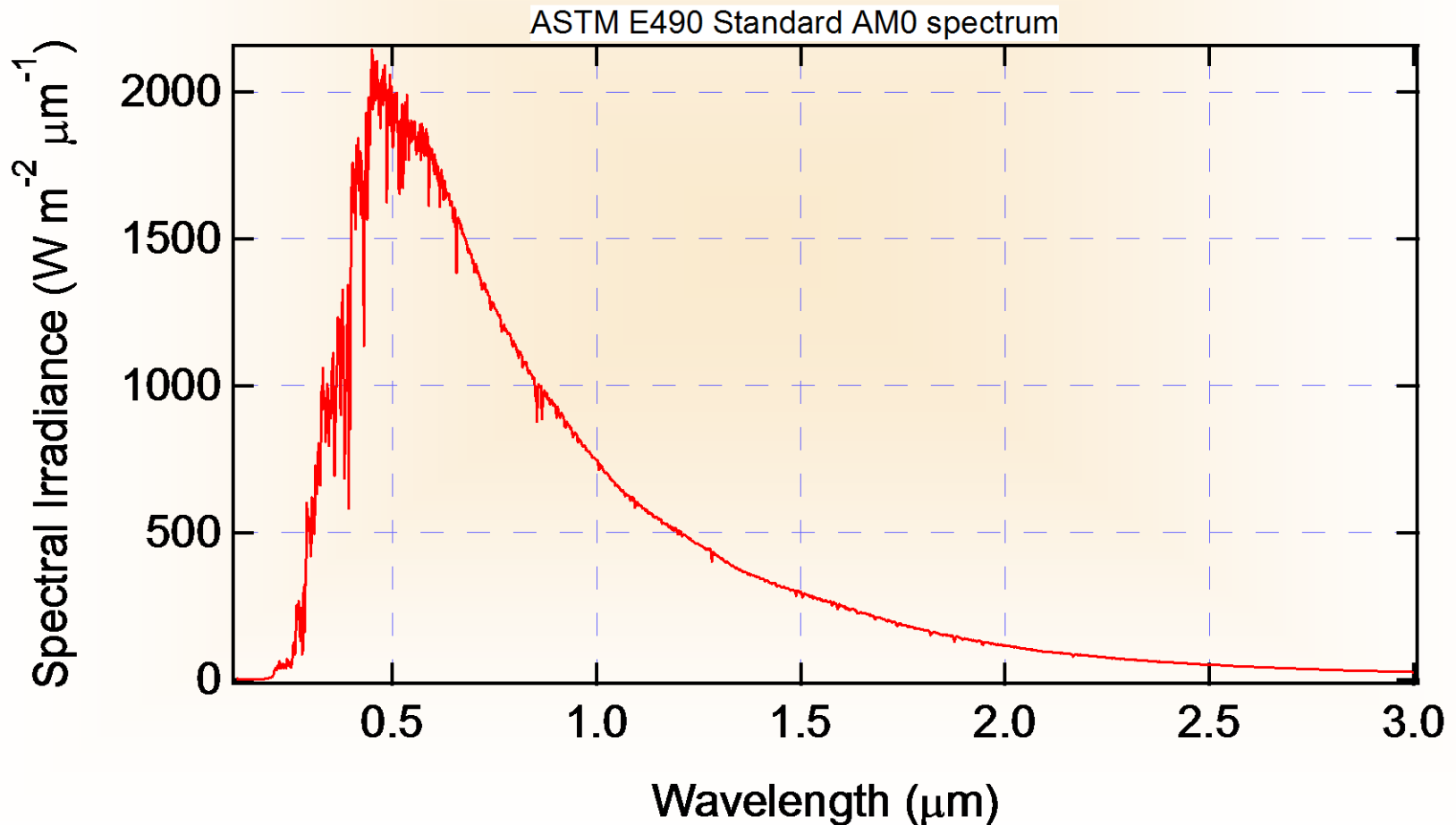
A more accurate equation for AM:

$$AM = \frac{1}{\cos \theta + 0.5057(96.07995 - \theta)^{-1.6364}}$$

## AM0: the spectrum above Earth's atmosphere

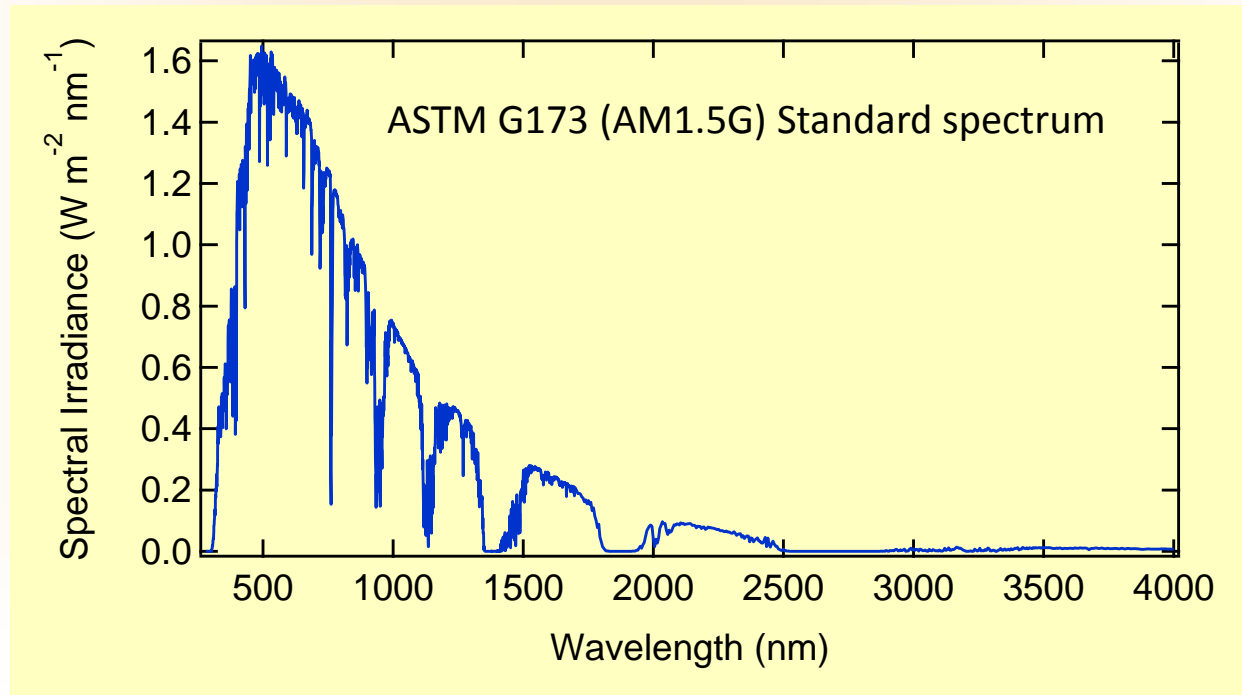
The AM0 spectrum applies to satellites and high-flying aircraft, which access the spectrum prior to any influence from Earth's atmosphere.

Integrated spectral irradiance = 1366 W/m<sup>2</sup>.



# AM1.5G: reference spectrum including direct and diffuse sunlight

- AM 1.5: From the equation provided for Air Mass, one calculates that  $\cos \theta = 0.667$ , so that  $\theta = 48.2^\circ$ . This represents the zenith angle, that is, the angle relative to the direction normal to Earth's surface.
- From the standard: "The receiving surface is defined in the standards as an inclined plane at  $37^\circ$  tilt toward the equator, facing the sun (i.e., the surface normal points to the sun, at an elevation of  $41.81^\circ$  above the horizon)." Note that  $41.8^\circ$  is the complement of  $48.2^\circ$ .
- Toledo latitude:  $41.6639^\circ$  N
- Integrating the energy within AM1.5G yields  $1000 \text{ W m}^{-2}$ .



# Standard Solar Reference Spectra

Where do these spectra come from (where can we get them)?

Start here: <http://rredc.nrel.gov/solar/spectra/>

The spectra most often referenced are the AM1.5G (technically referred to as the **ASTM G-173**) and the AM0 (technically known as the **ASTM E-490**).

Here's a link to versions of AM1.5G and AM0 spectra that are already adjusted for 1 nm spacing between data points (see January 24<sup>th</sup>):

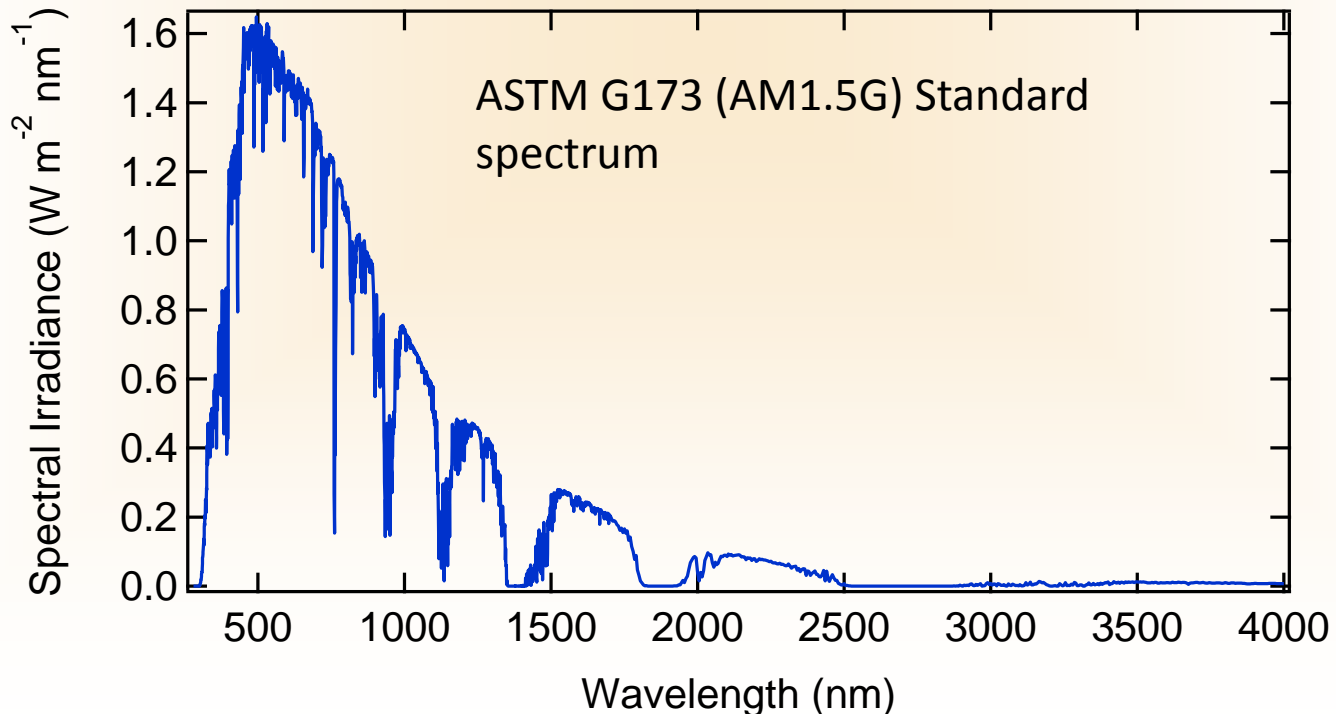
[http://astro1.panet.utoledo.edu/~relling2/teach/archives/4400.2013/spring2013\\_phys4400.html](http://astro1.panet.utoledo.edu/~relling2/teach/archives/4400.2013/spring2013_phys4400.html)



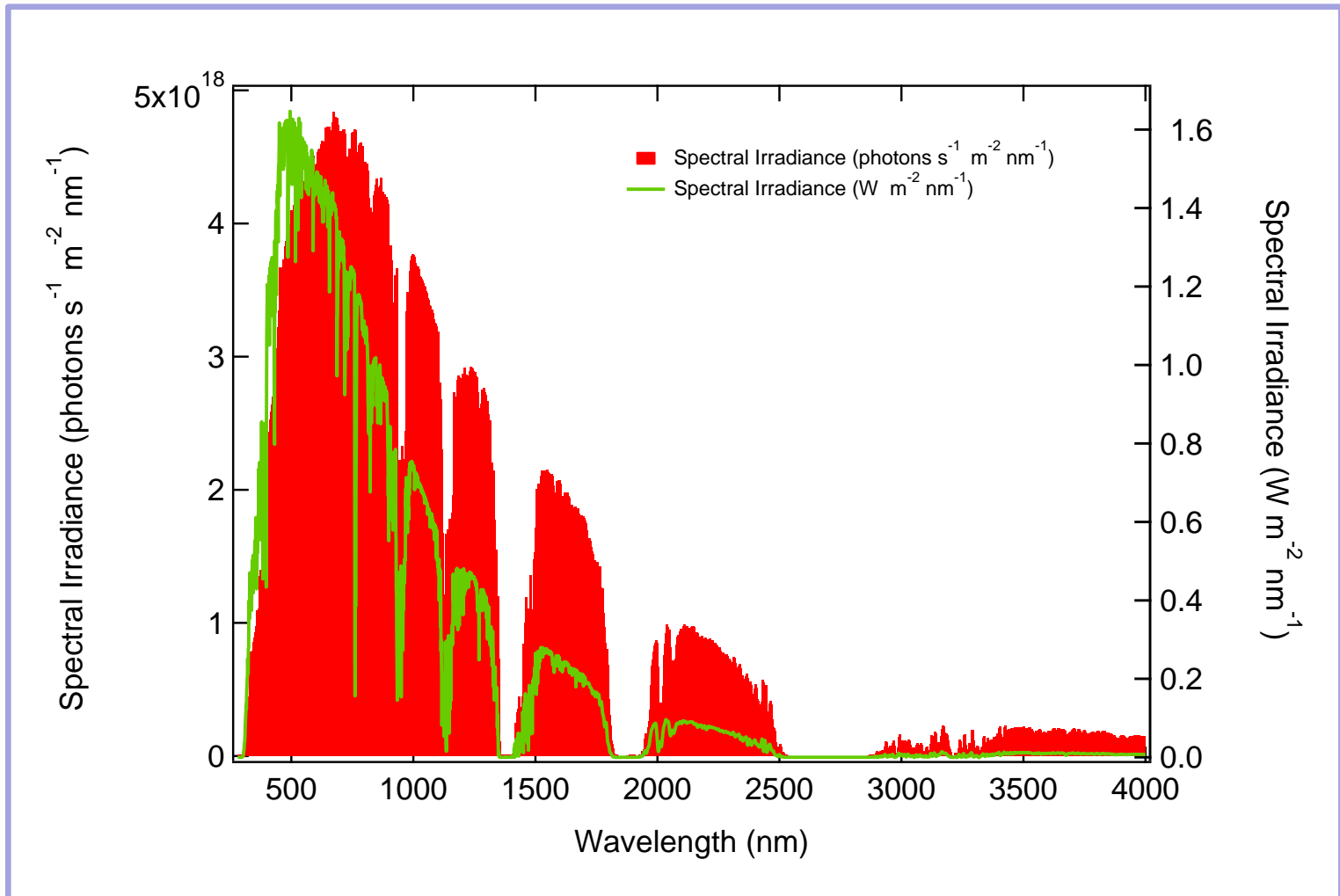
# What's in a square meter area of sunlight?

