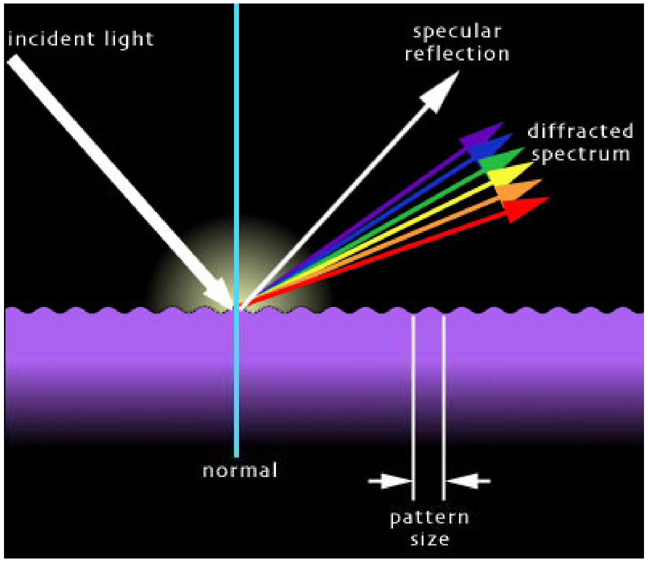
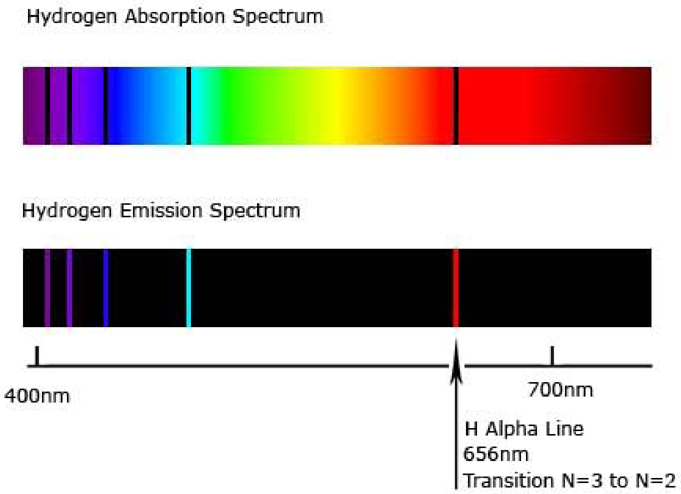
Grating Spectrometers and Atomic Spectroscopy of Hydrogen, Deuterium, and Helium

Week of October 25, 2010





Atomic and Nuclear Physics Laboratory (Physics 4780)

The University of Toledo
Instructor: Randy Ellingson

Atomic Structure: **Review of "Atomic" vs. "Nuclear" Physics**

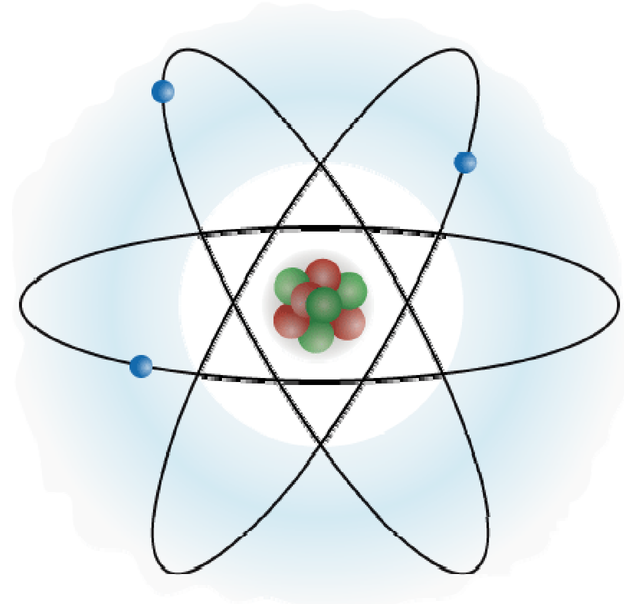
- Nucleus: positively charged core containing protons and neutrons;
- Electrons exist in specific energy levels (orbits) around the nucleus;
- Element determined by # of protons. Atoms of the same element which differ in number of neutrons are referred to as *isotopes*. Remember: # of protons is referred to as the *atomic number* (Z).
- A specific nuclide can be annotated as follows:

$$^{A}_{P}X$$

where *A* is the atomic mass number (# of protons + # of neutrons), *P* is the # of protons, and *X* is symbol for the element. Examples:

$^{1}_{1}H$ "hydrogen-1" $^{4}_{2}He$ "helium-4"

- Nuclear processes involve (surprise) the nucleus of the atom (e.g., radioactive decay including alpha and gamma decay); atomic processes involve (in general) electron processes (e.g. XRD, visible light emission).



Masses:

$m_e = 9.11 \times 10^{-31}$ kg

$m_n = 1.67 \times 10^{-27}$ kg

$m_p = 1.67 \times 10^{-27}$ kg

Revisiting Alpha Spectroscopy Experiments

- *Activity* refers to the number of decays of a specific source, per second. *Activity* has nothing to do with how far the source is from a detector, a person, etc. because in any measurement of the activity one corrects for the solid angle subtended by the detector. For a small uniform source of ^{241}Am, emission of alphas occurs in all directions uniformly.
- Everyone had trouble with Parts 3 and 4 of the alpha spectroscopy lab, so we'll rediscuss those and re-analyze our data to determine the <u>range</u> (at 1 atmosphere of pressure) of our ^{241}Am alphas in air, and in CO_2.

Atmospheric Pressure

- **Atmospheric pressure:** force per unit area against a surface resulting from the weight of air in the Earth's atmosphere above that surface.
- The standard atmosphere (1 atm): unit of pressure, defined as 101.325 kPa, or 760 mmHg (or 760 Torr), 29.92 inHg, 14.696 psi, 1013.25 millibars.
- Reducing the pressure in a chamber involves removing "atmosphere" or gas from that chamber. Removing a fraction 1/f of the gas in a chamber reduces the pressure in that chamber by the same fraction 1/f (creating a partial vacuum, unless f = 1).

Ideal Gas Law

- **Ideal Gas Law:** *PV = nRT*, where P is pressure, V is the volume of the gas, n is the number of moles of gas, R is the universal gas constant, and T is Temperature in Kelvin.

$$R = 8.314472 J \cdot mol^{-1} K^{-1}$$

- Taking n = m/M where m is the mass of the gas present and M is the molar mass, on can write *PV = mRT/M*. Bringing *V* over, one can write $P = \rho RT/M$, which shows us that for fixed volume, <u>pressure varies linearly with density</u>. Once again:

$$P = \frac{\rho RT}{M}$$

Our (Special) Vacuum Gauge

- "Trouble", if we were to give it a nickname. Why? 1. The gauge reads "0" when the chamber is at 1 atm = 760 mmHg, and 2. The gauge reads *larger* numbers as we reduce the pressure in the chamber (!), and 3. when we've reached our best vacuum, the gauge reads 745 mmHg (what's up with that?).
- Details: the gauge reads the pressure *relative to* 760 mmHg (the p values are inherently negative relative to air). We don't know whether the gauge is off at "low pressure" or not (is the "-745 mmHg" reading actually supposed to be "-760 mmHg", or are we in fact at 15 mmHg absolute pressure in the chamber? -- we don't know the answer).

What is meant by "Effective Length", anyway?

- One key goal of the alpha spectroscopy lab is to determine the "range" of 5.48 MeV alphas in air, and in CO_2, at 1 atm = 760 mmHg pressure. I.e., How far do our alphas travel in either air or CO_2 at 1 atm before they come to rest?
- The general idea: watch the alpha particles' energy as they travel a given distance (source-to-detector distance) of gas (air or CO_2) as a function of pressure. Since the density varies linearly with pressure, we can use an equation $R_0 = R_p(p/p_0)$ to compute, for any given pressure p, the equivalent length ("effective length) of gas *at 1 atm*. Examples: $R_0 = 0$ when $p = 0$; $R_0 = R_p$ when $p = p_0$. Note that R_p is the actual source-detector distance, and p_0 is standard atmospheric pressure (760 mmHg).

