

Integrating the Solar Spectrum

PHYS 4400, Principles and Varieties of Solar Energy

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Pop Quiz

Note: quiz does not count toward grade...

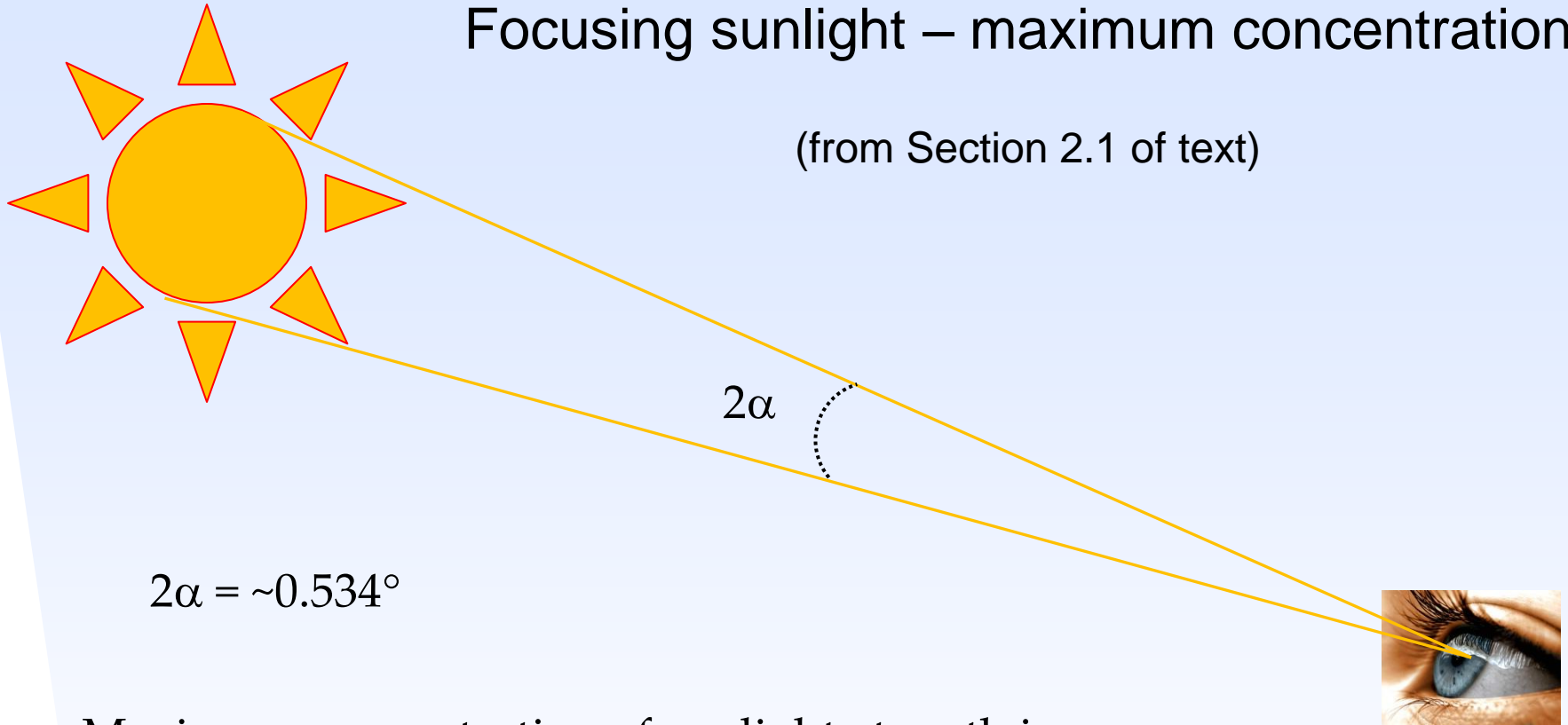
Write down the approximate wavelengths associated with these colors:

1. Green
2. Blue-green
3. Yellow
4. Red
5. Orange
6. Deep blue
7. Ultraviolet
8. Near-infrared
9. Infrared



Focusing sunlight – maximum concentration

(from Section 2.1 of text)



$$2\alpha = \sim 0.534^\circ$$

Maximum concentration of sunlight at earth is approximated by $C_{\max} = n^2 \cdot \sin^{-2}(\alpha) \rightarrow$

$$C_{\max} \cong 46,000$$

How do we calculate α ?