Assuming that the receiving area is normal to the incoming sunlight:

- Power, which when integrated with respect to time sums up to Energy. For example,  $1,366 \text{ W/m}^2$  of sunlight, integrated for 1 hour, gives  $(1366 \text{ W/m}^2) * (1 \text{ hr}) * (3600 \text{ s/hr}) * (1 \text{ J/s per W}) = 4.92 \times 10^6 \text{ J/m}^2$ .
- Photons. A very large number of photons per second (as we will find). Each photon with energy above a semiconductor's bandgap can be absorbed, boosting an electron from the valence band to the conduction band and contributing an electron to the *photocurrent* of the PV cell.

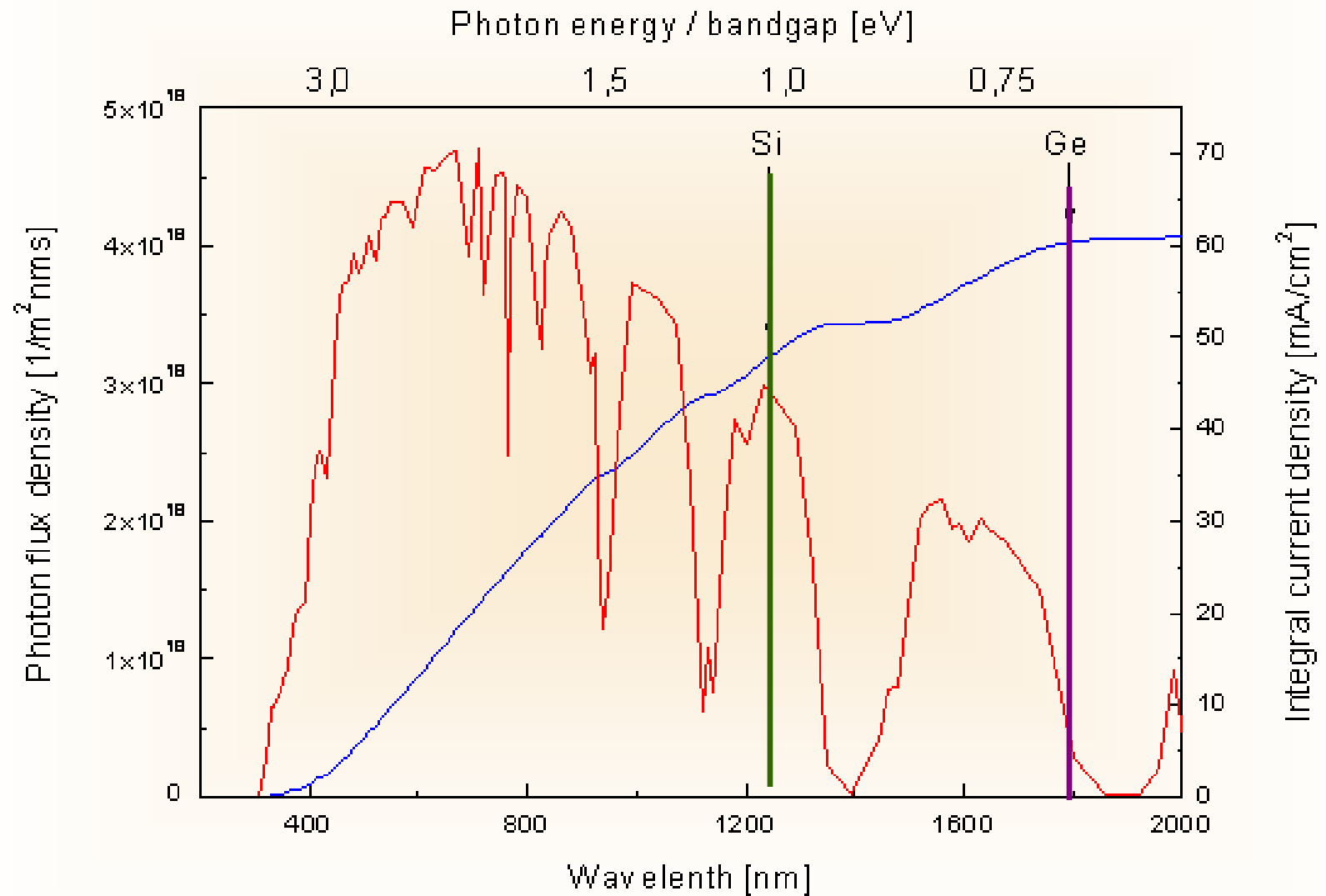


# Comparing shape of spectra, W vs. photons/s

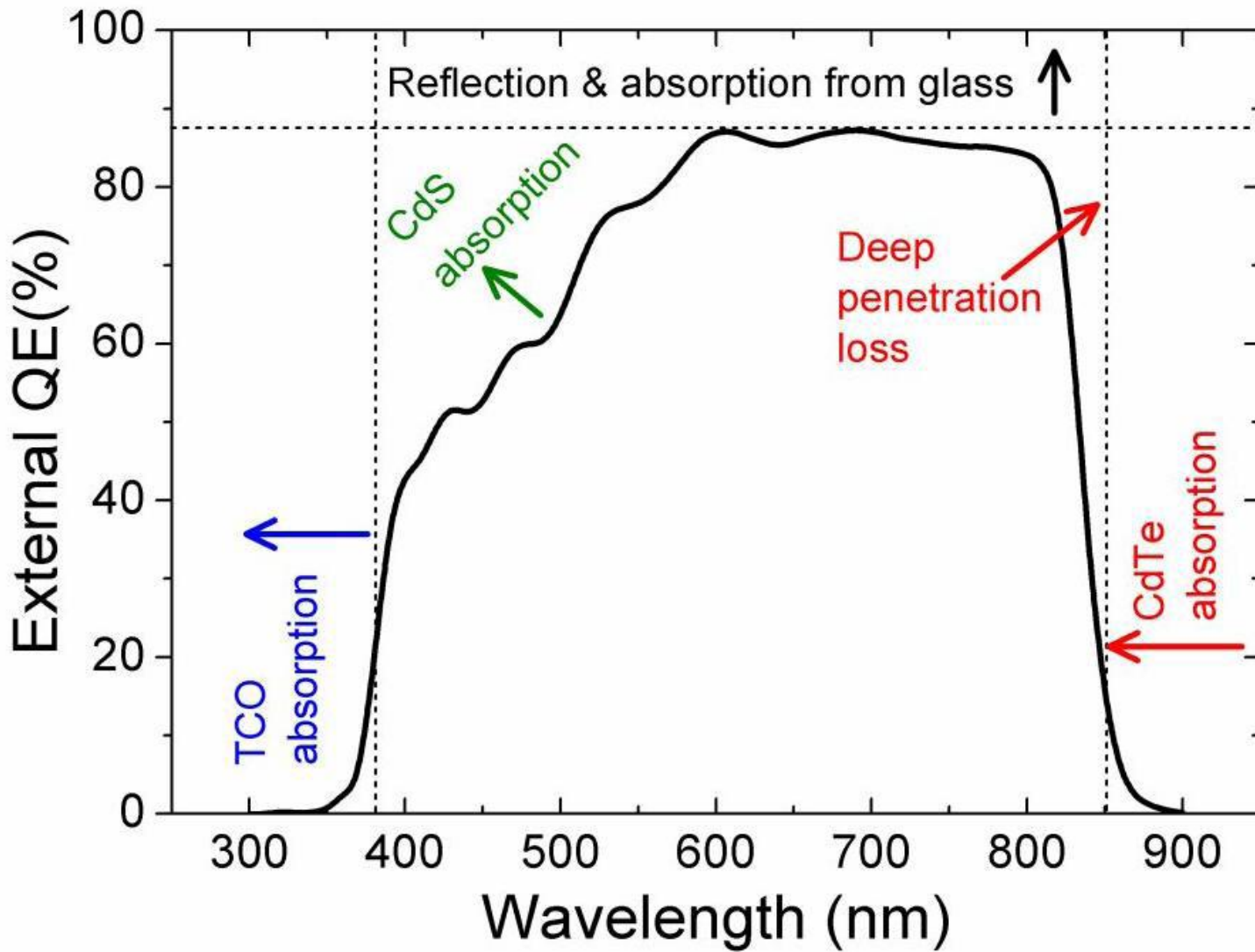


- Is this AM0 or AM1.5?
- What's going on here with the change in peak location?

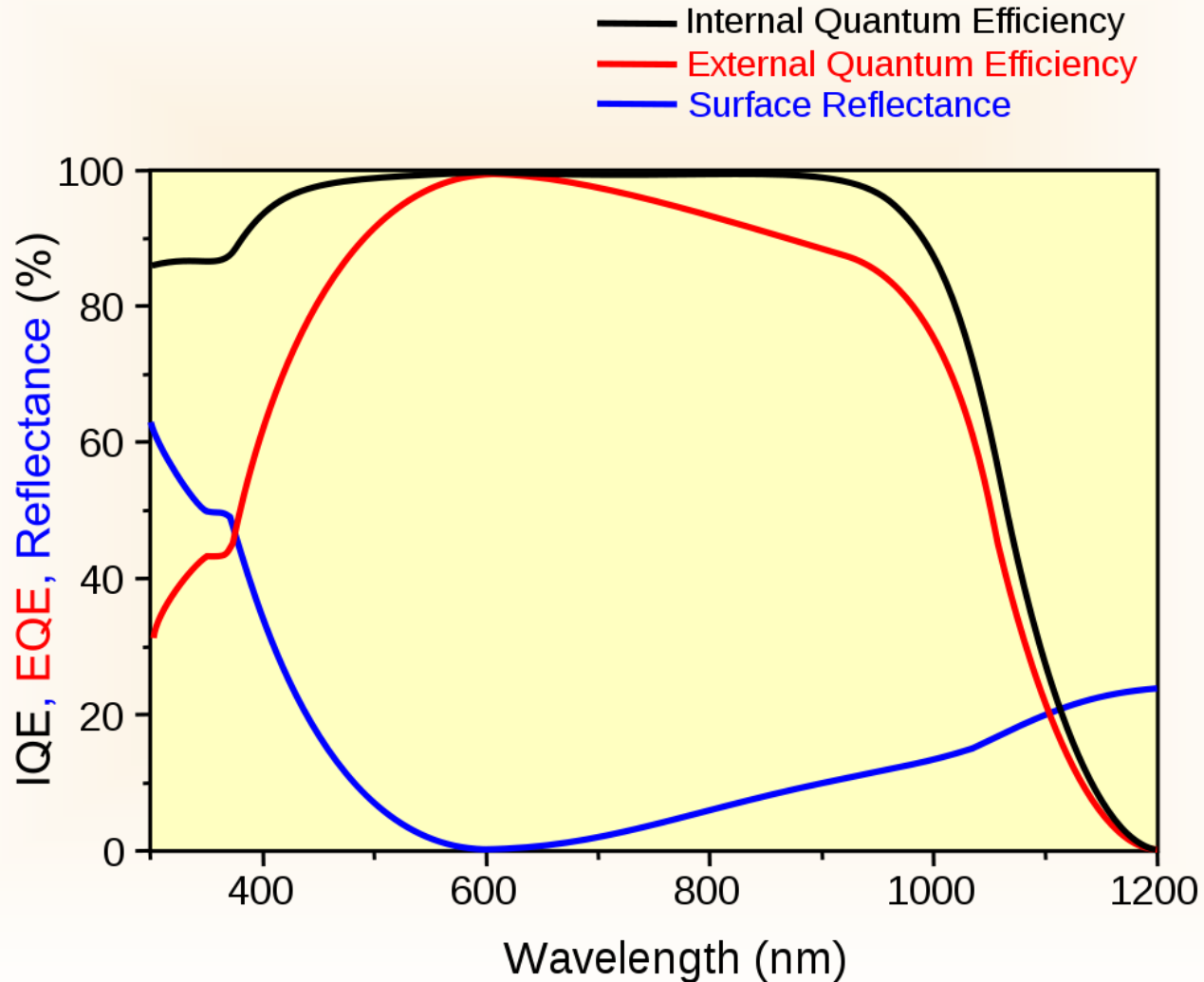
# Integrating the Solar Spectrum



# Spectral Response of a typical CdS/CdTe solar cell

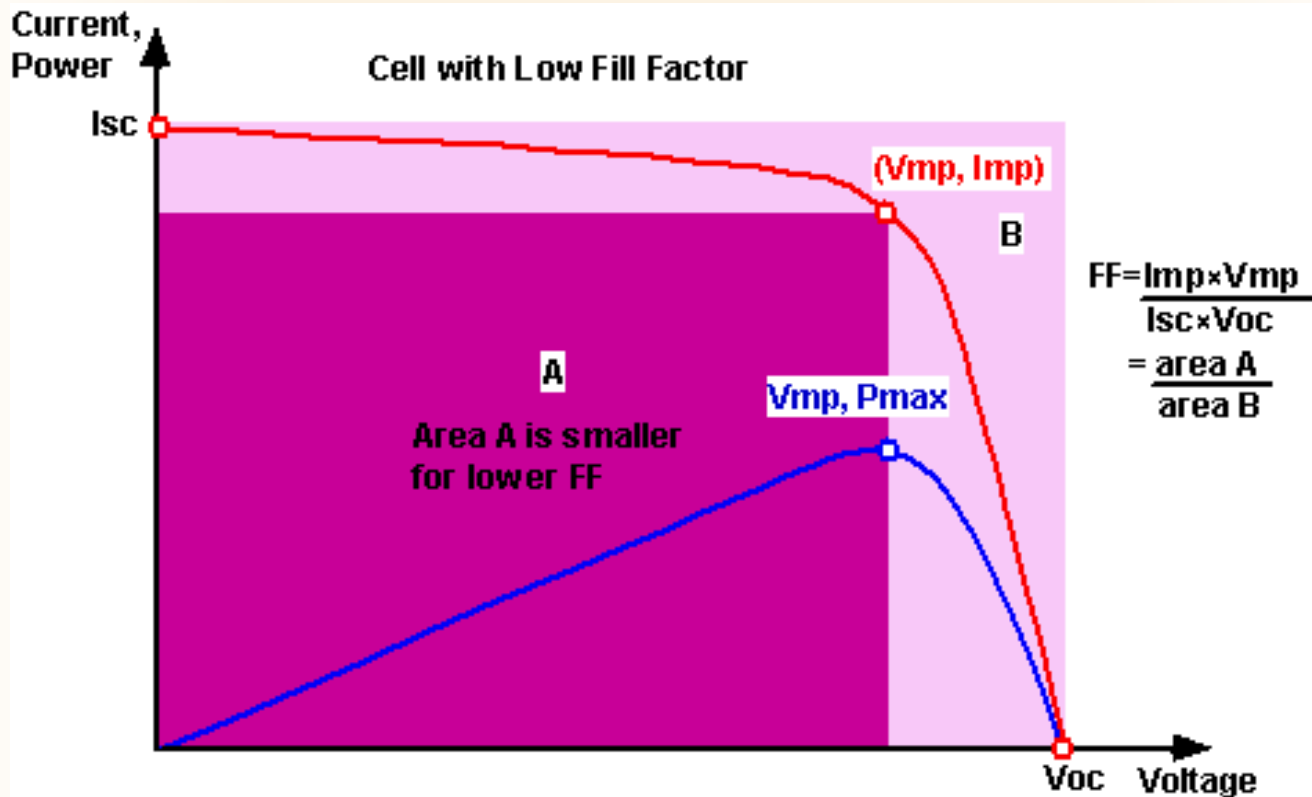


# Spectral Response of a typical c-Si solar cell



# Solar cell fill factor (FF)

At either "short circuit" ( $I_{sc}$ ) or "open circuit" ( $V_{oc}$ ), the power from the solar cell is zero. The "fill factor" (FF) is the parameter which, in conjunction with  $V_{oc}$  and  $I_{sc}$ , determines the maximum power from a solar cell. The FF is defined as the ratio of the maximum power from the solar cell to the product of  $V_{oc}$  and  $I_{sc}$ . Graphically, the FF is a measure of the "squareness" of the IV curve and is also the area of the largest rectangle which will fit in the IV curve, as illustrated below:



Graph of cell output current (red line) and power (blue line) as function of voltage. Also shown are the cell short-circuit current ( $I_{sc}$ ) and open-circuit voltage ( $V_{oc}$ ) points, as well as the maximum power point ( $V_{mp}$ ,  $I_{mp}$ ). Click on the graph to see how the curve changes for a cell with low FF.