Alpha particles at risk

- Back to the key question: How far do our alphas travel in either air or CO_2 at 1 atm before they come to rest?
  What eventually happens to an alpha particle? In our case, they eventually come to rest and recombine with a couple of electrons to create a He atom. When the alphas come to rest, they have essentially zero energy (OK, perhaps kT or 0.025 eV, almost zero compared to 5.48×10^6 eV they start with).
- So, by figuring out the effective length of gas at which they come to rest, we can assign that length as the "range of (5.48 MeV) alpha particles" in that gas at 1 atmosphere
- B-I-N-G-O! …

Your Data: worthless or valuable? How do we get that "range" value?

- All of you acquired data in the form of Channel # (of the alphas peak) vs. pressure reading from the gauge. Interesting, right? Something like this (happens to be for air, and for R_p = 5 cm):

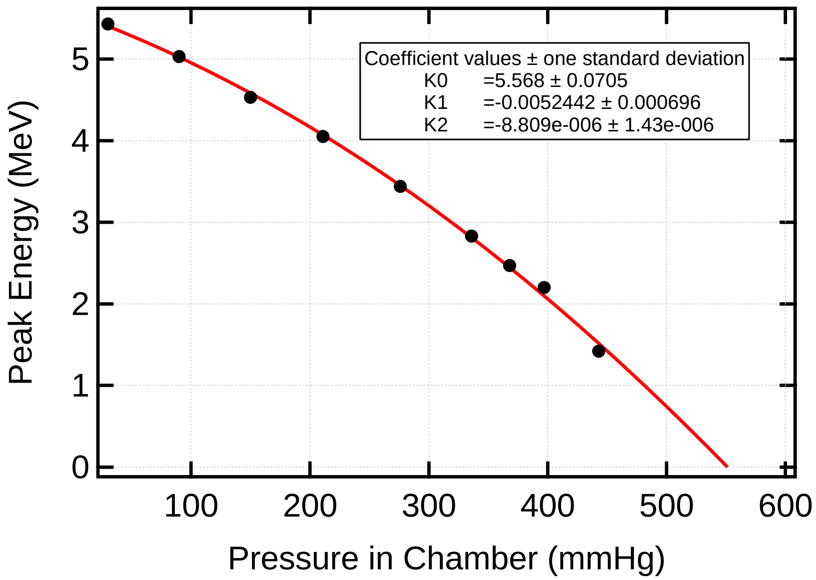
Peak Location (ch. #)	Gauge Pressure (mmHg): P_{gauge}
2177	730
2015	670
1812	610
1616	549
1369	484
1125	424
980	392
870	363
554	317

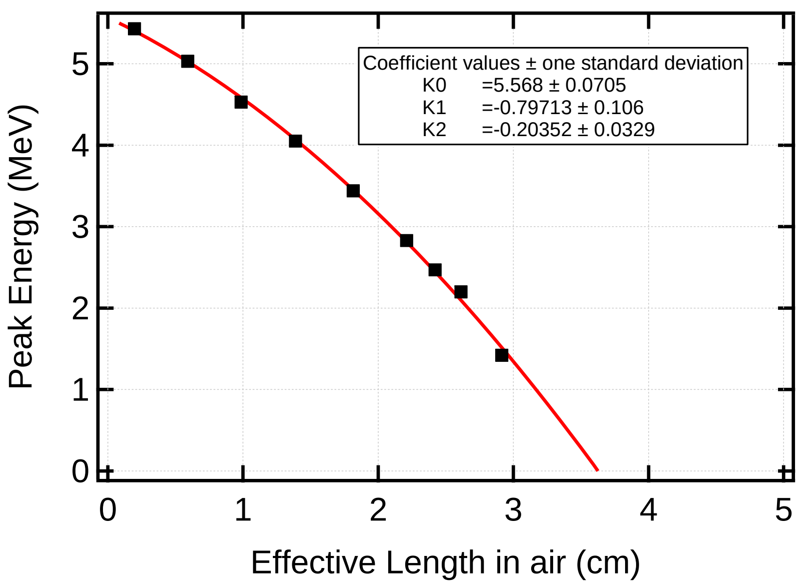
- Make the data useful by (a) converting the pressure to the pressure in the chamber instead of the pressure value read off the gauge: Pressure in the chamber = 760 mmHg – pressure read off gauge:

$$P_{chamber}=760 \text{ mmHg} - P_{gauge}$$

- Convert your pressure ($P_{chamber}$) to an effective length by $R_0 = R_p(P_{chamber}/760 \text{ mmHg})$. For example, P_{gauge} = 730 mmHg indicates an *effective range* of R_0 = 4 cm (730/760) = 3.84 cm (assuming your R_p = 4 cm).
- Convert the channel # to an energy (MeV) using the calibration you established in Part 1 of the lab.

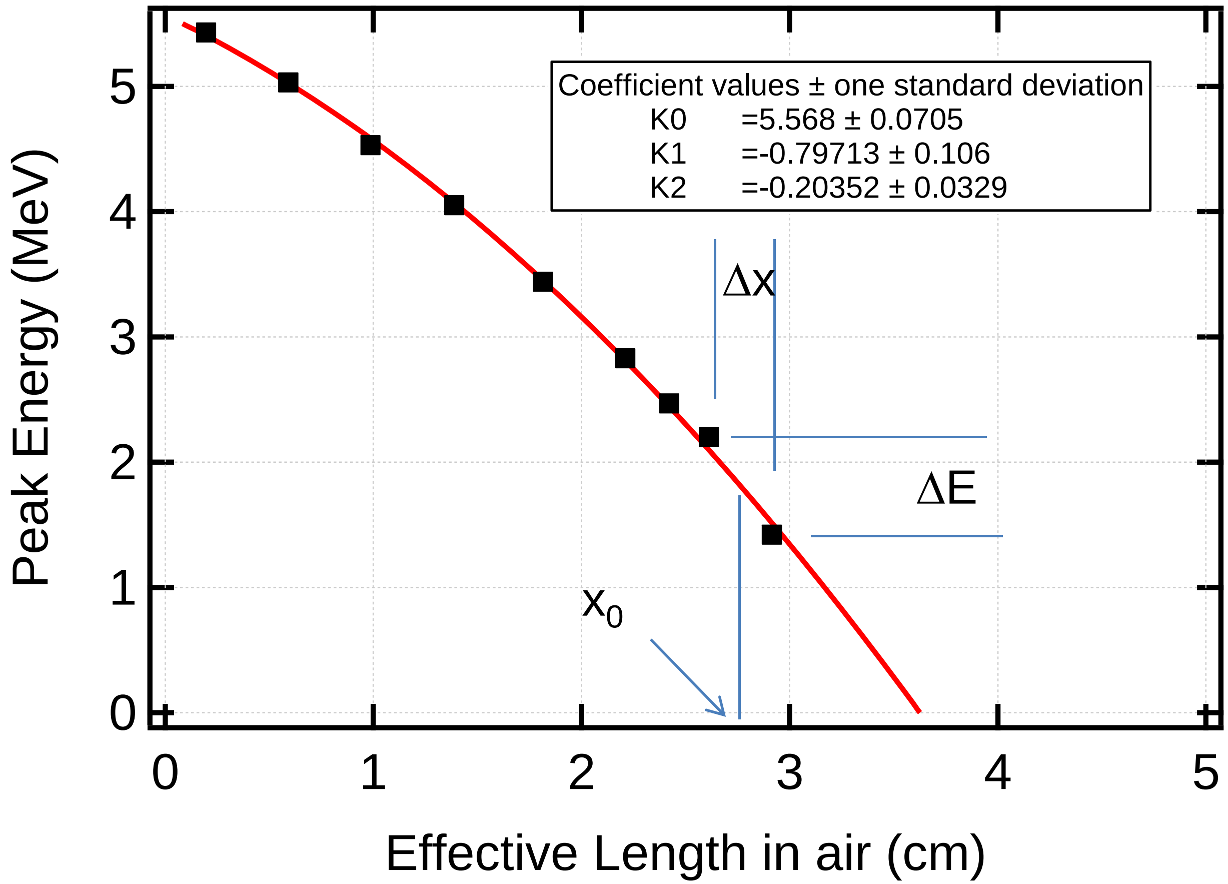
Graphing your data





- Top graph shows the alpha peak energy vs. chamber pressure, with a 2nd-order (3-term) polynomial fit (very easy in Igor Pro – use Output Option, X Range Full Width of Graph for the fit). Extrapolating to zero energy (when the alphas have come to rest) tells us the pressure at which the alphas just don't quite arrive at the detector. That pressure can be converted to an effective length in the gas.
- The bottom graph shows the same data, except plotted for Effective Length. Extrapolating to zero energy tells us the Effective Length in the gas at which the alphas come to rest (and therefore don't quite make it to the detector) – i.e., the "range" in that gas.
- Note: Your data may differ somewhat since these measurements were made for a source-detector distance R_p of 5 cm 9and you likely used 4 cm).