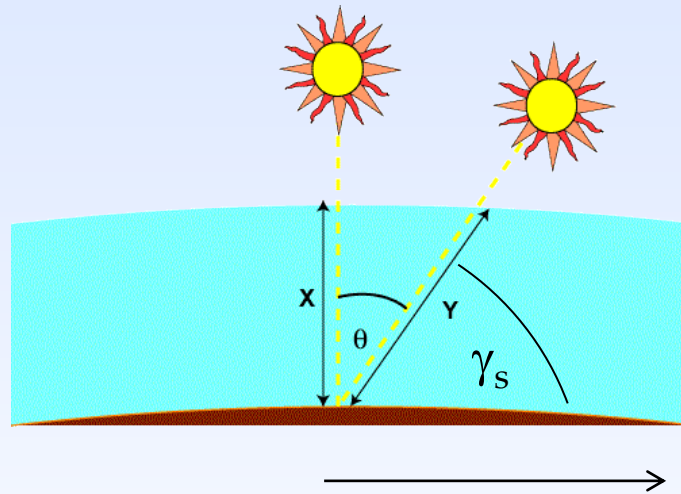


Air Mass – examples (Toledo, Ohio)

At 1 pm today (January 30), the sun will be at an elevation angle of $\gamma_s = 30.5^\circ$.

$$AM = \frac{1}{\cos \theta}$$

$$AM = \csc(\gamma_s) = \frac{1}{\sin(\gamma_s)}$$



Therefore, on Jan. 30 at 1 pm, assuming clear weather: $AM = 1.97$.
How about June 21, at noon? In that case, $\gamma_s = 70.3^\circ$, so that $AM = 1.06$.

What about the intensity (Direct, and Global) of the sunlight expected June 21?

$$I_D = 1353 \times 0.7^{(AM^{0.678})} = 933.6 \text{ W} \cdot \text{m}^{-2}$$

$$I_G = 1.1 \times I_D = 1,027 \text{ W} \cdot \text{m}^{-2}$$

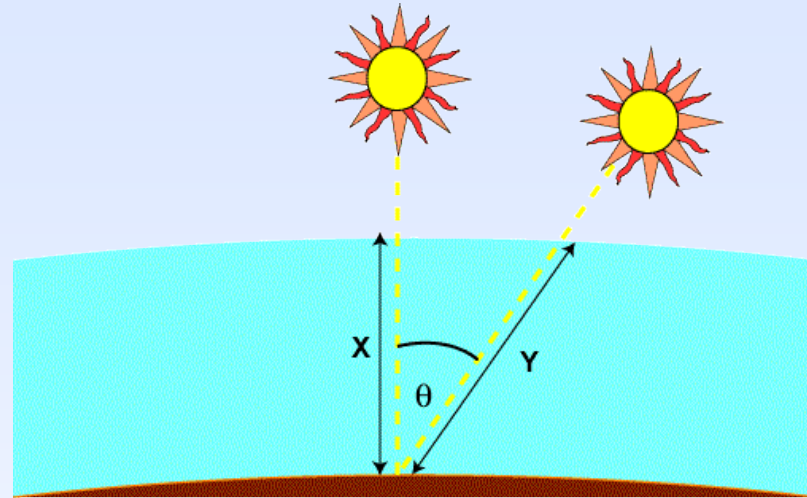
→ 1,022 W/m² on June 21



Air Mass (continued)

$$AM = \frac{1}{\cos \theta}$$

$AM \neq \infty$ when $\theta = 90^\circ$



$$AM = \frac{1}{\cos \theta + 0.5057(96.07995 - \theta)^{-1.6364}}$$

In-class exercises:

- Calculate the intensity of sunlight in Toledo for January 29 at 1:00 pm using $AM = 1.97$.
- Calculate intensity again using the AM as calculated by the equation accounting for Earth's curvature.