# Solar cell efficiency

The efficiency of a solar cell (known also as the power conversion efficiency, or PCE, and often abbreviated  $\eta$ ) represents the ratio where the output electrical power at the maximum power point on the IV curve is divided by the incident light power – typically using a standard AM1.5G simulated solar spectrum.

The efficiency of a solar cell is determined as the fraction of incident power which is converted to electricity and is defined as:

$$P_{\max} = V_{OC} I_{SC} FF$$

$$\eta = \frac{V_{OC} I_{SC} FF}{P_{inc}}$$

where  $V_{oc}$  is the open-circuit voltage;

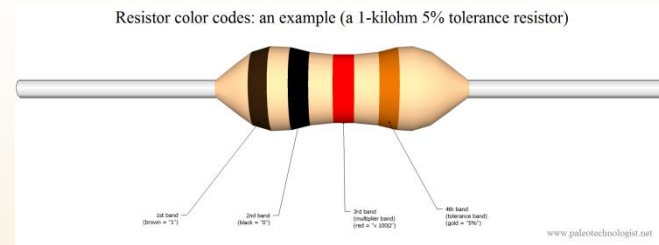
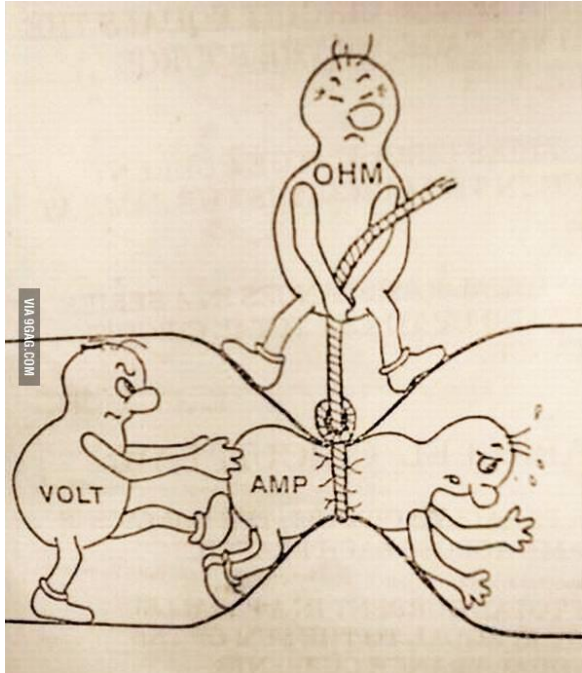
where  $I_{sc}$  is the short-circuit current; and

where  $FF$  is the fill factor

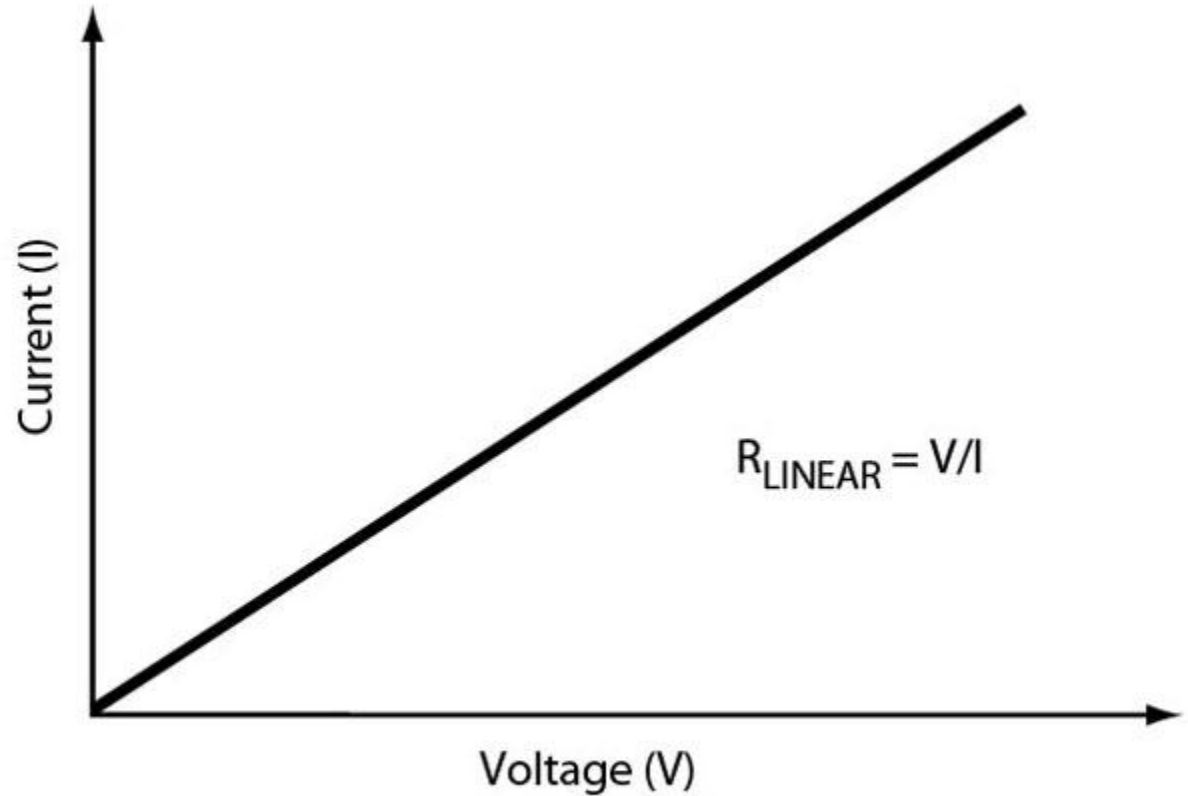
where  $\eta$  is the efficiency.

In a  $10 \times 10 \text{ cm}^2$  cell the input power at AM1.5G is  $100 \text{ mW/cm}^2 \times 100 \text{ cm}^2 = 10 \text{ W}$ .

# Ohm's Law



A typical resistor shows a resistance independent of current and voltage over a wide range of values.



According to Ohm's law, the common current-voltage relationship has a linear, positive slope.



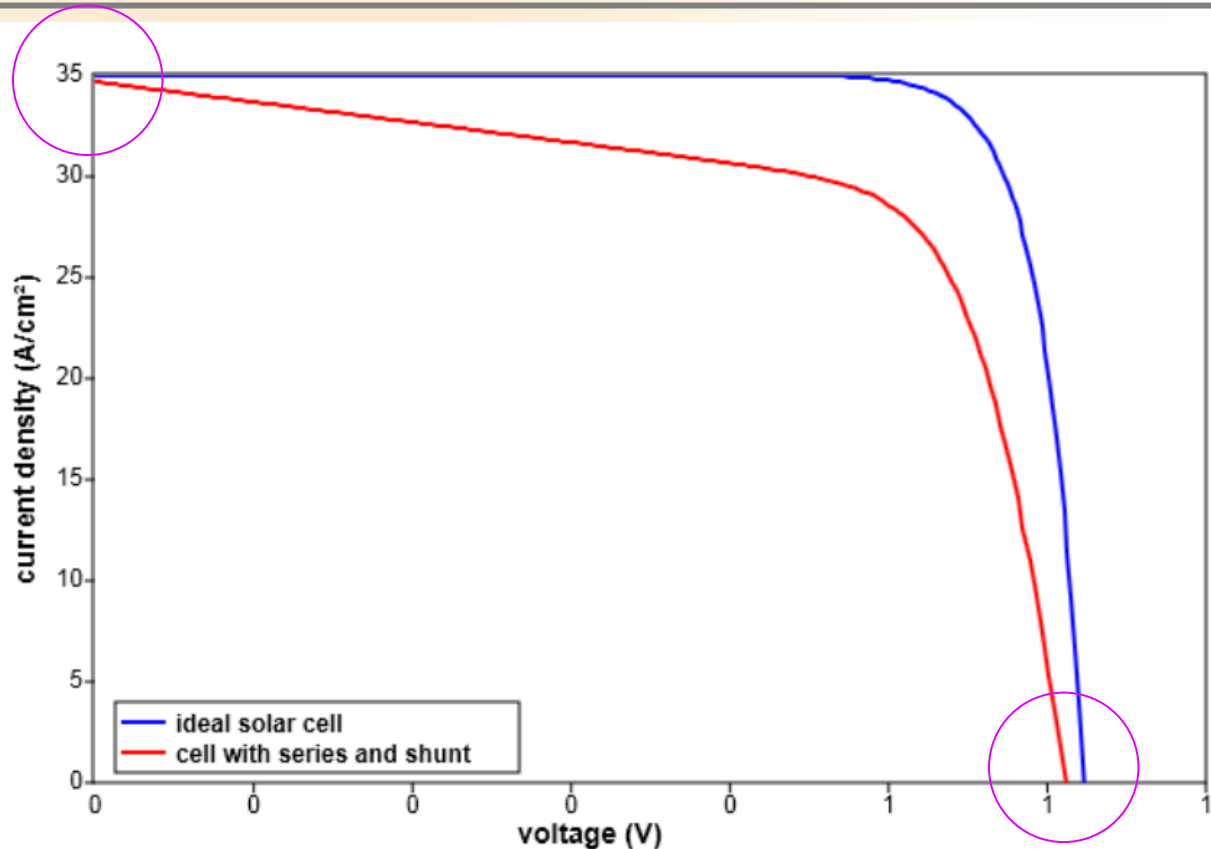
# Solar cell series and shunt resistance

From <http://www.pveducation.org/pvcdrom/solar-cell-operation/series-resistance>

**Series resistance** ( $R_s$ ) in a solar cell has three causes: (1) the movement of current through the front contact and the semiconductor absorber region of the solar cell; (2) contact resistance between the metal contact and the silicon; and (3) resistance of the top and rear metal contacts. A high series resistance reduces the fill factor, and excessively high values may also reduce the short-circuit current.

Significant power losses caused by the presence of a **shunt resistance** ( $R_{sh}$ ) are typically due to manufacturing defects, rather than poor solar cell design. Low shunt resistance causes power losses in solar cells by providing an alternate current path for the light-generated current.

We have measured  $I$  vs.  $V$ , so that for  $I$  in Amps and  $V$  in Volts, the apparent resistance ( $\Omega$ ) at any point on the curve is given by:  $(-1)/\text{slope}$ . The shunt resistance is defined at  $V = 0$  V, and the series resistance is defined at  $V = V_{OC}$ . For optimal power generation, solar cells should have a large  $R_{sh}$  and a small  $R_s$ .



# External and internal quantum efficiency

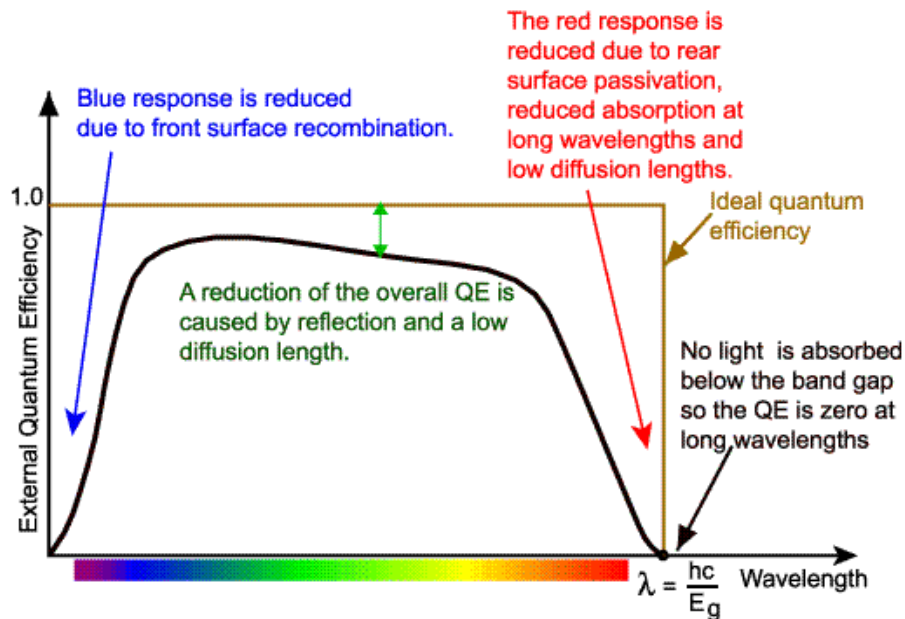
Internal and external quantum efficiency are functions of wavelength, i.e., EQE( $\lambda$ ) and QE( $\lambda$ ):

External quantum efficiency (EQE):

$$EQE(\lambda) = \frac{\text{Electrons collected as photocurrent, per second}}{\text{Photons incident, per second}}$$

Internal quantum efficiency (QE):

$$QE(\lambda) = \frac{\text{Electrons collected as photocurrent, per second}}{\text{Photons absorbed, per second}}$$



## $I_{SC}$ VS. $J_{SC}$

The current produced by a solar cell depends on several parameters, such as the incident light power and spectral distribution, the quantum efficiency of the solar cell as a function of wavelength, and the area of the solar cell.

To remove the effect of the area of a solar cell, one can divide the current (such as the short-circuit current,  $I_{SC}$ ) by the area of the solar cell typically measured in  $\text{cm}^2$ . In doing so, one “calibrates” the response so that two different solar cells (different materials, different areas) measured under the same spectrum can be directly compared regarding current generation and conversion efficiency.

The result is that while the short circuit current (expressed as  $I_{SC}$ ) has units of milliAmps (mA), the short circuit current density (expressed as  $J_{SC}$ ) has units of  $\text{mA}/\text{cm}^2$ .

Lab Goals [Time Period: Nov. 5 - 19, 2013]; **refer to Lab Guide**

- *Determine the external quantum efficiency (EQE, or spectral response) as a function of wavelength (note that you need not measure the reflectance spectrum to obtain EQE).*
- *Calculate the predicted power that would be generated under AM 1.5G and AM0 illumination.*
- *Qualitatively discuss the shape of the EQE curve (spectral response) and suggest ways to improve the performance of the solar cell.*

# X-ray Diffraction and Crystal Structures

November 5, 2013

**PHYS 4580, PHYS 6/7280**

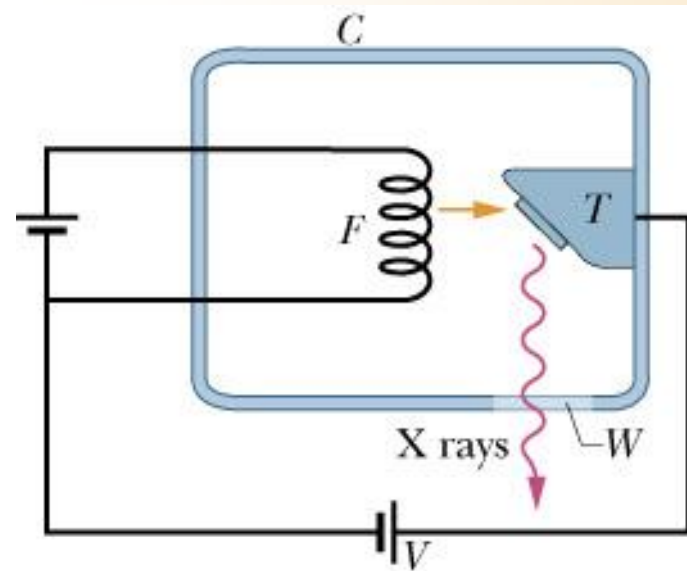
The University of Toledo  
Instructors: R. Ellingson, M. Heben



# X-Ray Generation

X-rays are electromagnetic radiation with wavelength  $\sim 1 \text{ \AA} = 10^{-10} \text{ m}$   
(visible light  $\sim 5.5 \times 10^{-7} \text{ m}$ )

X-ray generation: electrons are emitted from the cathode and accelerated toward the anode. Here, Bremsstrahlung radiation occurs as a result of the “braking” process – X-ray photons are emitted.