What about dE/dx?



Generate a simple dE/dx plot by taking each pair of consecutive data points and figuring out ΔE and Δx where Δx is the changing in the effective length of gas traveled. Use the center 'x' value (x_0) for each value of dE/dx. Note that dE/dx values are negative. Plot those values on a new graph (not shown here) and comment on the trend of the rate of energy loss vs. distance traveled.

What about Part 4) anyway?

Part 4) of the lab guide requested analysis of the same type of "range" for alphas in air and in CO_2, but instead of tracking energy of the alphas (via the channel # of the peak), we were supposed to connect the detector output directly to the counter to counmt the total pulses (each pulse was an alpha particle detection).

As the pressure was methodically and gradually increased, the counts would rop to zero. Right as they dropped to zero, the pressure in the chamber would indicate the effective length at which the alphas no longer reach the detector (i.e., the "range" of the alphas in the gas being studied.

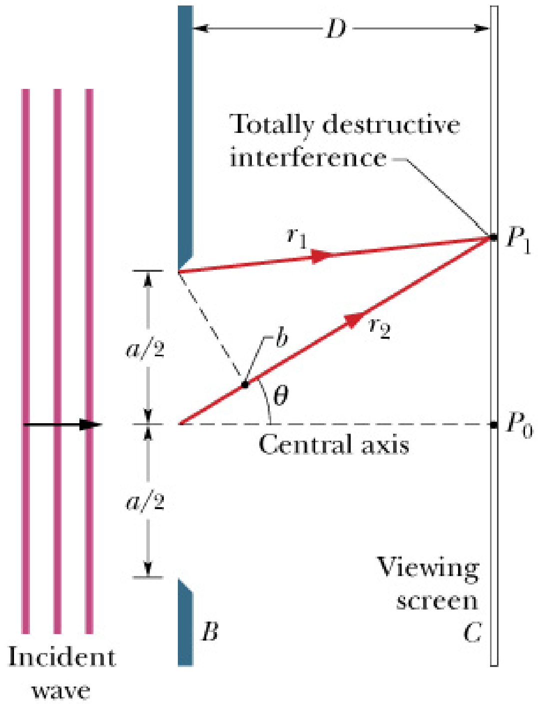
Fixing your data analysis for Part 3 of the alpha particle spectroscopy lab (don't worry about Part 4)

- Prepare a concise and careful report on Part 3 of the alpha spectroscopy lab, following the examples and guidance provided in the previous slides.
- Starting with your data acquired in lab for air and for CO_2 (confirm the validity of the data with your peers and/or TA if necessary), re-analyze your data to properly determine the approximate range for ^{241}Am alphas in 1 atm of each of those gases.
- Include a table of your data for air and for CO_2 (pressure reading (mmHg), actual pressure in chamber (mmHg), peak channel #, and peak energy (MeV)).
- Include graphs of Peak Energy (MeV) vs. Pressure (in the chamber, mmHg) and Peak Energy (MeV) vs. Effective Length (cm) for each gas, and show the fit from which you extrapolate the zero energy values for P and for effective length.
- Include graphs of dE/dx for air and for CO_2 and discuss which gas has a larger "stopping power" for these alphas (i.e., which gas shows a shorter range at 1 atm).

• ***Note: this correction should be an "easier" lab report, so ask questions as necessary to make sure you understand the physics and the experiment. This "correction" report will be optional, unweighted,* and worth a maximum of 5 points. It is due Nov. 5th at 5 pm (for this report only, late submission will be worth zero points).***

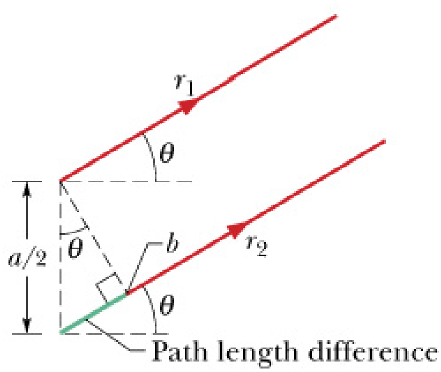
****In other words, these points will add to the sum of your lab report scores.***

Diffraction by a Single Slit: Locating the *Minima*



(a)

When the path length difference between rays r_1 and r_2 is $\lambda/2$, the two rays will be out of phase when they reach P_1 on the screen, resulting in destructive interference at P_1. The path length difference is the distance from the starting point of r_2 at the center of the slit to point b.

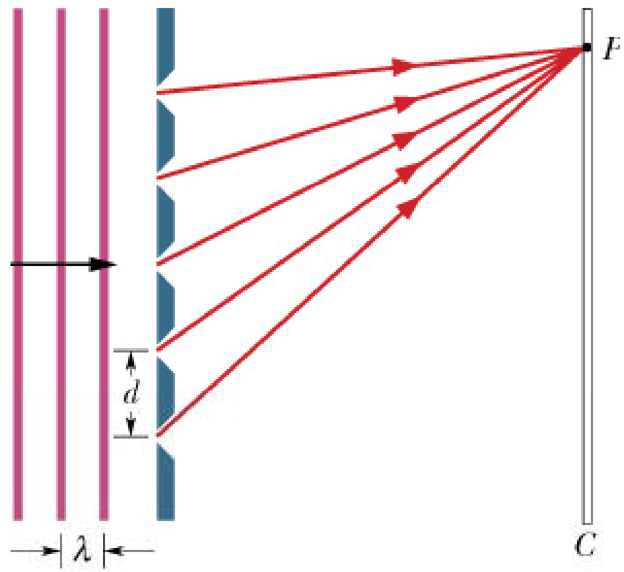


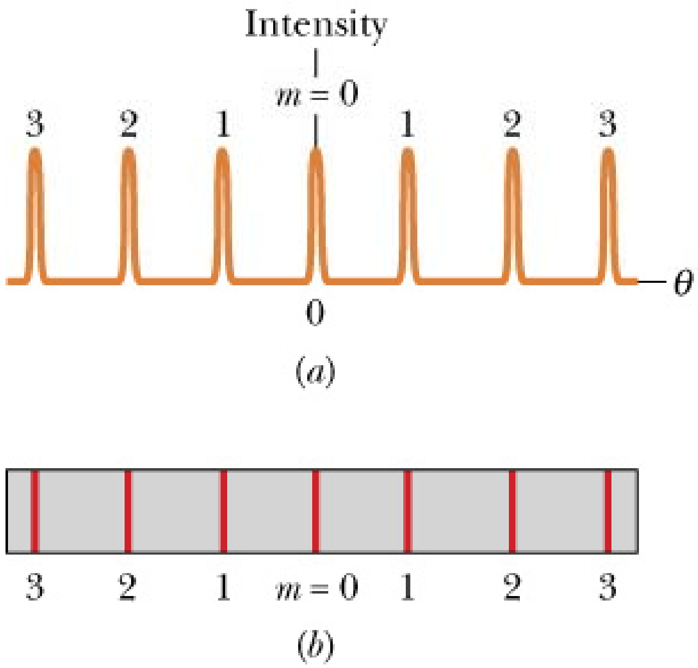
(b)

For $D \gg a$, the path length difference between rays r_1 and r_2 is $(a/2) \sin \theta$.

Diffraction Gratings

Device with *N* slits (rulings) can be used to separate different wavelengths of light that are contained in a single beam. How does a diffraction grating affect monochromatic light?