Air Mass (continued)

In-class exercise:

- Calculate the intensity of sunlight for AM1.5.

$$AM = \frac{1}{\cos \theta + 0.5057(96.07995 - \theta)^{-1.6364}}$$

Solar intensity vs zenith angle z and airmass coefficient AM

z	AM	range due to pollution ^[12]	formula (I.1)	ASTM G-173 ^[11]
degree		W/m ²	W/m ²	W/m ²
-	0	1367 ^[15]	1353	1347.9 ^[16]
0°	1	840 .. 1130 = 990 ± 15%	1040	
23°	1.09	800 .. 1110 = 960 ± 16% ^[17]	1020	
30°	1.15	780 .. 1100 = 940 ± 17%	1010	
45°	1.41	710 .. 1060 = 880 ± 20% ^[17]	950	
48.2°	1.5	680 .. 1050 = 870 ± 21% ^[17]	930	1000.4 ^[18]
60°	2	560 .. 970 = 770 ± 27%	840	
70°	2.9	430 .. 880 = 650 ± 34% ^[17]	710	
75°	3.8	330 .. 800 = 560 ± 41% ^[17]	620	
80°	5.6	200 .. 660 = 430 ± 53%	470	
85°	10	85 .. 480 = 280 ± 70%	270	
90°	38		20	

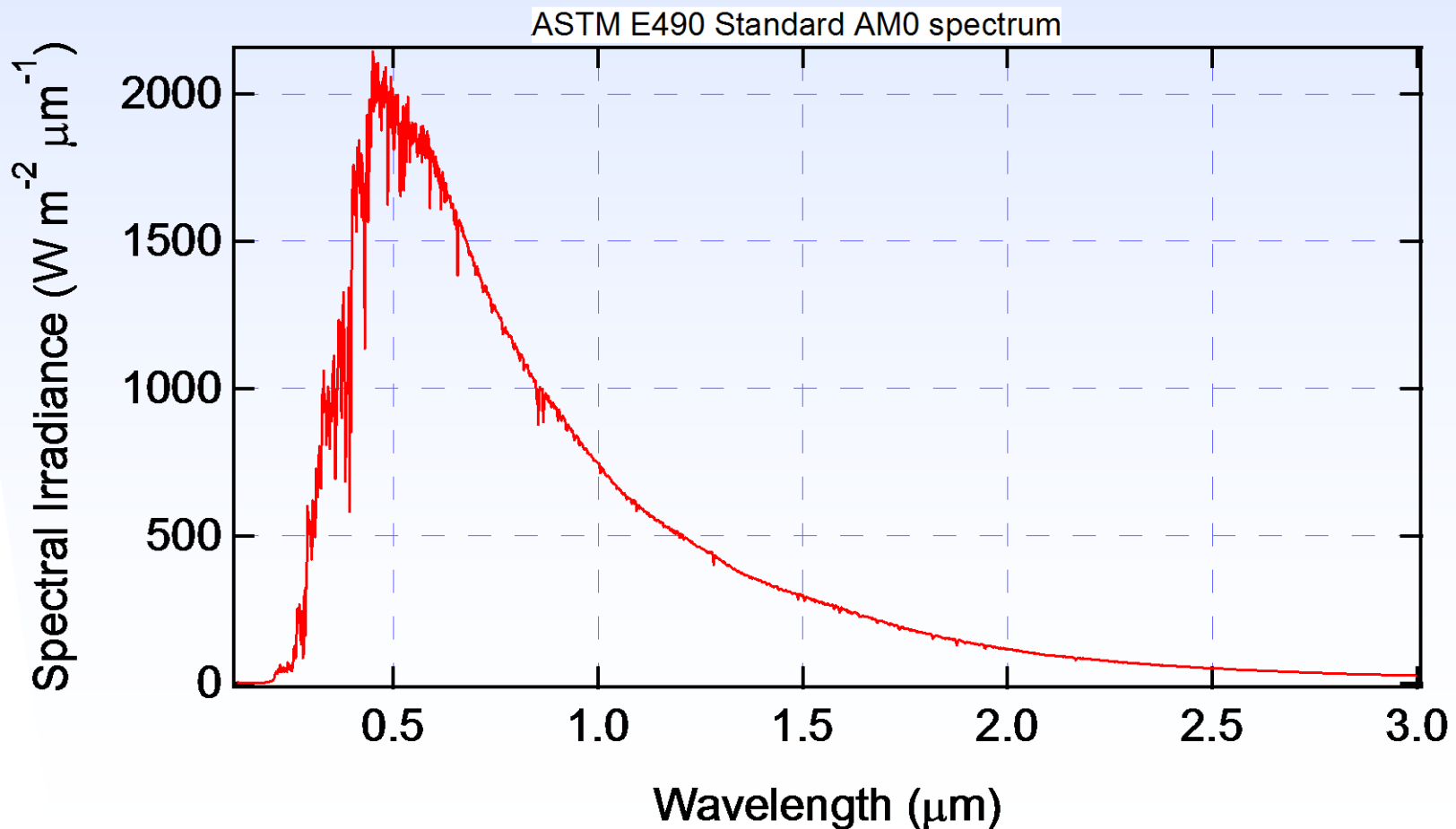
This illustrates that significant power is available at only a few degrees above the horizon.