X-ray wavelengths too short to be resolved by a standard optical grating

$$\theta = \sin^{-1} \frac{m\lambda}{d} = \sin^{-1} \frac{(1)(0.1 \text{ nm})}{3000 \text{ nm}} = 0.0019^\circ$$

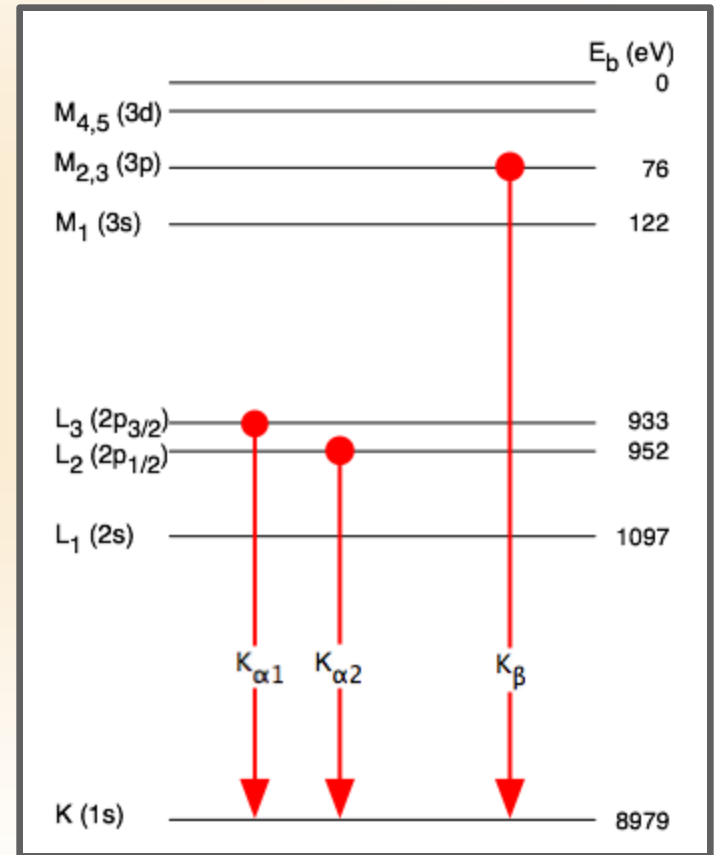
# X-Ray Generation

The most common metal used is copper, which can be kept cool easily, due to its high thermal conductivity, and which produces strong  $K_\alpha$  and  $K_\beta$  lines. The  $K_\beta$  line is sometimes suppressed with a thin ( $\sim 10 \mu\text{m}$ ) nickel foil.

- **K-alpha ( $K_\alpha$ )** emission lines result when an electron transitions to the innermost "K" shell (principal quantum number 1) from a 2p orbital of the second or "L" shell (with principal quantum number 2).
- The  $K_\alpha$  line is actually a doublet, with slightly different energies depending on spin-orbit interaction energy between the electron spin and the orbital momentum of the 2p orbital.

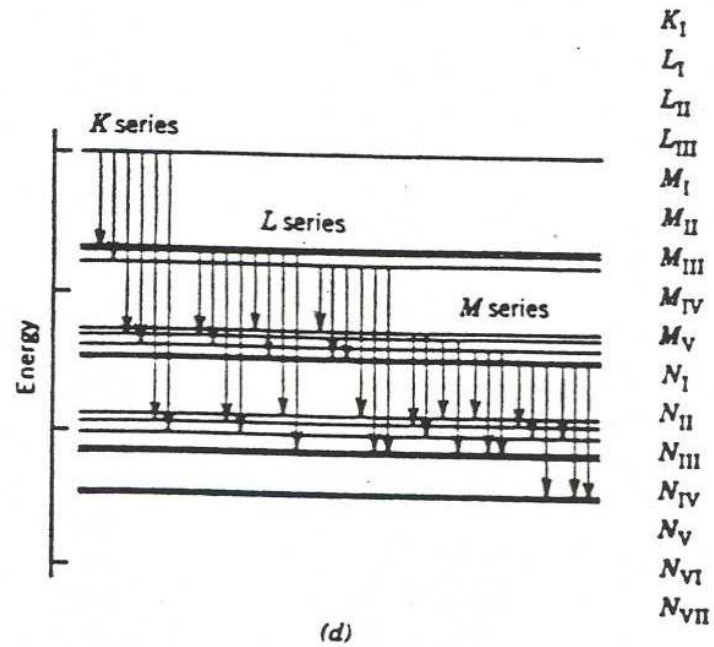
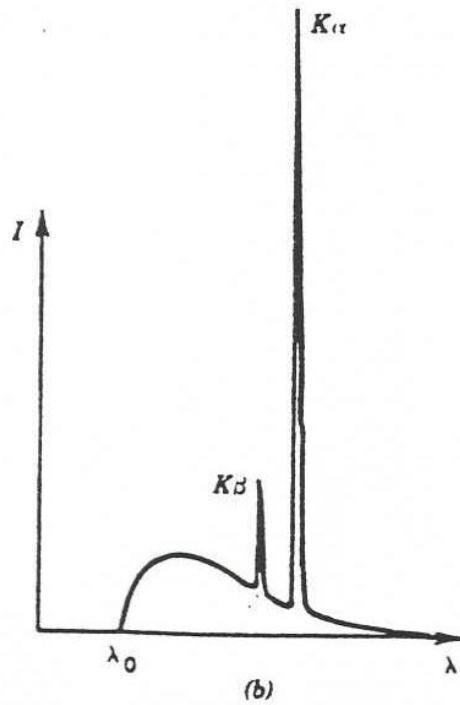
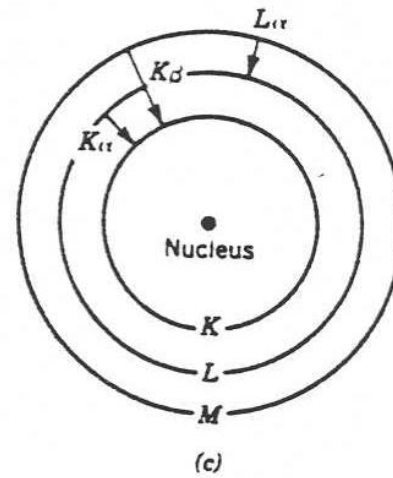
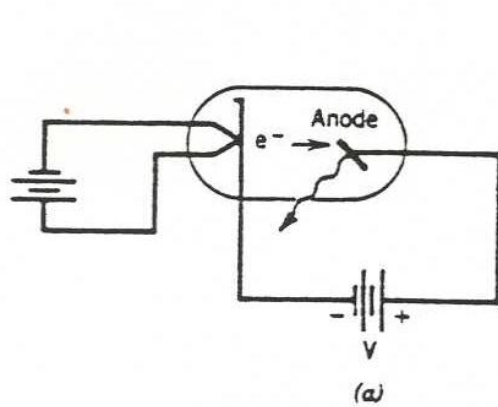
$$\lambda(K_\alpha) = 0.154 \text{ nm}$$

$$\lambda(K_\beta) = 0.139 \text{ nm}$$



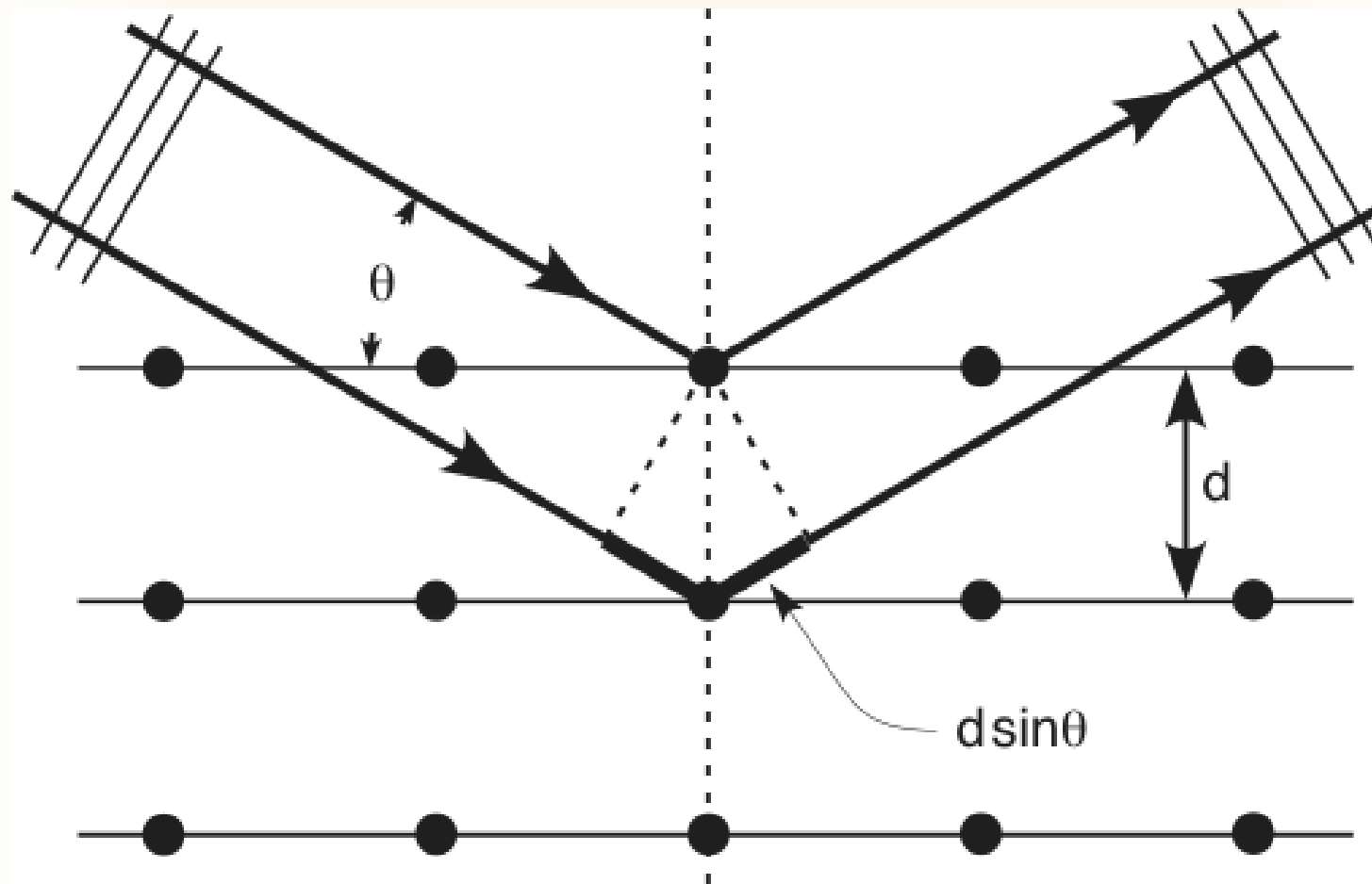
Atomic levels involved in copper  $K_\alpha$  and  $K_\beta$  emission.

# $K_{\alpha}$ and $K_{\beta}$ X-ray lines

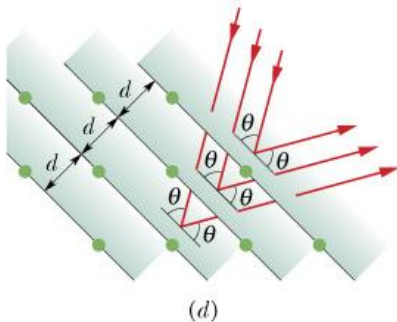
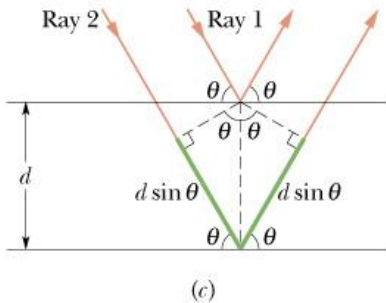
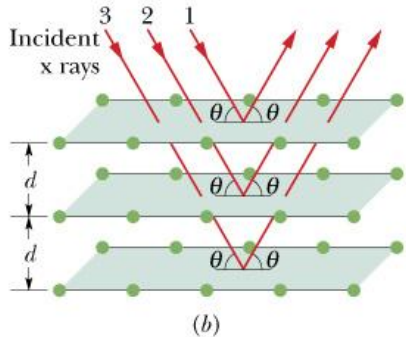
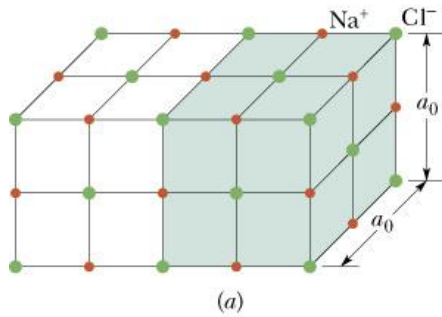




# X-Ray diffraction



# X-Ray Diffraction -- Bragg's Law



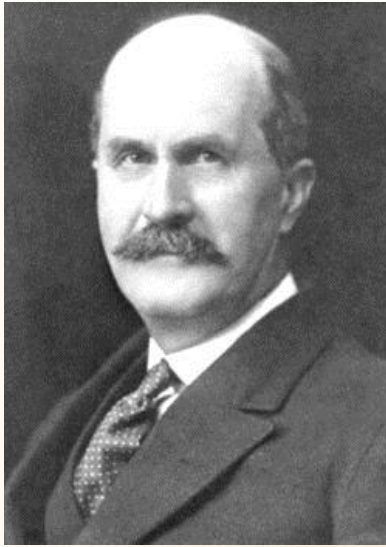
Diffraction of x-rays by crystal: spacing  $d$  of adjacent crystal planes on the order of 0.1 nm  
→ three-dimensional diffraction grating with diffraction maxima along angles where reflections from different planes interfere constructively

$$2d \sin \theta = m\lambda \text{ for } m = 0, 1, 2, \dots$$

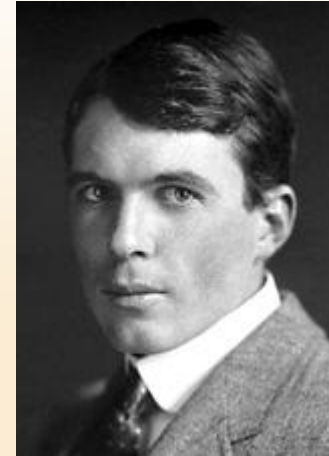
**Bragg's Law**

Note that your measured XRD spectra will most likely reveal only 1<sup>st</sup> order diffracted lines (i.e., those for which  $m = 1$ ).