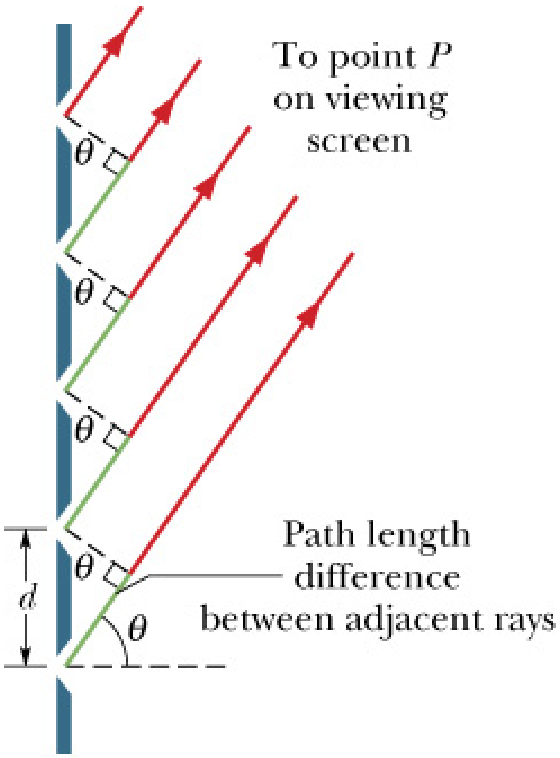


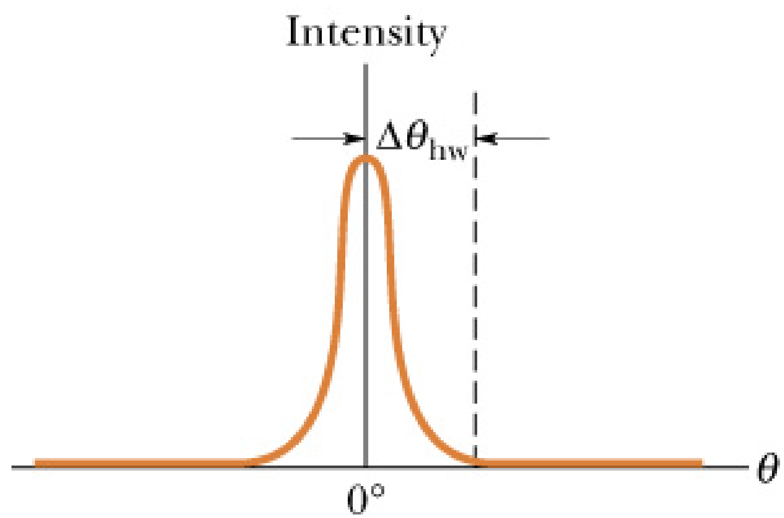


$$d \sin\theta = m\lambda \quad \text{for } m = 0, 1, 2 \ldots \quad \text{(maxima-lines)}$$

Width of Lines

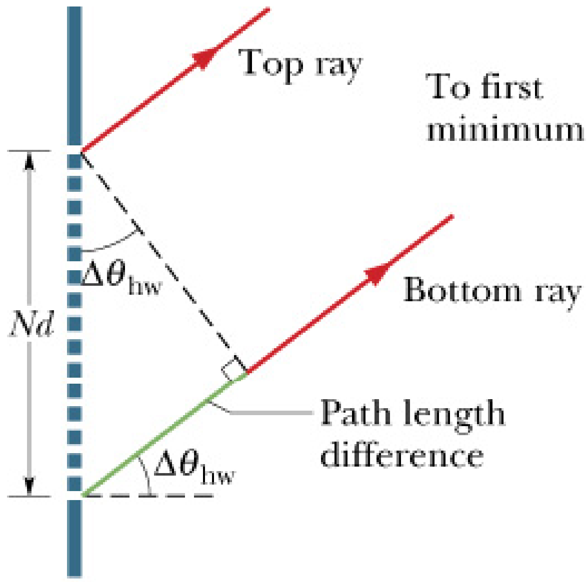
The ability of the diffraction grating to resolve (separate) different wavelengths depends on the width of the lines (maxima)





Width of Lines, cont'd

A wave is roughly defined as any longitudinal wave (energy moving along the direction of wave propagation).



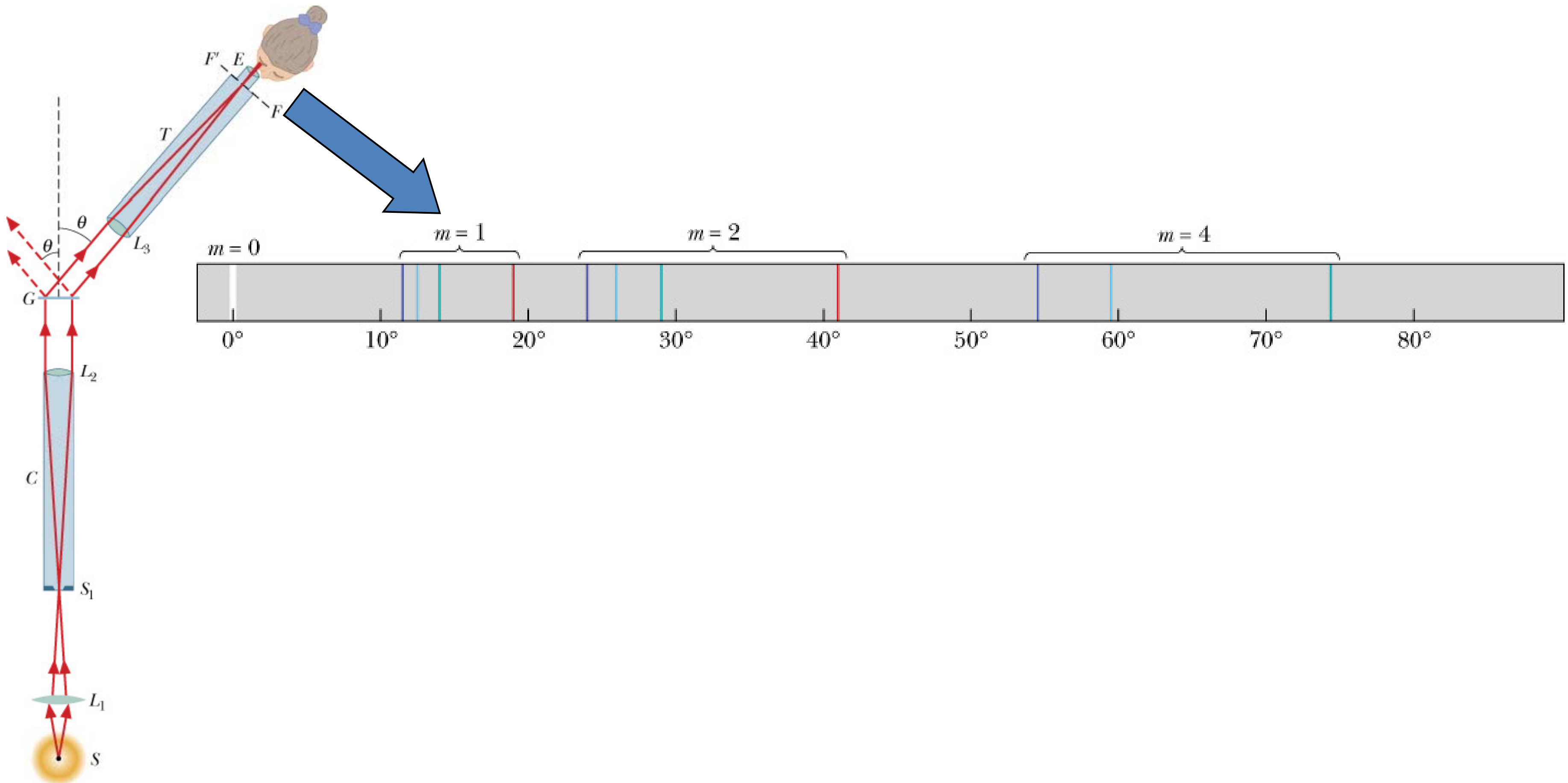
$$Nd \sin \Delta\theta_{\text{hw}} = \lambda \quad , \qquad \sin \Delta\theta_{hw} \approx \Delta\theta_{hw}$$

$$\Delta\theta_{\text{hw}} = \frac{\lambda}{Nd} \quad \text{(half width of central line)}$$