AM0: the spectrum above Earth's atmosphere

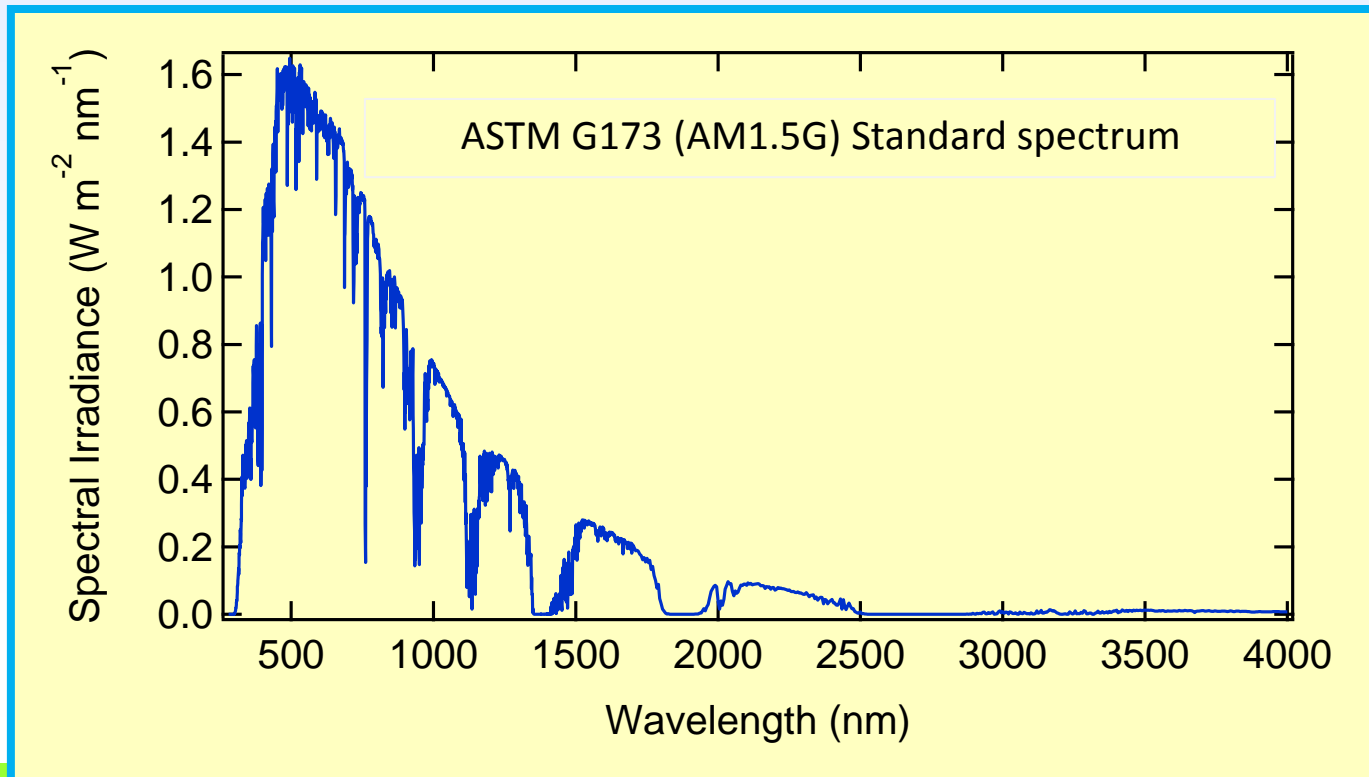
The AM0 spectrum applies to satellites and high-flying aircraft, which access the spectrum prior to any influence from Earth's atmosphere.

Integrated spectral irradiance = 1366 W/m^2 .