# The Braggs (Bragg's Law)



**Sir William Henry Bragg**  
**1862-1942**

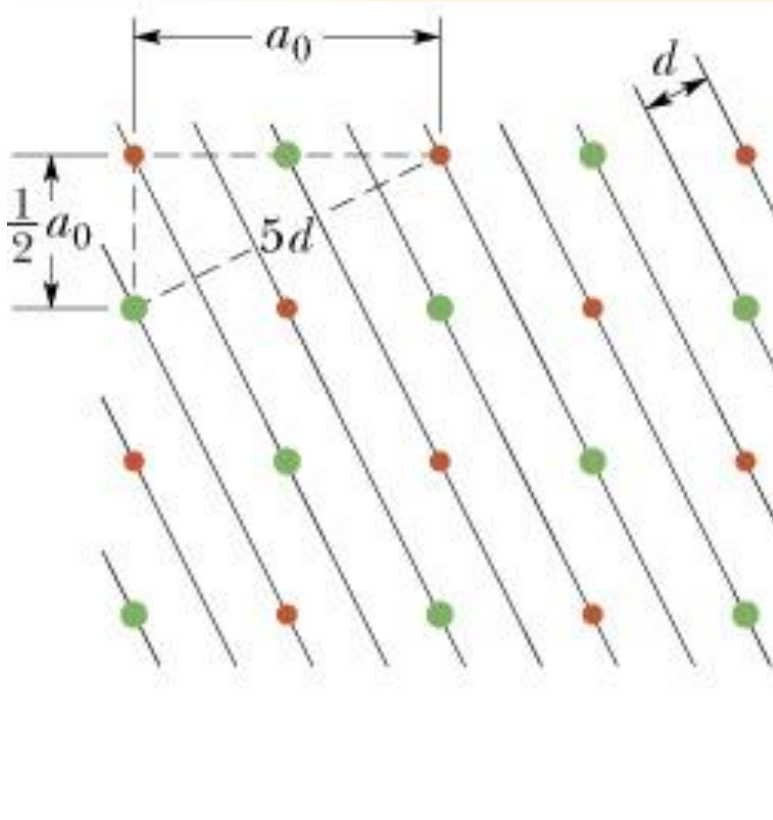


**William Lawrence Bragg**  
**1890-1971**

Bragg occupied the Cavendish chair of physics at the University of Leeds from 1909. He continued his work on X-rays with much success. He invented the X-ray spectrometer and with his son, William Lawrence Bragg, then a research student at Cambridge, founded the new science of X-ray analysis of crystal structure.

In 1915 father and son were jointly awarded the Nobel Prize in Physics for their studies, using the X-ray spectrometer, of X-ray spectra, X-ray diffraction, and of crystal structure.

## X-Ray Diffraction, cont'd



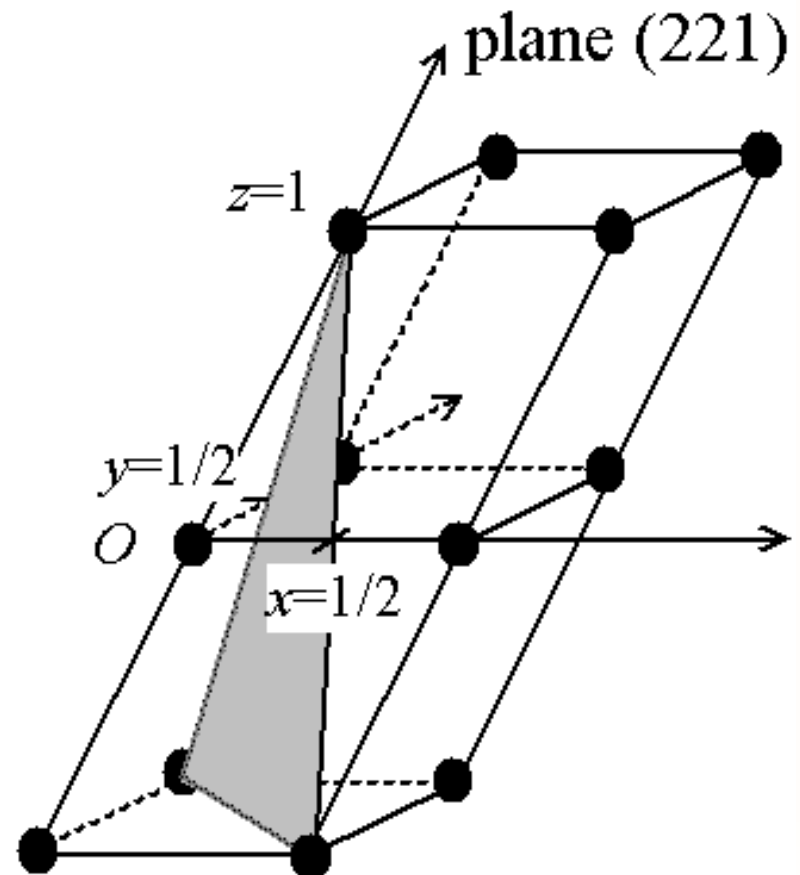
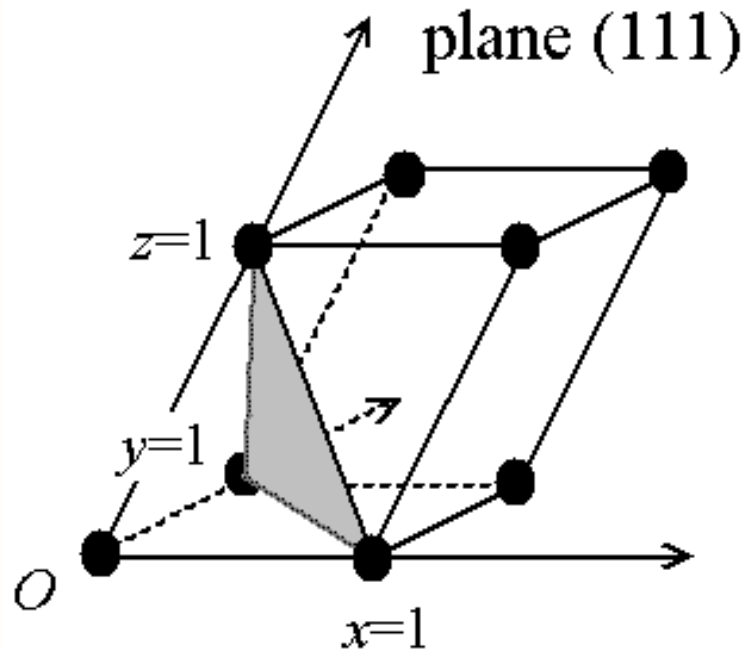
Interplanar spacing  $d$  is related to the unit cell dimension  $a_0$

$$5d = \sqrt{\frac{5}{4} a_0^2} \quad \text{or} \quad d = \frac{a_0}{20} = 0.2236a_0$$

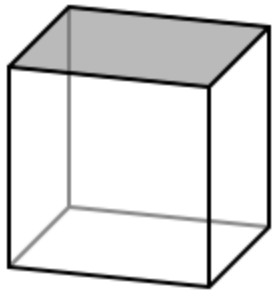
Not only can crystals be used to separate different x-ray wavelengths, but x-rays in turn can be used to study crystals, for example determine the type of crystal ordering and  $a_0$ .

# Crystal structure, lattice planes, and Miller indices

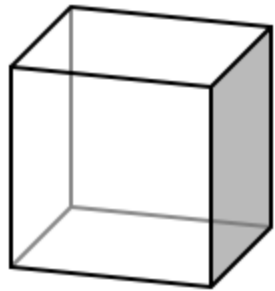
Planes with different Miller indices in cubic crystals. The *inverse* of these fractional intercepts yields the Miller indices  $h, k, l$ .



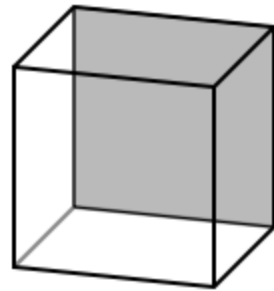
# Crystal structure and Miller indices



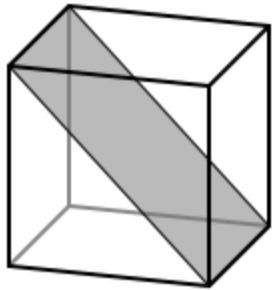
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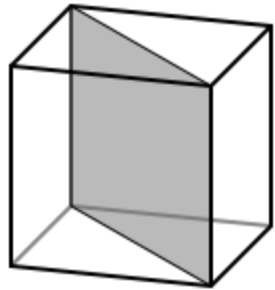
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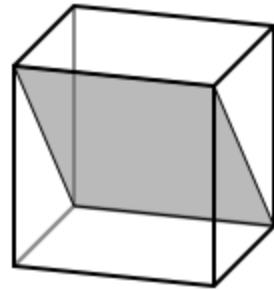
(010)



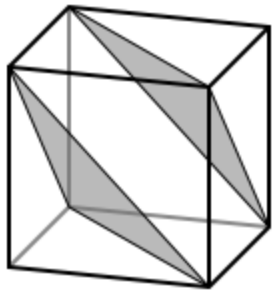
(101)



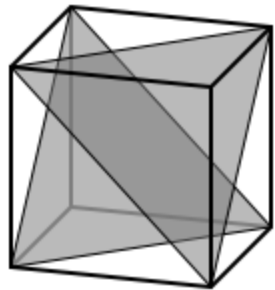
(110)



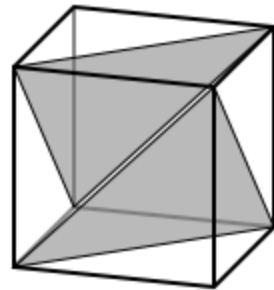
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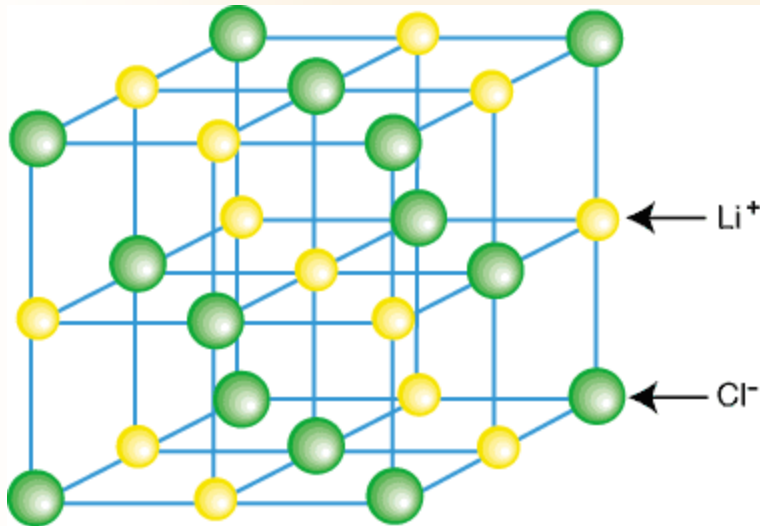
(11̄1)



(1̄11)

Planes with different Miller indices in cubic crystals.

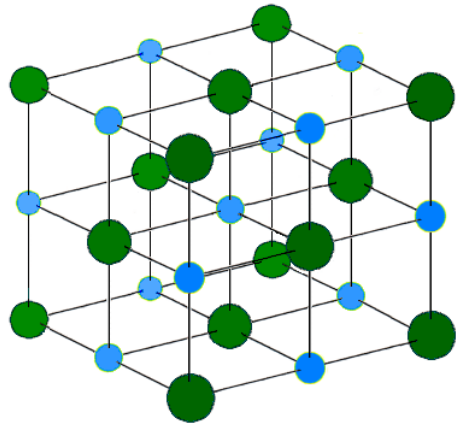
# Crystal structure and Miller indices



 UNIVERSITY OF  
CAMBRIDGE

**Indexing lattice  
planes**

## Rock salt (cubic) crystal structure



$$d_{hkl} = \frac{a_0}{\sqrt{h^2 + k^2 + l^2}}$$

**Structure factor for NaCl:**

$$F = \left[ f_{Na} + f_{Cl} e^{i\pi(h+k+l)} \right] \left[ 1 + e^{i\pi(h+k)} + e^{i\pi(h+l)} + e^{i\pi(k+l)} \right]$$

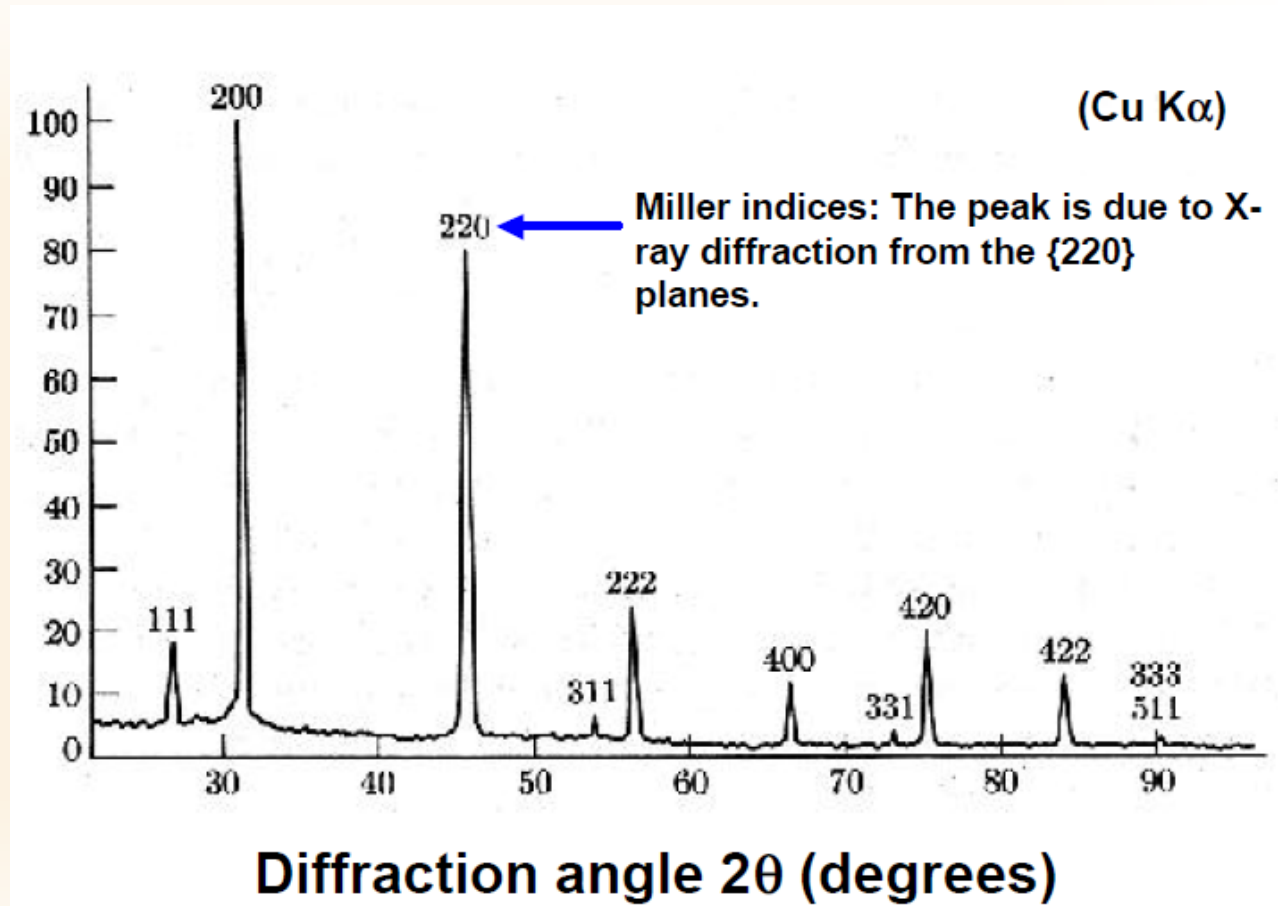
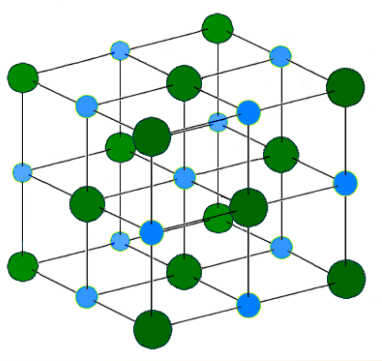
$$F = 4(f_{Na} + f_{Cl}) \quad \text{if } h, k, l \text{ are even}$$

$$F = 4(f_{Na} - f_{Cl}) \quad \text{if } h, k, l \text{ are odd}$$

$$F = 0 \quad \text{if } h, k, l \text{ are mixed}$$



# X-Ray diffraction (XRD) pattern (diffractogram) from NaCl



$$d_{hkl} = \frac{a_0}{\sqrt{h^2 + k^2 + l^2}}$$

The value of  $d$ , the distance between adjacent planes in the set  $(hkl)$ , may be found from the following equations.