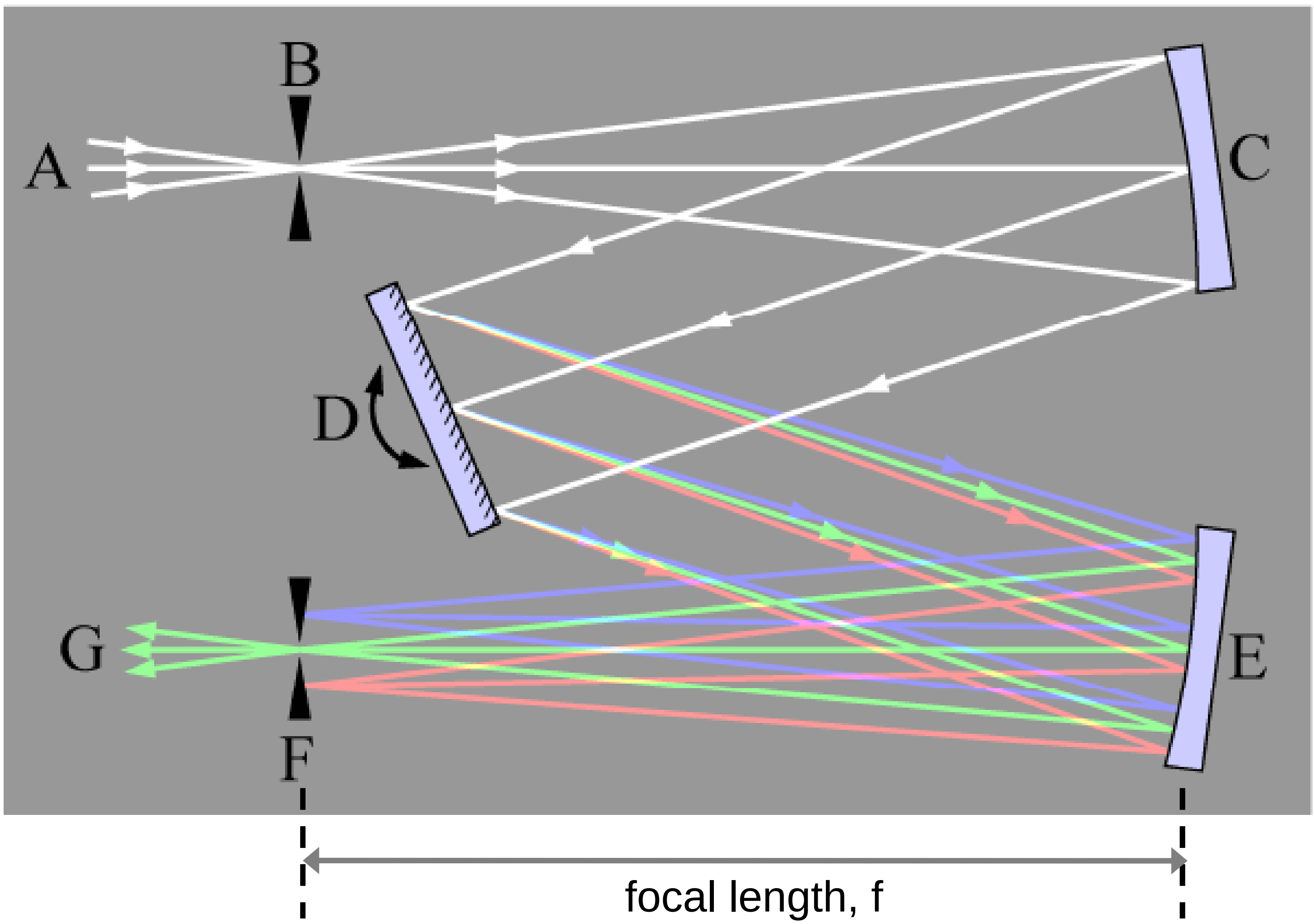
$$\boxed{\Delta\theta_{\text{hw}} = \frac{\lambda}{Nd \cos\theta} \quad \text{(half width of line at } \theta\text{)}}$$

Grating Spectroscope

Separates different wavelengths (colors) of light into distinct diffraction lines (this image shows a transmission diffraction grating, while most spectrometers use reflective diffraction gratings).