AM1.5G: reference spectrum including direct and diffuse sunlight

- AM 1.5: From the equation provided for Air Mass, one calculates that $\cos \theta = 0.667$, so that $\theta = 48.2^\circ$. This represents the zenith angle, that is, the angle relative to the direction normal to Earth's surface.
- From the standard: "The receiving surface is defined in the standards as an inclined plane at 37° tilt toward the equator, facing the sun (i.e., the surface normal points to the sun, at an elevation of 41.81° above the horizon)." Note that 41.8° is the complement of 48.2° .
- Toledo latitude: 41.6639° N
- Integrating the energy within AM1.5G (ASTM G-173) yields 1000 W m^{-2} .



Standard Solar Reference Spectra

Where do these spectra come from (where can we get them)?

Start here: <http://rredc.nrel.gov/solar/spectra/>

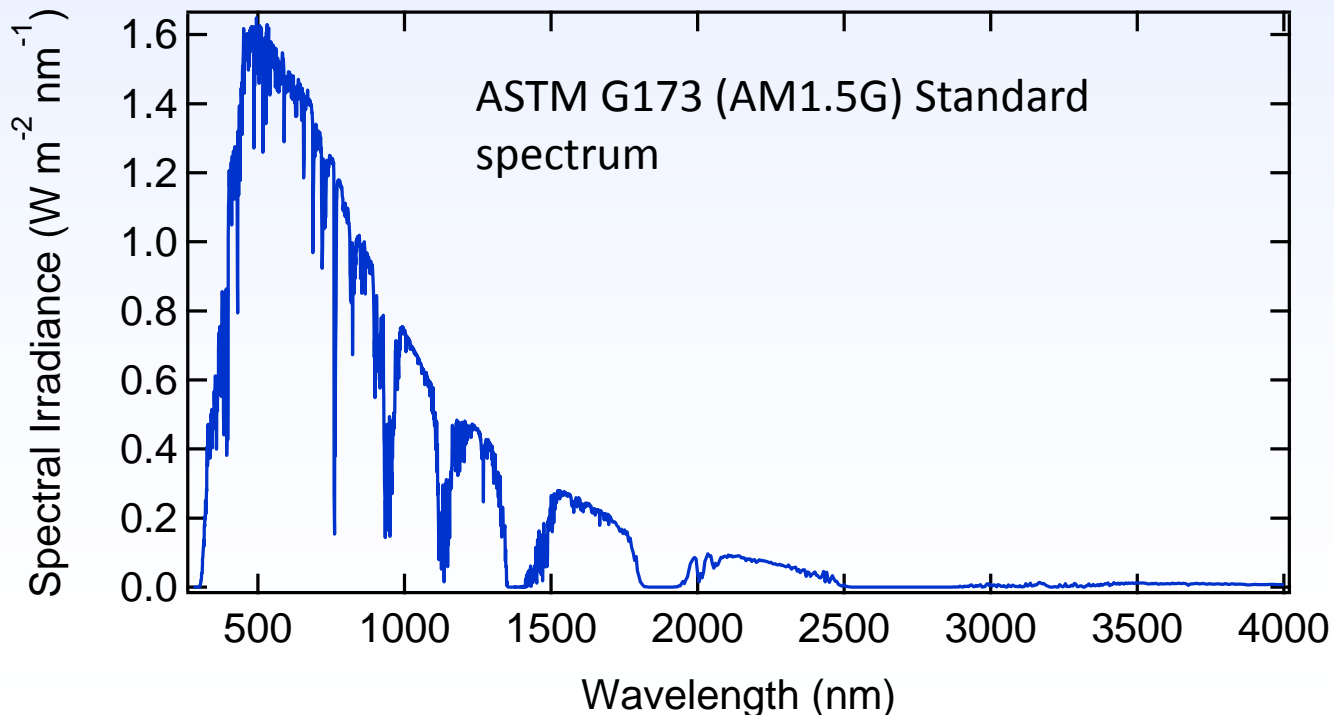
The spectra most often referenced are the AM1.5G (technically referred to as the **ASTM G-173**) and the AM0 (technically known as the **ASTM E-490**).



What's in a square meter area of sunlight?

Assuming that the receiving area is normal to the incoming sunlight:

- Power, which when integrated with respect to time sums up to Energy. For example, $1,366 \text{ W/m}^2$ of sunlight, integrated for 1 hour, gives $(1366 \text{ W/m}^2) * (1 \text{ hr}) * (3600 \text{ s/hr}) * (1 \text{ J/s per W}) = 4.92 \times 10^6 \text{ J/m}^2$.
- Photons. A very large number of photons per second (as we will find). As a look ahead, each photon with energy above a semiconductor's bandgap can be absorbed, boosting an electron from the valence band to the conduction band and contributing an electron to the *photocurrent* of the PV cell.