$$\text{Cubic:} \quad \frac{1}{d^2} = \frac{h^2 + k^2 + l^2}{a^2}$$

$$\text{Tetragonal:} \quad \frac{1}{d^2} = \frac{h^2 + k^2}{a^2} + \frac{l^2}{c^2}$$

$$\text{Hexagonal:} \quad \frac{1}{d^2} = \frac{4}{3} \left( \frac{h^2 + hk + k^2}{a^2} \right) + \frac{l^2}{c^2}$$

*Rhombohedral:*

$$\frac{1}{d^2} = \frac{(h^2 + k^2 + l^2) \sin^2 \alpha + 2(hk + kl + hl)(\cos^2 \alpha - \cos \alpha)}{a^2(1 - 3 \cos^2 \alpha + 2 \cos^3 \alpha)}$$

$$\text{Orthorhombic:} \quad \frac{1}{d^2} = \frac{h^2}{a^2} + \frac{k^2}{b^2} + \frac{l^2}{c^2}$$

$$\text{Monoclinic:} \quad \frac{1}{d^2} = \frac{1}{\sin^2 \beta} \left( \frac{h^2}{a^2} + \frac{k^2 \sin^2 \beta}{b^2} + \frac{l^2}{c^2} - \frac{2hl \cos \beta}{ac} \right)$$

$$\text{Triclinic:} \quad \frac{1}{d^2} = \frac{1}{V^2} (S_{11}h^2 + S_{22}k^2 + S_{33}l^2 + 2S_{12}hk + 2S_{23}kl + 2S_{13}hl)$$

In the equation for triclinic crystals,

$V$  = volume of unit cell

$$S_{11} = b^2c^2 \sin^2 \alpha,$$

$$S_{22} = a^2c^2 \sin^2 \beta,$$

$$S_{33} = a^2b^2 \sin^2 \gamma,$$

$$S_{12} = abc^2(\cos \alpha \cos \beta - \cos \gamma),$$

$$S_{23} = a^2bc(\cos \beta \cos \gamma - \cos \alpha),$$

$$S_{13} = ab^2c(\cos \gamma \cos \alpha - \cos \beta).$$

# d spacings for tetragonal, hexagonal, orthorhombic crystals

**Bragg's Law (1):**

$$d = \frac{\lambda}{2 \sin \theta_c} \quad (1)$$

$$d_{hkl} = \frac{a}{\sqrt{h^2 + k^2 + l^2}} \quad (2) \text{ Plane spacing for cubic crystals}$$

Combined (1) and (2):

$$\left(\frac{n\lambda}{2a}\right)^2 = \frac{\sin^2 \theta}{h^2 + k^2 + l^2} \text{ or } \sin^2 \theta = \frac{\lambda^2}{4a^2} (h^2 + k^2 + l^2) \quad (3)$$

For a particular incident x-ray wavelength and cubic crystal of unit cell size  $a$ , this equation predicts all possible Bragg angles at which diffraction can occur from planes (hkl).

→ Diffraction planes are determined solely by the shape and size (lattice parameters) of the unit cell.

Plane spacings for:

Tetragonal:  $\frac{1}{d^2} = \frac{h^2 + k^2}{a^2} + \frac{l^2}{c^2} \quad (4)$

Hexagonal:  $\frac{1}{d^2} = \frac{4}{3} \left( \frac{h^2 + hk + k^2}{a^2} \right) + \frac{l^2}{c^2} \quad (5)$

Orthorhombic:  $\frac{1}{d^2} = \frac{h^2}{a^2} + \frac{k^2}{b^2} + \frac{l^2}{c^2} \quad (6)$

If crystal is **tetragonal** with  $a \neq c$  then (1) and (4) become:

$$\sin^2 \theta = \frac{\lambda^2}{4a^2} (h^2 + k^2) + \frac{\lambda^2}{4c^2} (l^2) \quad (7)$$

If crystal is **hexagonal** with  $a \neq c$  then (1) and (5) become:

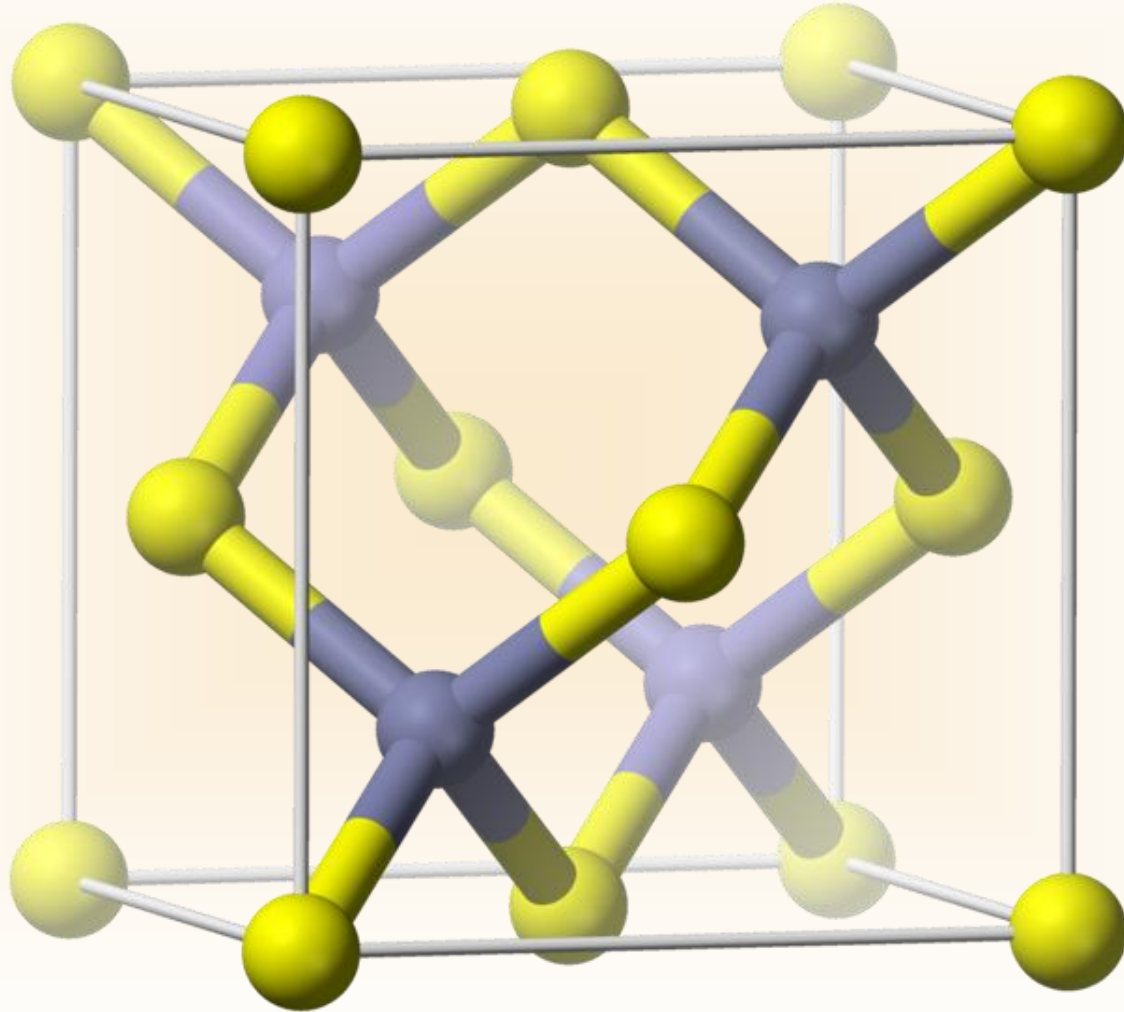
$$\sin^2 \theta = \frac{\lambda^2}{3a^2} (h^2 + k^2 + hk) + \frac{\lambda^2}{4c^2} (l^2) \quad (8)$$

If crystal is **orthorhombic** with  $a \neq b \neq c$  then (1) and (6):

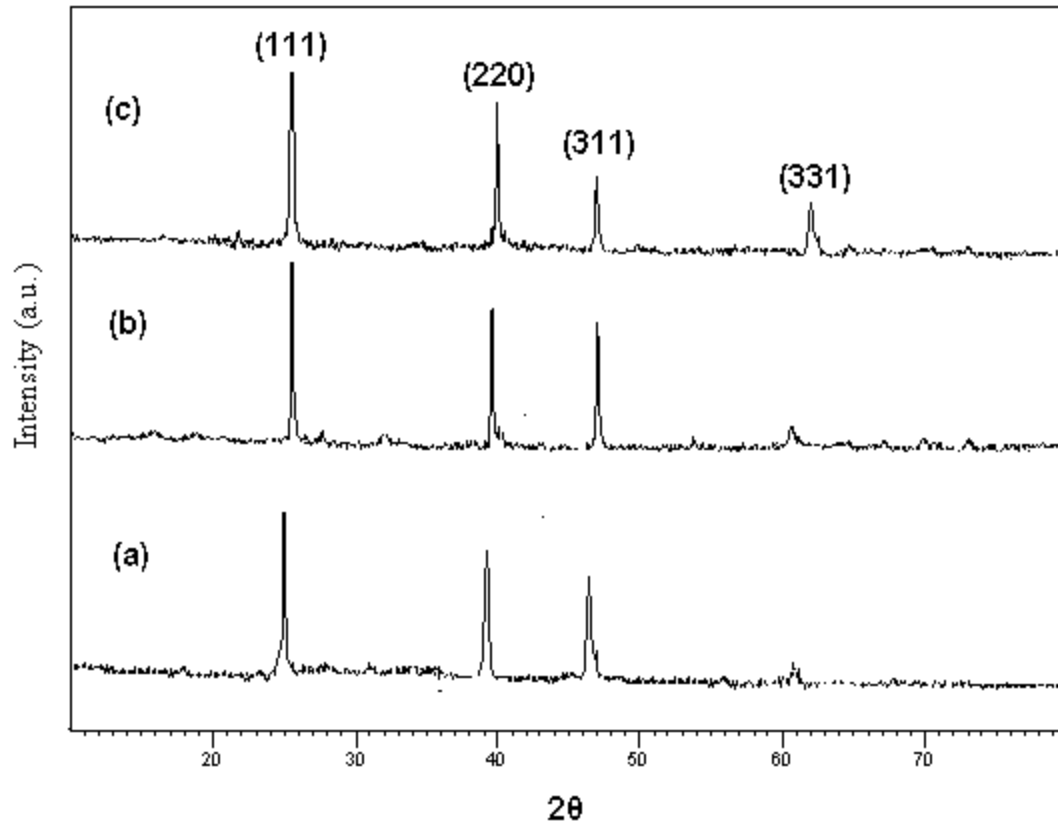
$$\sin^2 \theta = \frac{\lambda^2}{4a^2} (h^2) + \frac{\lambda^2}{4b^2} (k^2) + \frac{\lambda^2}{4c^2} (l^2) \quad (9)$$

# CdTe crystal structure (zincblende)

$a_0 = 0.648 \text{ nm}$

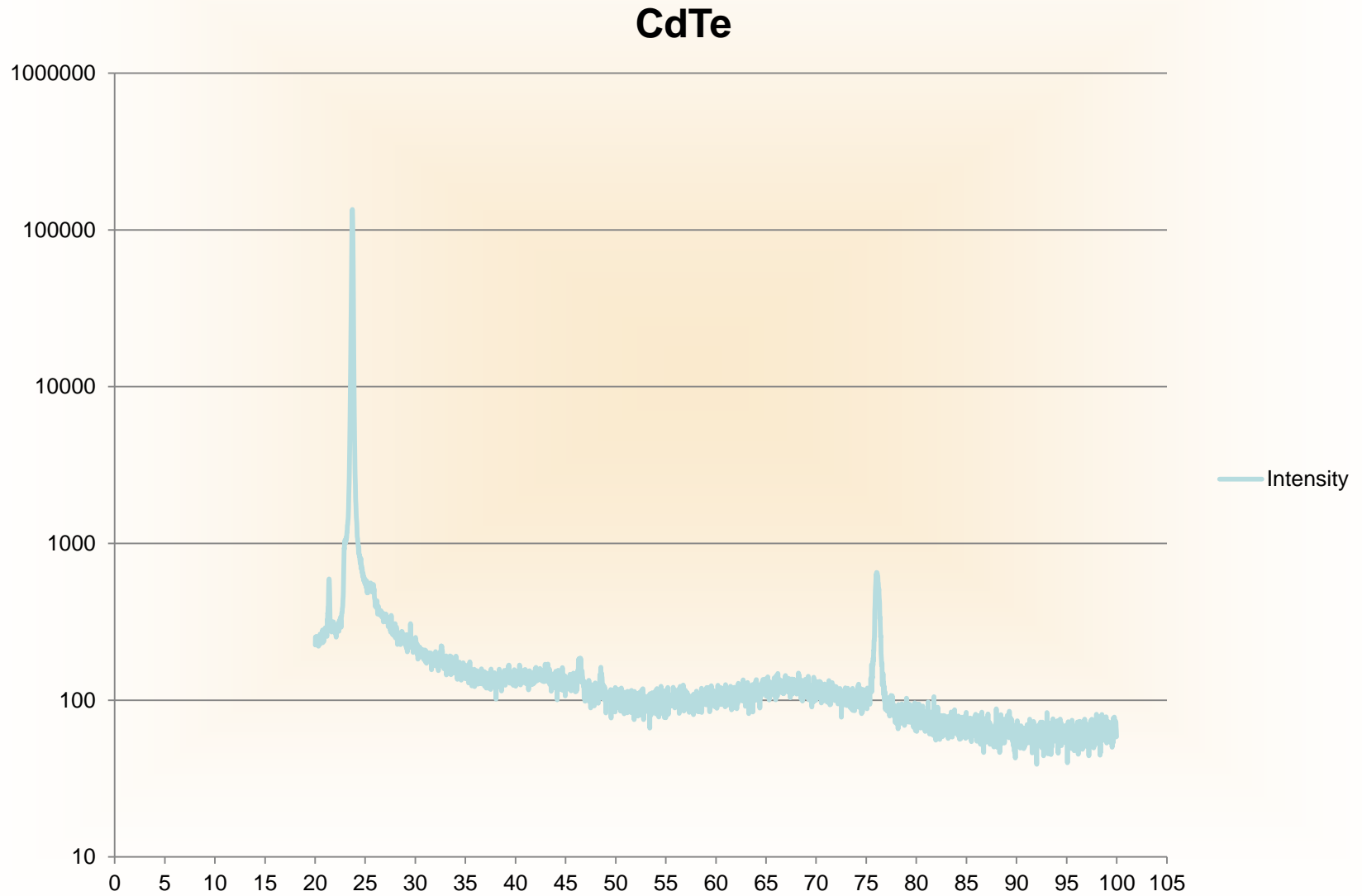


# CdTe XRD pattern



*X-ray diffractograms of thin films at annealing temperatures of a) 350 C, b) 400 C and c) 450 C.*