Czerny-Turner Monochrometer



Grating equation

http://www.cis.rit.edu/class/simg401/DiffractionGratingEquation.pdf

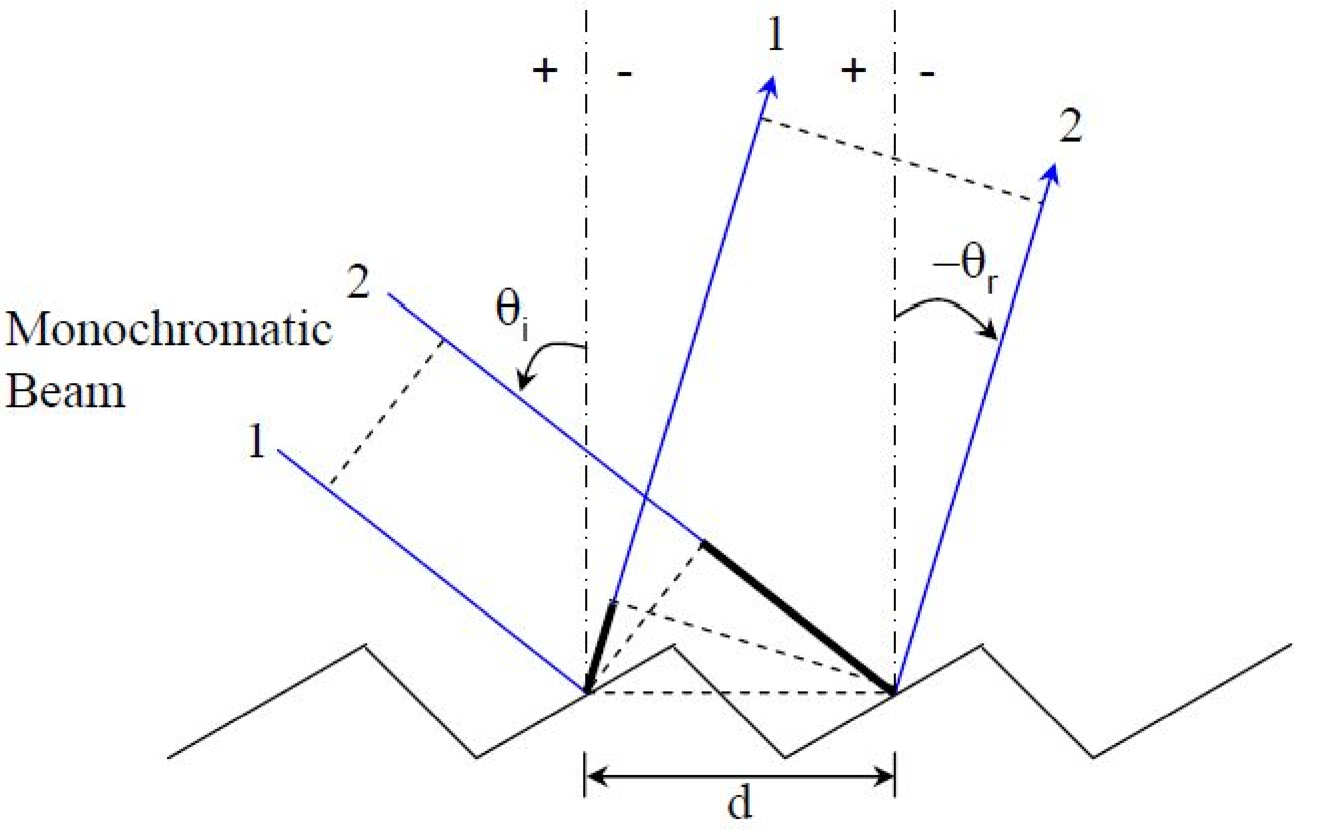
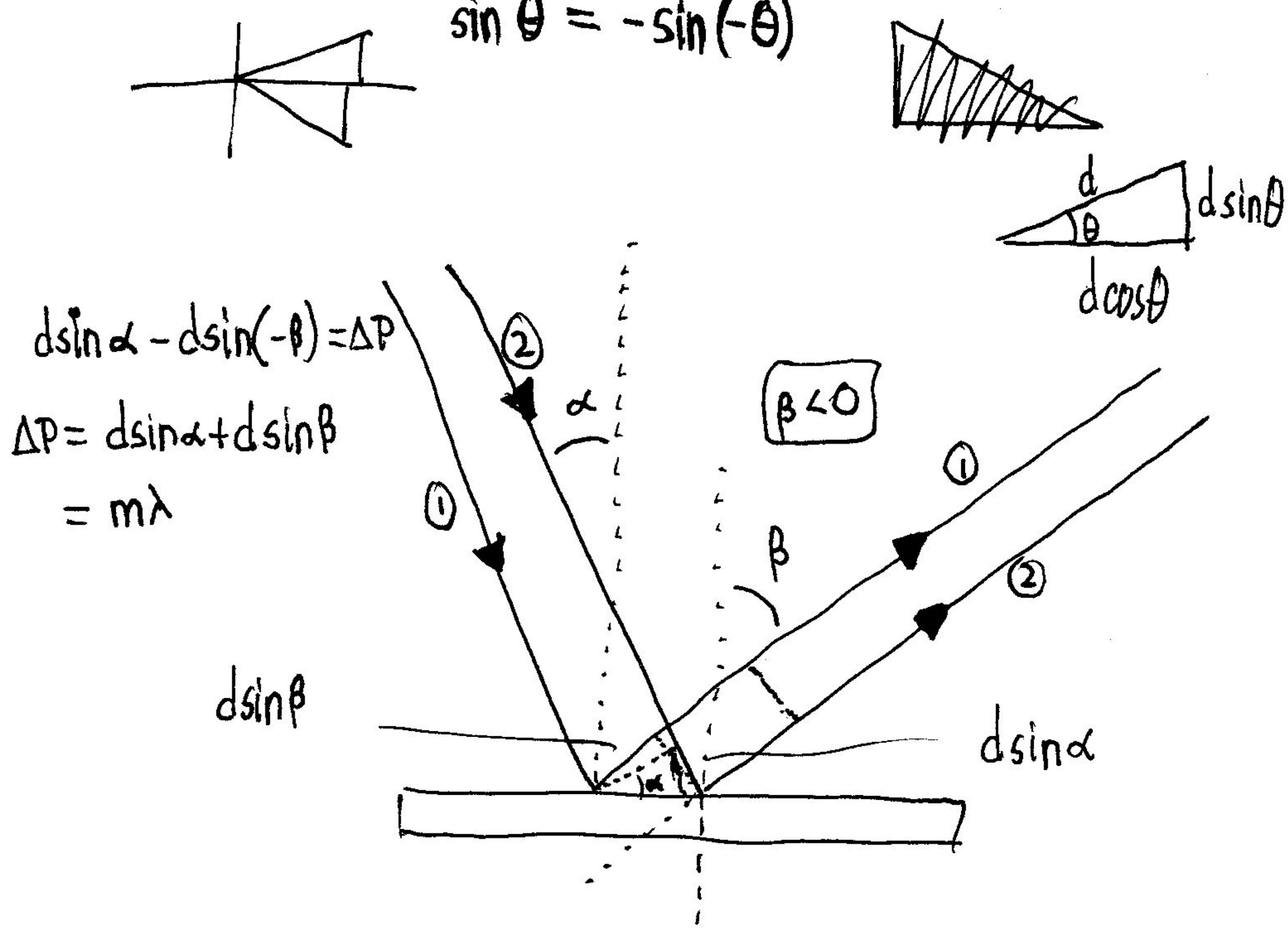
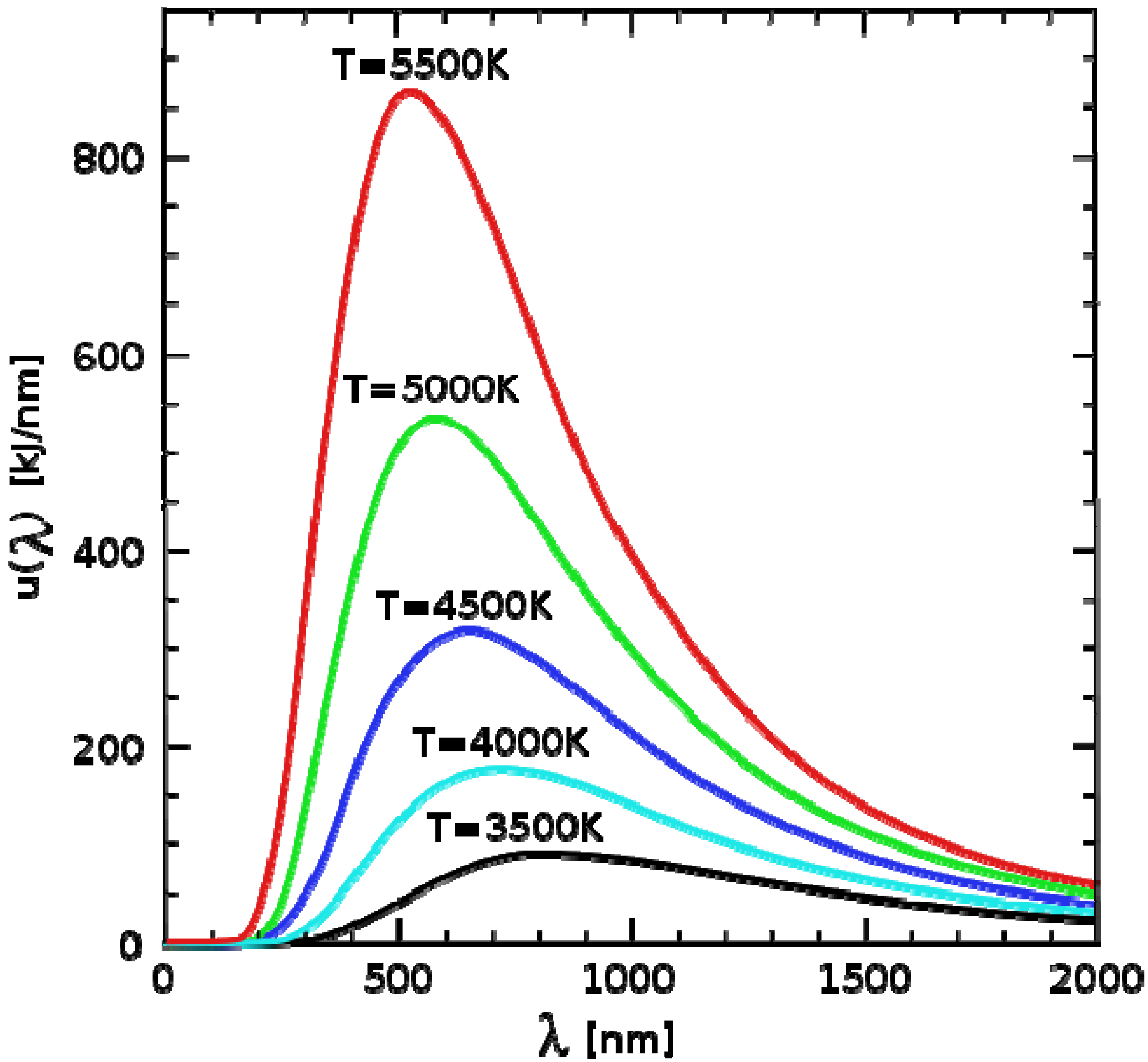


Figure 1: Monochromatic beam incident on (blazed) diffraction grating at angle θ_i and diffracted at angle $-\theta_r$. The blaze spacing is d.