Why does it matter what's in a square meter area of sunlight?

Photovoltaic solar cells respond differently depending on the energy of the incident photons.

Each photon absorbed within a PV cell generates one electron which can be collected as photocurrent.

Only those photons with energy greater than the bandgap energy of the semiconductor are absorbed; a semiconductor is typically transparent to photons of energy lower than the band gap energy.

<http://pveducation.org/pvcdrom/pn-junction/band-gap>

<http://pveducation.org/pvcdrom/pn-junction/absorption-of-light>



In-class exercises: integration of the solar spectra

How can we add up the power contained within the AM0 or AM1.5G spectra? *Answer: add up the values for the spectral irradiance – but we must do so carefully.*

If we have values for the Spectral Irradiance with each data point corresponding to a 1 nm spectral width (e.g., from 500 nm to 501 nm), the problem is simpler. However, upon examining the data contained in either of these spectra:

http://rredc.nrel.gov/solar/spectra/am0/E490_00a_AM0.xls

<http://rredc.nrel.gov/solar/spectra/am1.5/ASTMG173/ASTMG173.xls>

we find that the wavelength values start at 0.5 nm spacing, then move the 1 nm spacing, and ultimately go to 5 nm or larger spacing between points.

To properly integrate the spectrum, we can either break up the data into regions depending on the wavelength increment between data points, or we can interpolate the data to “fix it so that it is spaced by 1 nm for all wavelength regions.

Let’s interpolate, after which we can simply add up the values, effectively multiplying each value by the 1 nm of spectral bandwidth to which it applies...

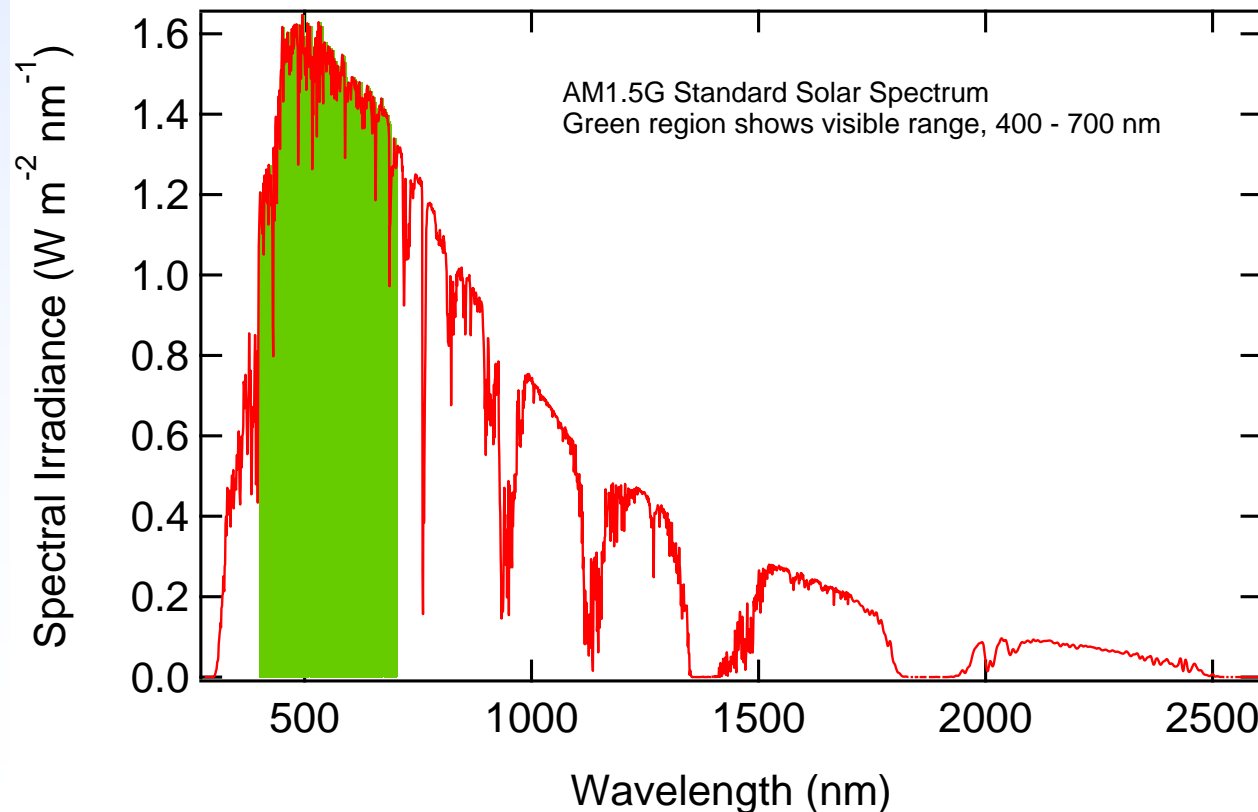
The interpolated data for AM0 and AM1.5G is provided on the course web site:

http://astro1.panet.utoledo.edu/~relling2/teach/4400.2014/spring2014_phys4400.html



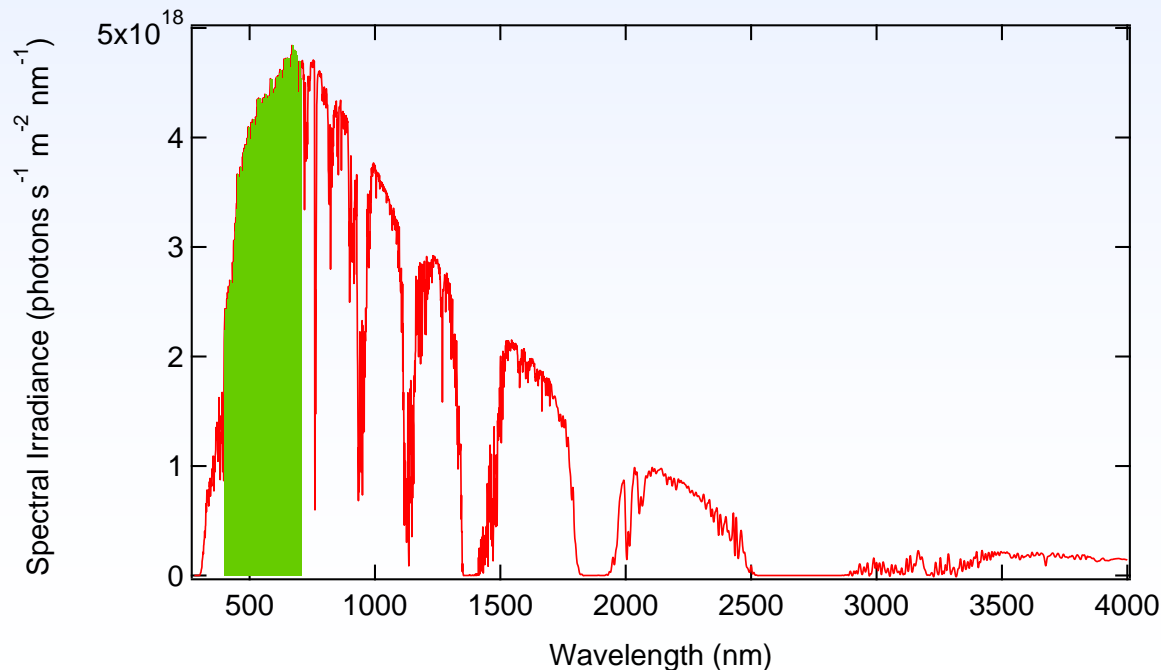
In-class exercise: integration of the AM1.5G solar spectrum

- How much power per unit area is contained within the AM1.5 spectrum? *This is much more easily answered using the interpolated spectra – summing from 400 nm to 700 nm yields 431.03 W m^{-2} .*
- What fraction of the total power of the AM1.5G spectrum falls within the visible range (400-700 nm)? *Summing the AM1.5G over the full range from 280 to 4000 nm gives 1000.36 W m^{-2} , so the fraction of power contained in the “visible” = 43.1 %.*