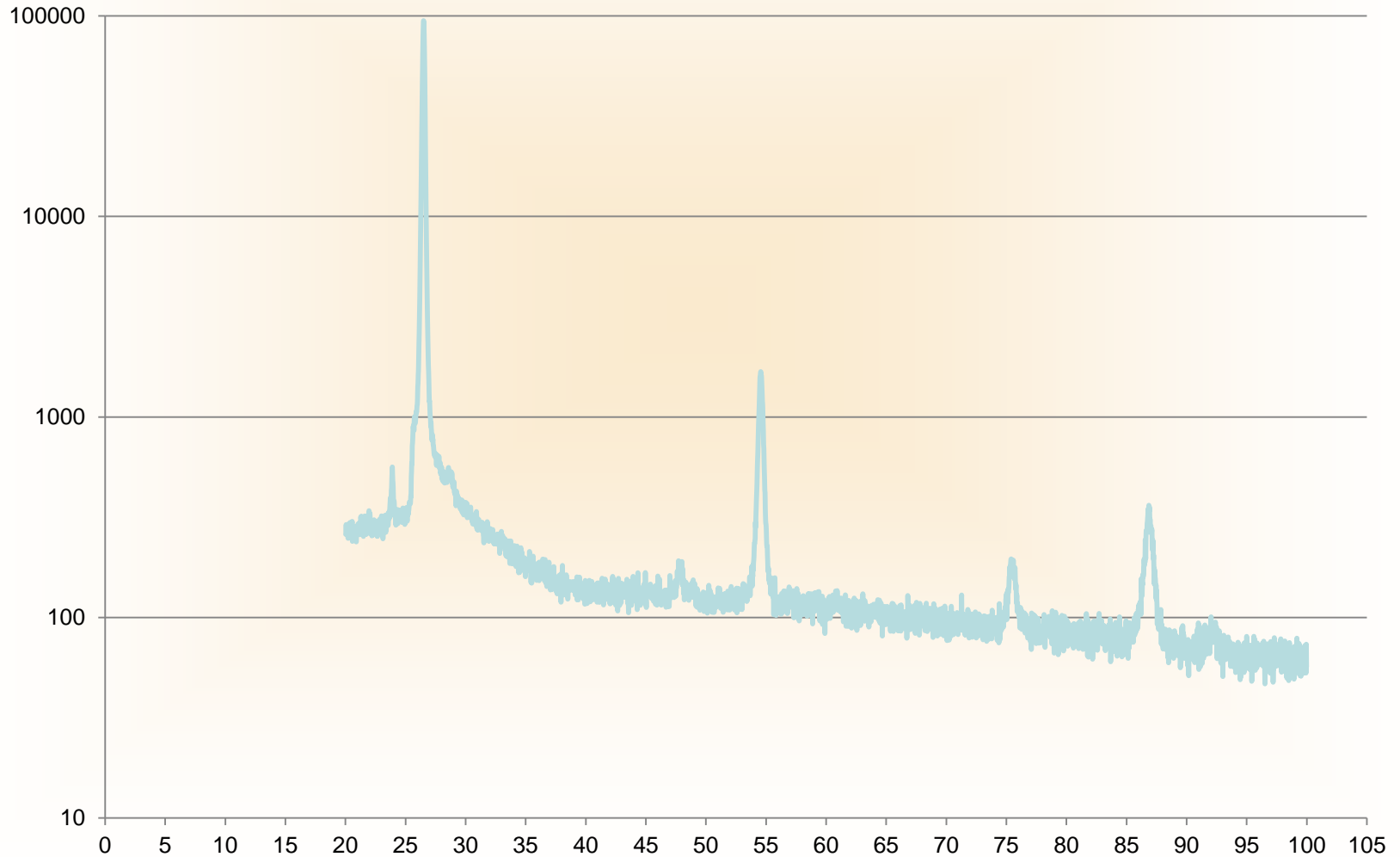
# CdTe XRD pattern (intensity vs. $2\theta$ )



# CdS XRD pattern (intensity vs. $2\theta$ )

$a_0 = 0.5832$  nm for zincblende  
 $a=4.160$ ;  $c=6.756$  for wurtzite

**CdS**



# Scherrer Equation (relationship to Shape Factor)

$$\tau = \frac{K\lambda}{\beta \cos \theta}$$

$K$  is the shape factor,  $\lambda$  represents the x-ray wavelength used for the measurement,  $\beta$  is the line width (FWHM) in radians,  $\theta$  is the Bragg angle (note, this is not the  $2\theta$  angle, just  $\theta$ ), and  $\tau$  is the mean size of the crystalline domains. The formula yields a lower bound on the possible particle size.

The shape factor enables one to determine the average size of crystal grains within a polycrystalline thin film. Assuming a Gaussian function to fit the peak, the shape factor is 0.9, so that

$$\tau = \frac{0.9\lambda}{\beta \cos \theta}$$

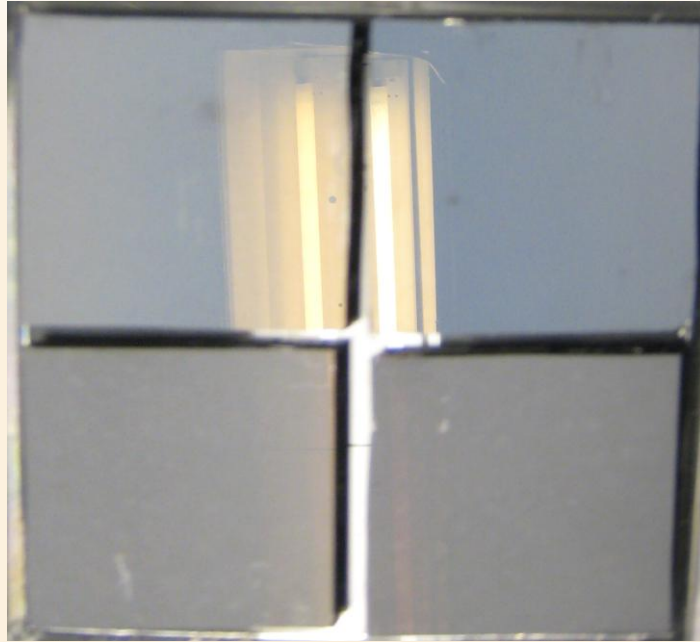




# CdTe XRD

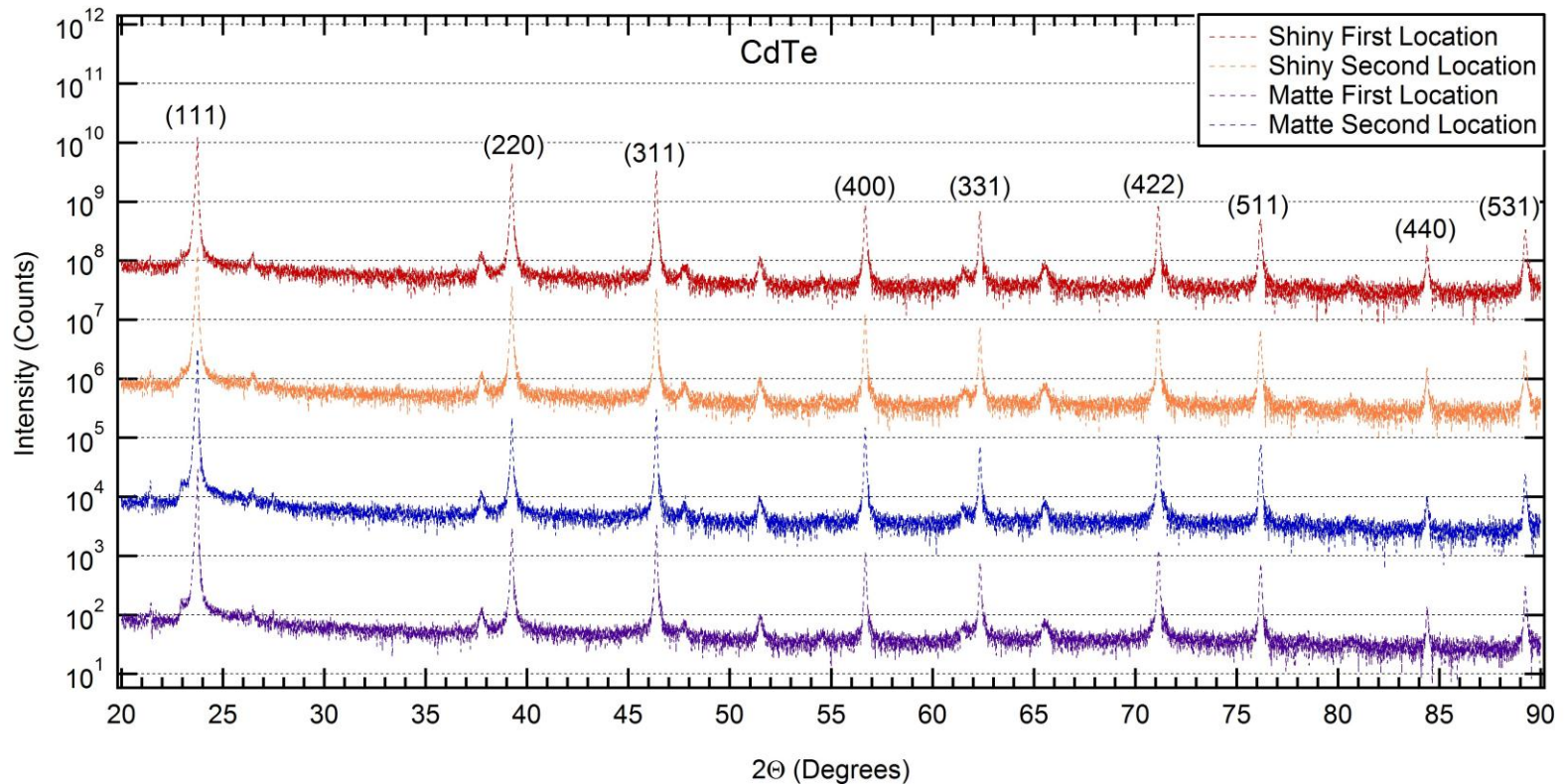
*Meghan Mapes*  
*February 6, 2012*

# Motivation



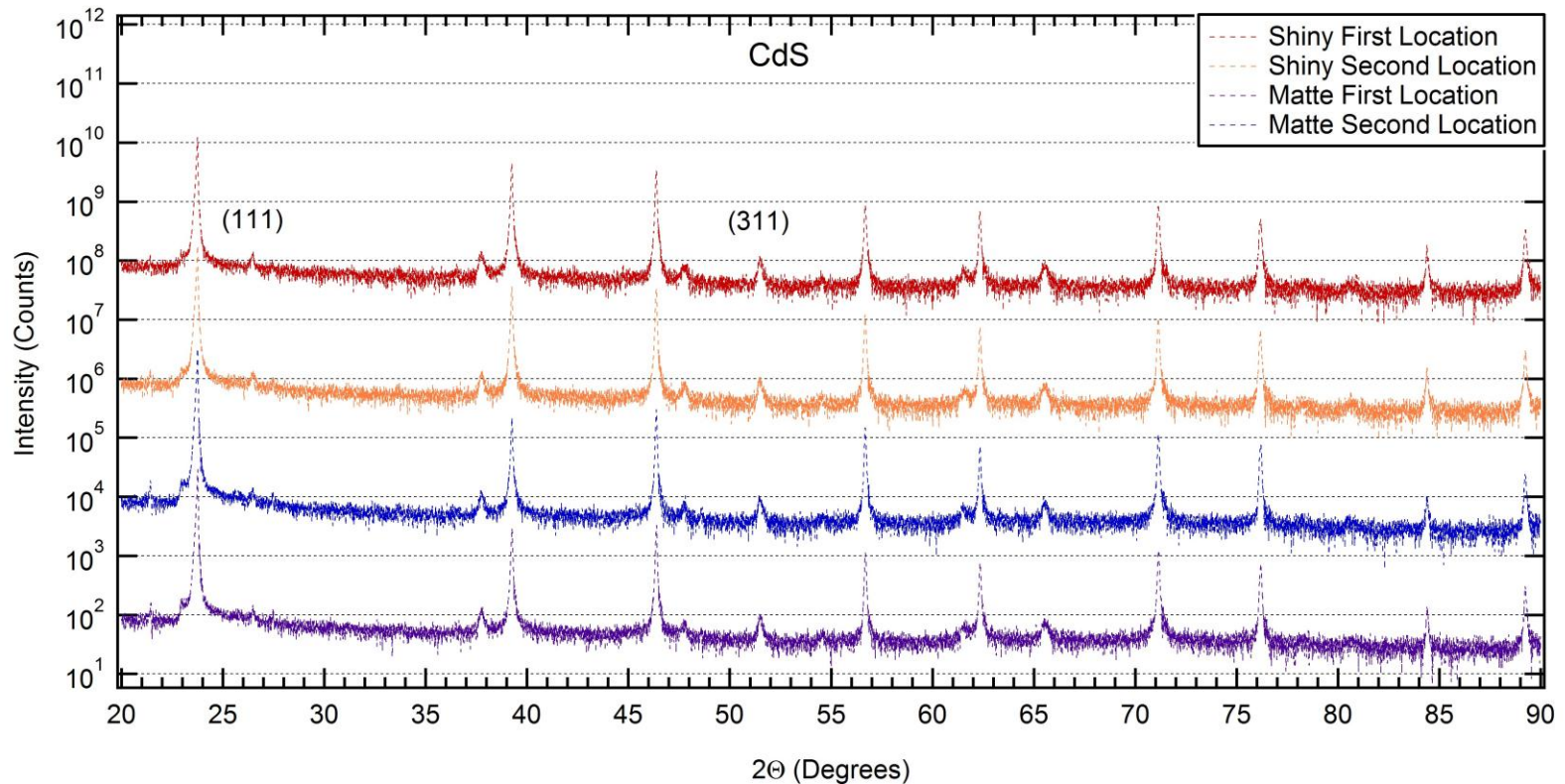
Determine why some samples appear shiny, and some appear matte.