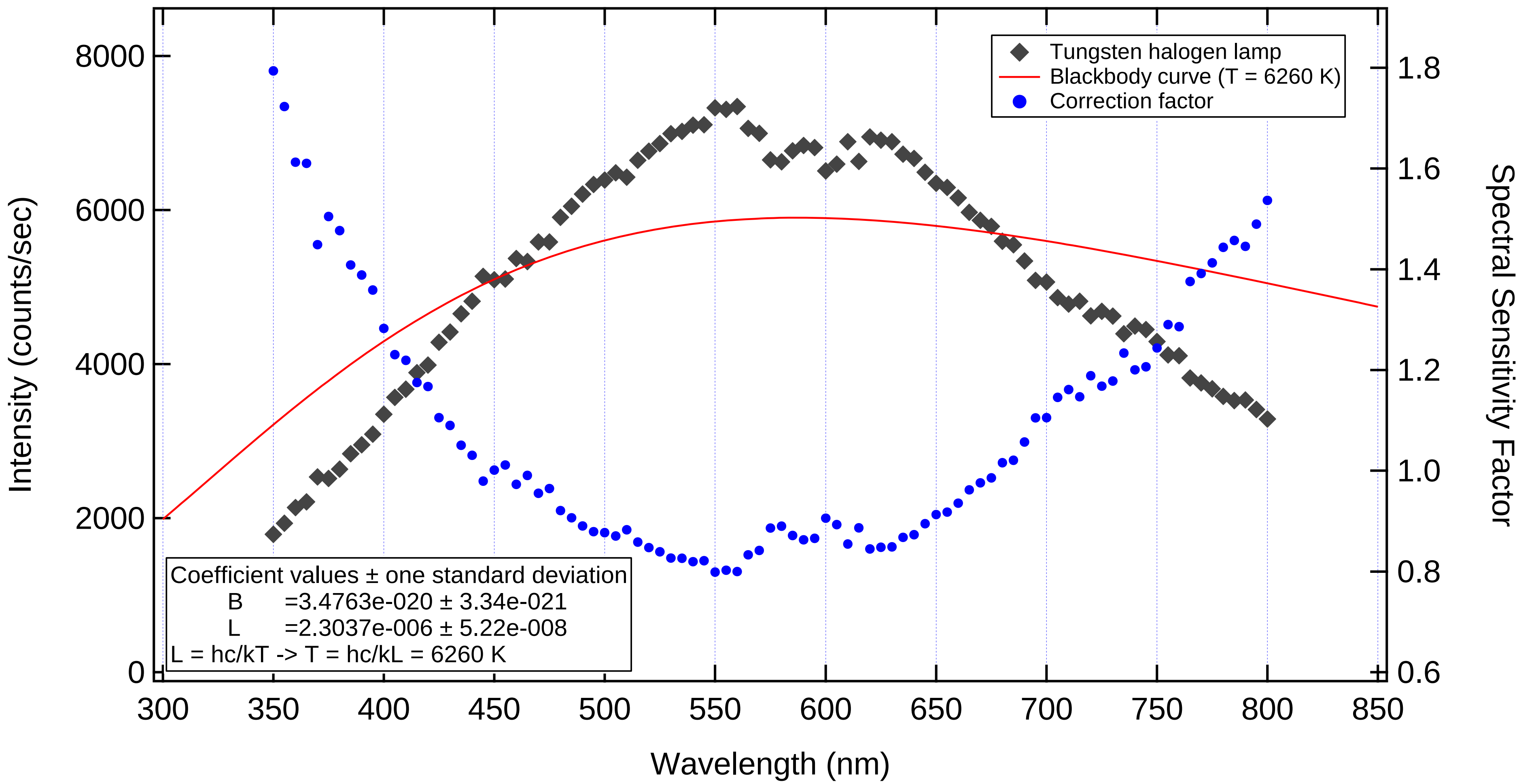
Grating equation



Spectrometer sensitivity calibration: black body radiation, grating efficiency, PMT sensitivity



Spectrometer sensitivity calibration (continued)



spectral photon flux (units of *photons/(time-area-Δλ)*):

$$\boxed{\frac{dN}{dt} \propto \frac{B}{\lambda^4 \left(e^{hc/\lambda kT} - 1\right)}}$$

Hydrogen atom (Bohr model)

$$\frac{m_e v^2}{r} = \frac{Z k_e e^2}{r^2}$$

Centripetal force is equal to the Coulomb force (Z is the atomic number).

$$E = \frac{1}{2} m_e v^2 = -\frac{k_e e^2}{2r}$$

Total energy at any radius

$$v = \sqrt{\frac{Z k_e e^2}{m_e r}}$$

Solve for velocity

$$m_e v r = n\hbar$$

Quantization of angular momentum

$$r_n = \frac{n^2 \hbar^2}{Z k_e e^2 m_e}$$

Substitute expression for velocity and solve for r_n

$$E_n = -\frac{\left(k_e e^2\right)^2 m_e}{2\hbar^2 n^2}$$

Energy of n^{th} level

Hydrogen atom (Bohr model)

$$E_n = -\frac{k_e{}^2 e^4 m_e}{2\hbar^2}\frac{1}{n^2}$$

Hydrogen atom energy levels, assuming infinitely massive nucleus (m_p/m_e = 1836.15). k_e is Coulomb's constant = 8.987551787 x 10^9 N·m²·C⁻². $k_e = 1/(4\pi\varepsilon_0)$

$$\alpha = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{\hbar c} \cong \frac{1}{137}$$

Fine structure constant (note that '≅' is not the same as '=') .

$$E_n = -\frac{\mu\alpha^2 c^2}{2}\frac{1}{n^2}$$

Incorporating reduced mass

$$\mu = \frac{m_e M}{m_e + M} = \frac{m_e m_p}{m_e + m_p}$$

Reduced mass due to finite mass of nucleus (m_p, for H-atom)

Hydrogen atom Balmer Series wavelength calculations

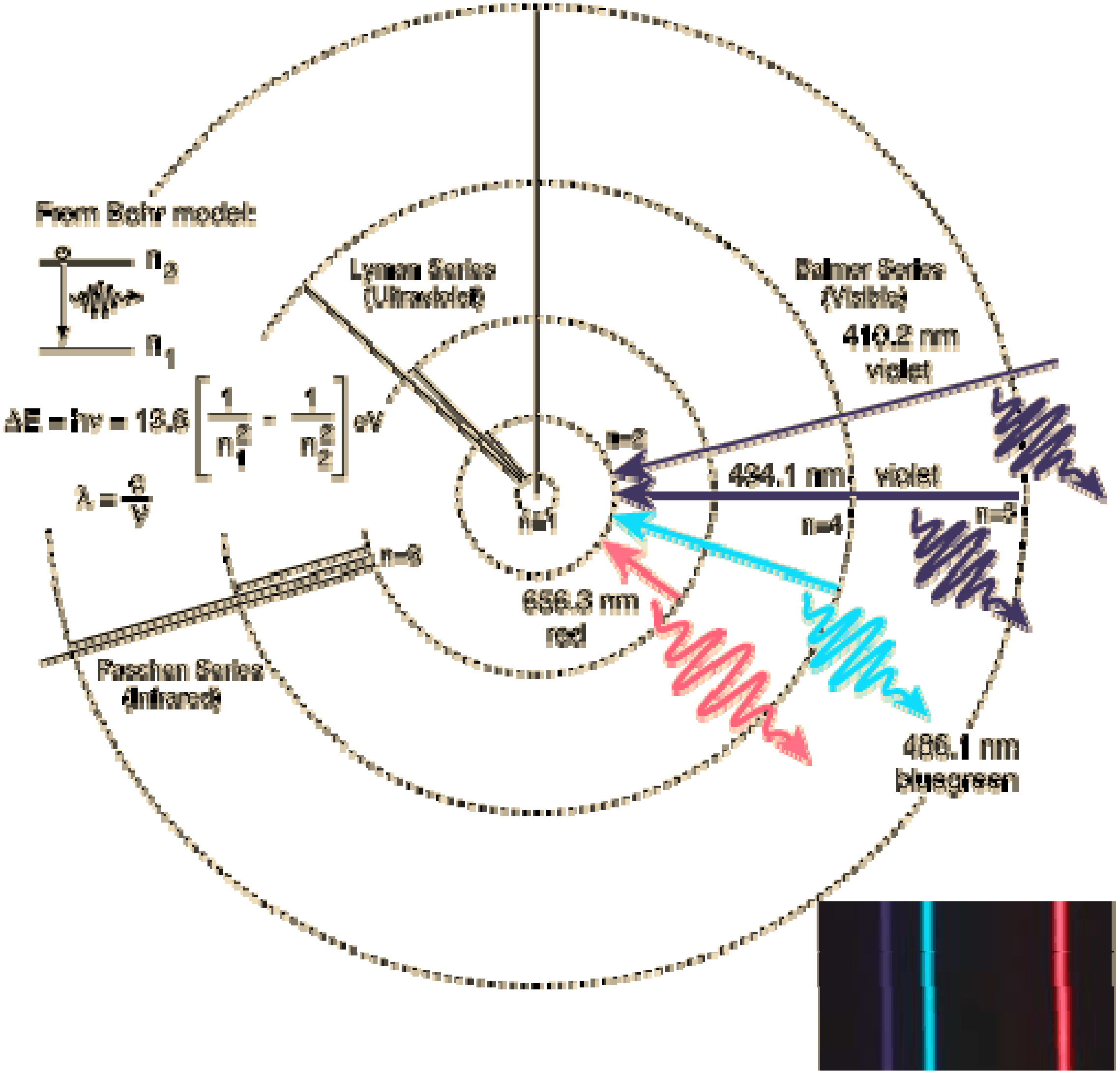
$$E_n = -\frac{k_e{}^2 e^4 \mu}{2\hbar^2}\frac{1}{n^2}$$