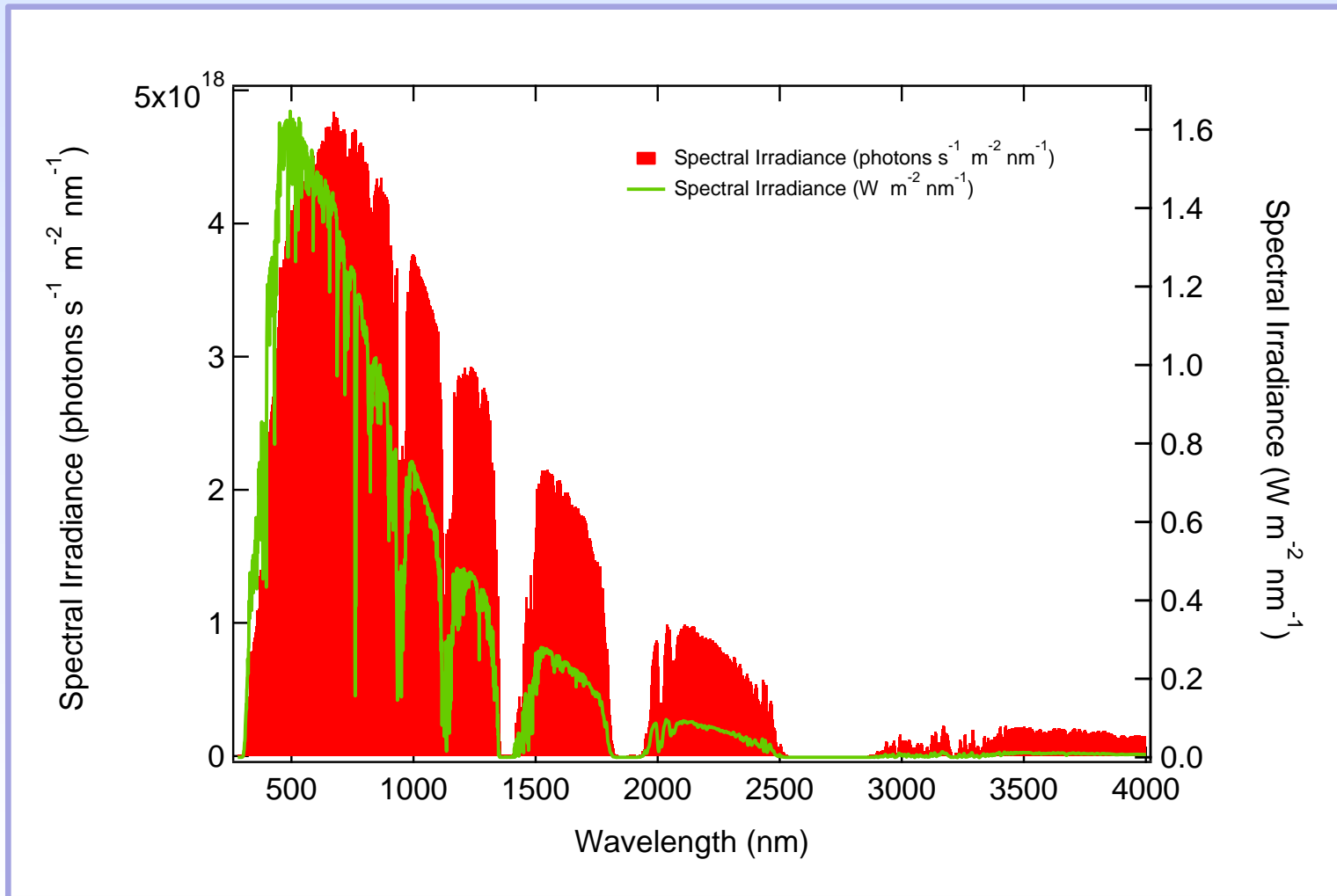


In-class exercises: integration of the solar spectra

- Conversion of spectral irradiance from ($\text{W m}^{-2} \text{nm}^{-1}$) to ($\text{photons s}^{-1} \text{m}^{-2} \text{nm}^{-1}$) is required to calculate photon flux within solar spectra.
- Accomplished by dividing the ($\text{W m}^{-2} \text{nm}^{-1}$) values by the photon energy, to convert W to photons/second...
- How many photons per second per unit area are incident within the visible portion of the AM1.5G spectrum? *For 400 nm to 700 nm range, we calculated 1.19×10^{21} photons $\text{s}^{-1} \text{m}^{-2}$.*
- What fraction of the total AM1.5G photons (from 280 to 4000 nm) lie in this visible range? *We calculate a total irradiance of 4.30×10^{21} photons $\text{s}^{-1} \text{m}^{-2}$, so the fraction in the visible is $1.19/4.30 = 27.7\%$.*



Comparing shape of spectra, W vs. photons/s



- What's going on here?