# Raw Data



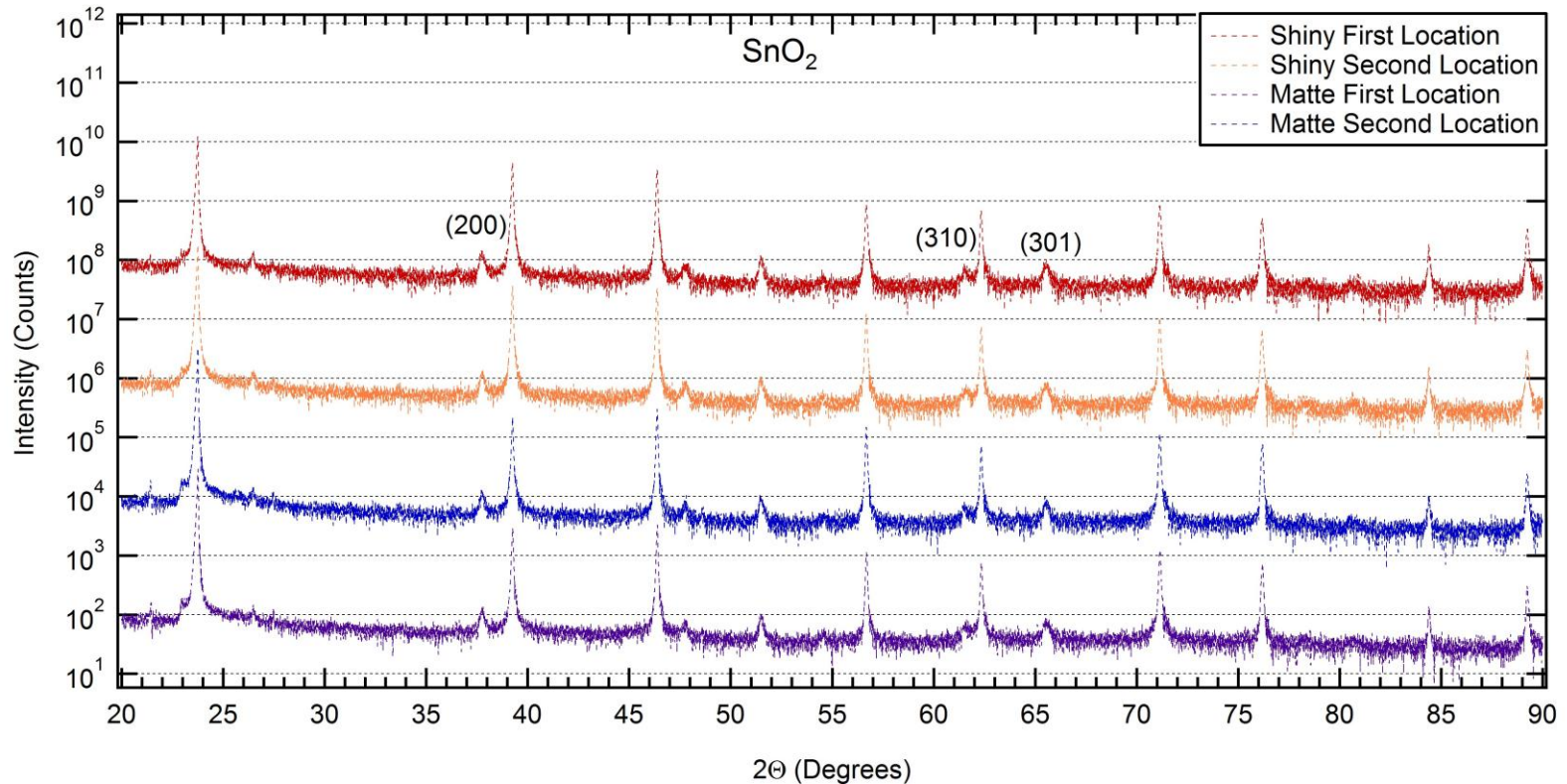
Peaks were considered if they were known CdTe peaks. Peaks from other layers (ex. CdS) were not included.

# Raw Data



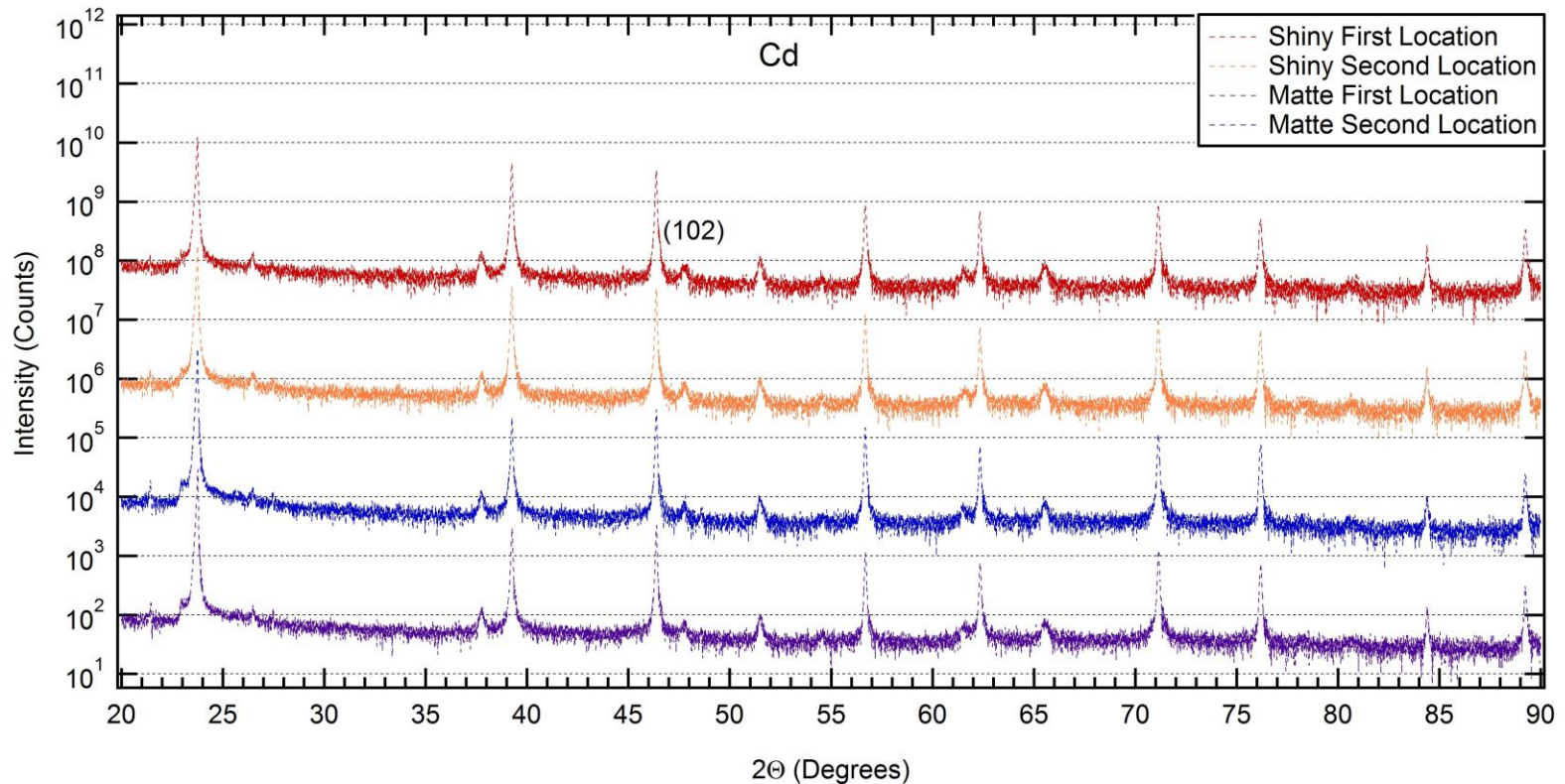
Peaks were considered if they were known CdTe peaks. Peaks from other layers (ex. CdS) were not included.

# Raw Data



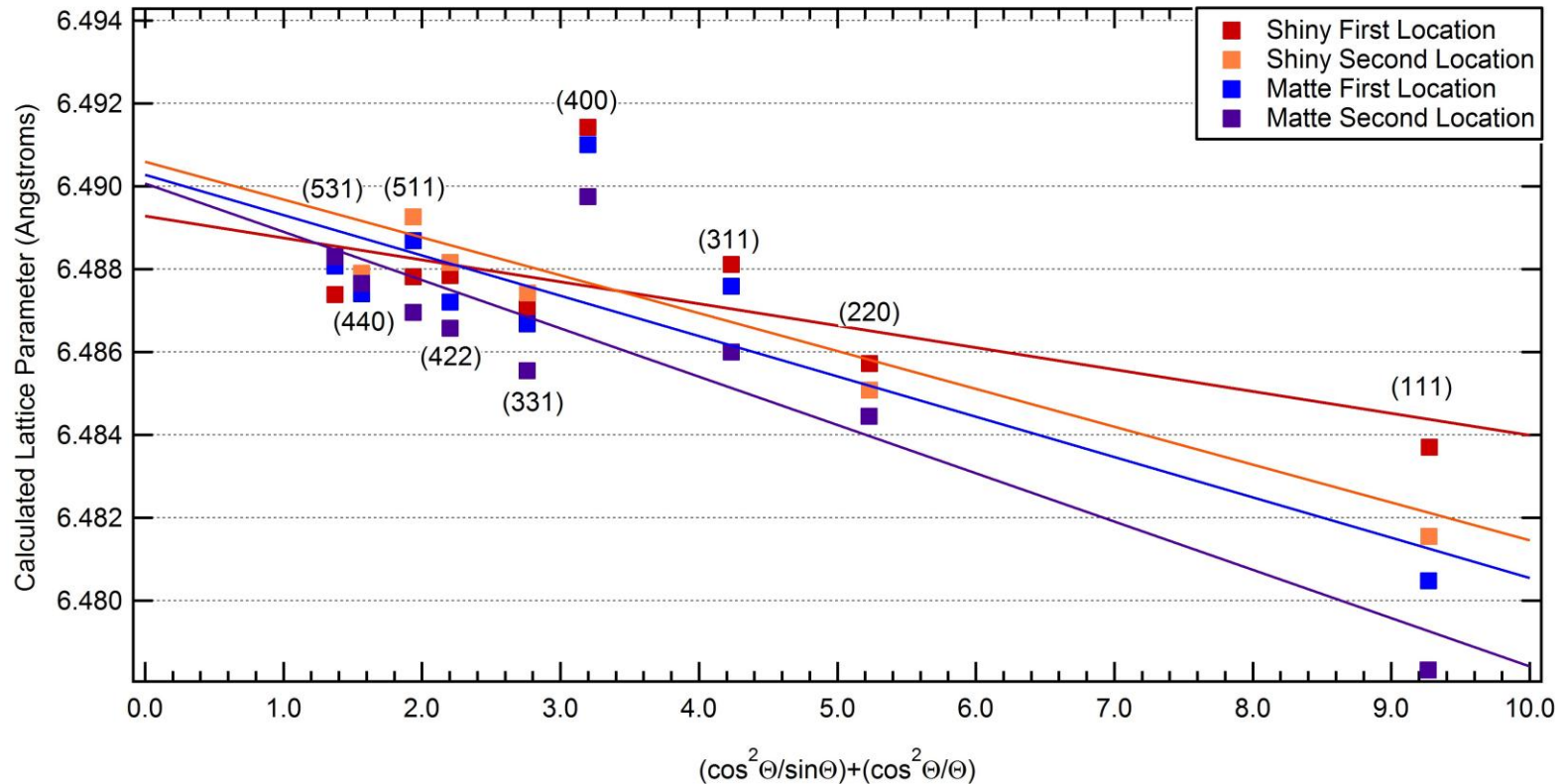
Peaks were considered if they were known CdTe peaks. Peaks from other layers (ex. CdS) were not included.

# Raw Data



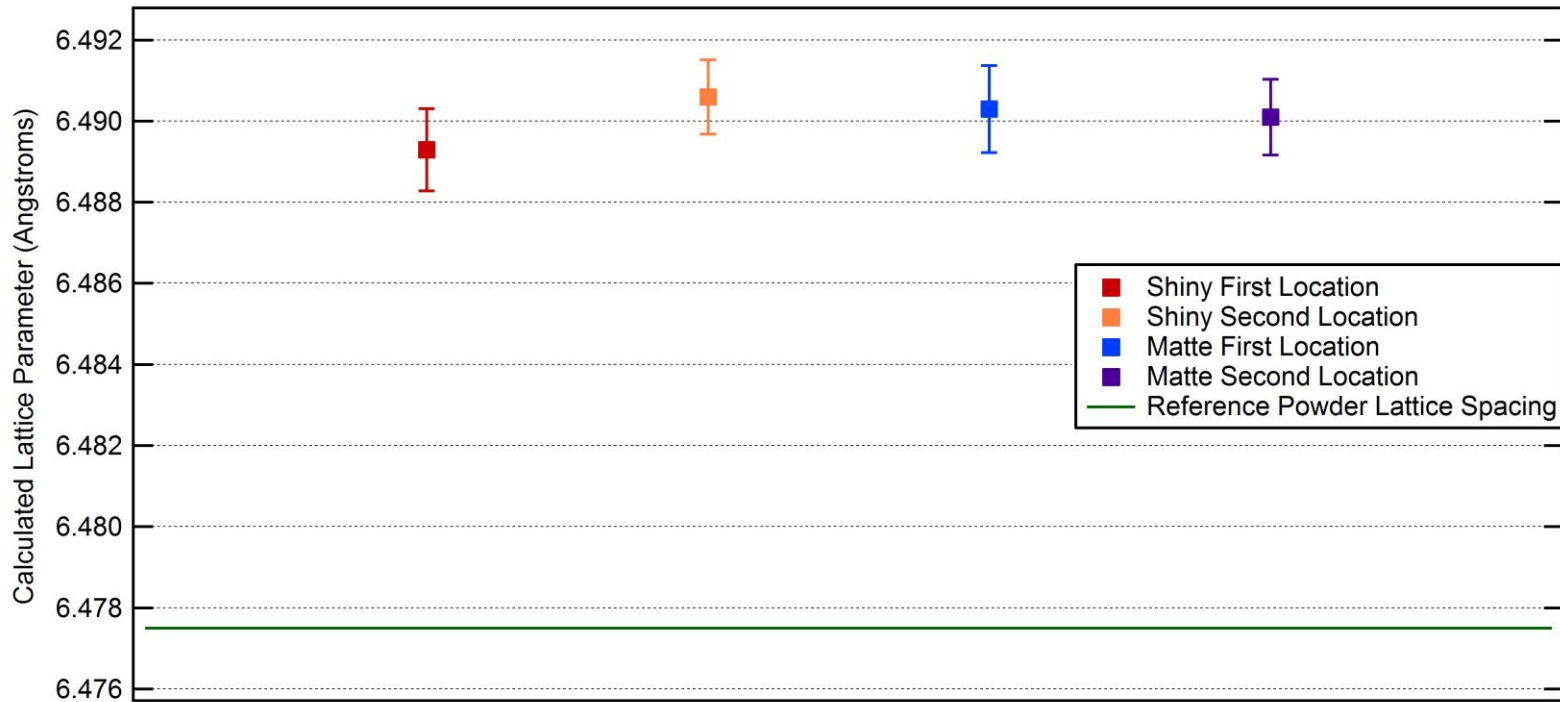
Peaks were considered if they were known CdTe peaks. Peaks from other layers (ex. CdS) were not included.

# Calculated Lattice Parameter



Data points calculated using: 
$$a = \frac{\lambda \sqrt{h^2 + k^2 + l^2}}{2 \sin\theta}$$

# Lattice Parameter



Orientation factor calculated using:  $p = \frac{I}{\Sigma I} \times \frac{\Sigma I'}{I'}$  Grain size calculated using:  $\tau = \frac{K\lambda}{\beta \cos\theta}$

	Shiny First Location	Shiny Second Location	Matte First Location	Matte Second Location
Orientation Factor $p$ for (111) Orientation	1.32	1.60	2.00	1.90
Average Grain size $\tau$ (nanometers)	264.86	302.70	302.70	325.99
Lattice Parameter $a$ (angstroms)	6.4893 ± 0.0010	6.4906 ± 0.0009	6.4903 ± 0.0011	6.4901 ± 0.0009