- The Balmer series refers to those transitions for which the H (or D) atom ends up in the n = 2 state, so they include n=3 to n=2, n=4 to n=2, and n=5 to n=2.
- The general expression for the energy of a photon emitted during such a transition is given by:

$$\Delta E = -\frac{k_e{}^2 e^4 \mu}{2\hbar^2}\left(\frac{1}{n_i^2} - \frac{1}{2^2}\right)$$

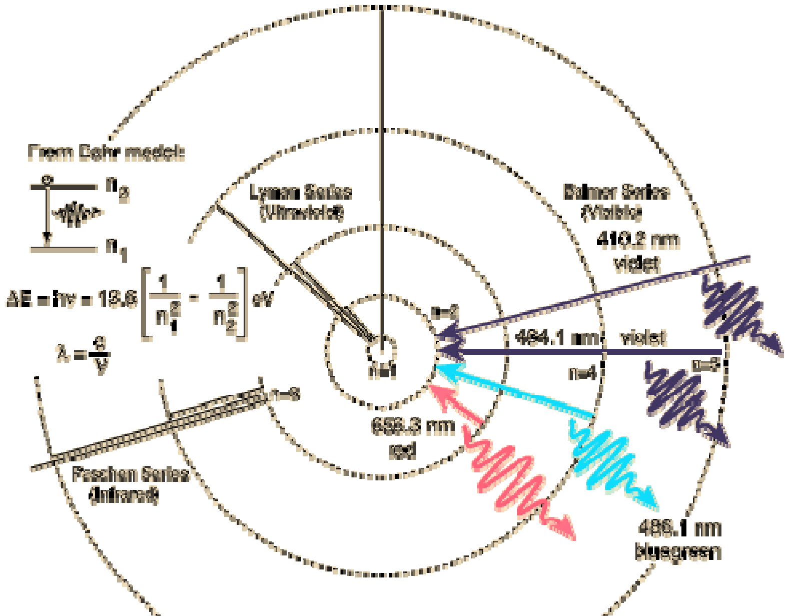
where n_i *is 3, 4, 5, …*

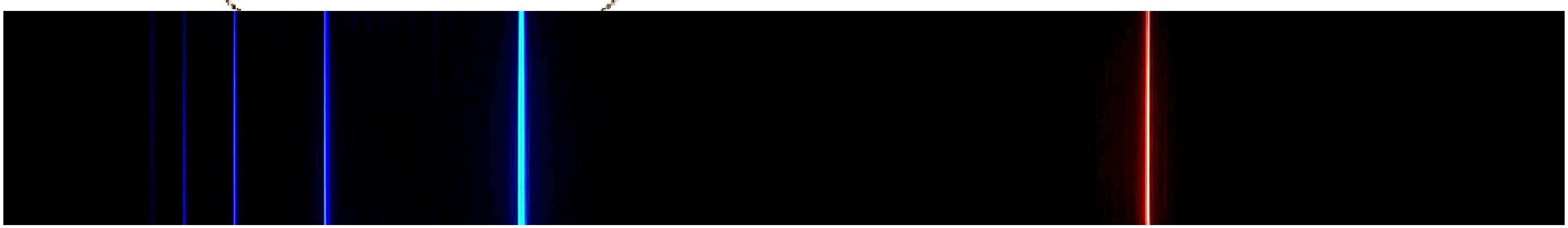
Hydrogen spectrum



From http://hyperphysics.phy-astr.gsu.edu/hbase/hyde.html#c4

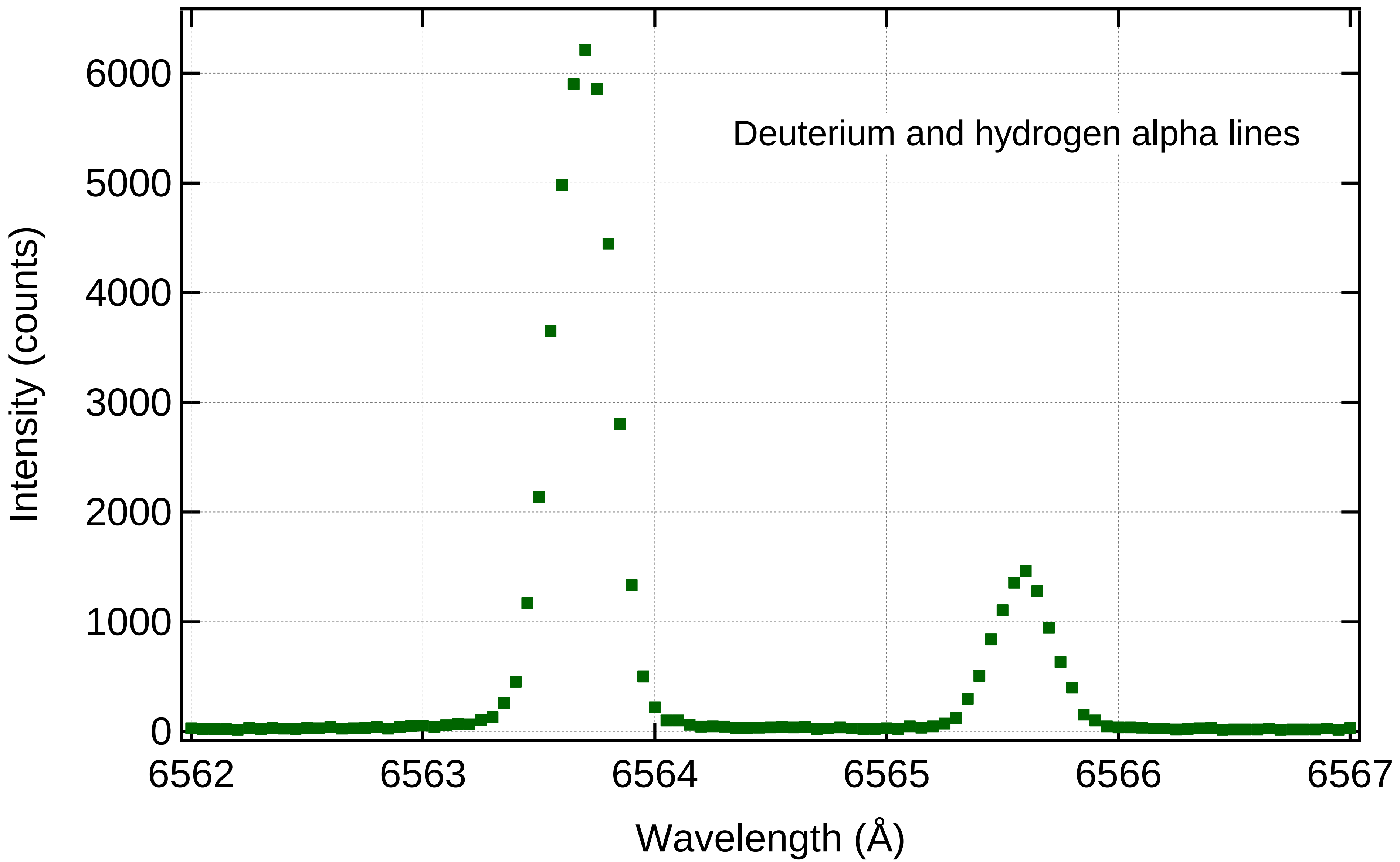
Hydrogen spectrum





Balmer series (transitions ending at n = 2)

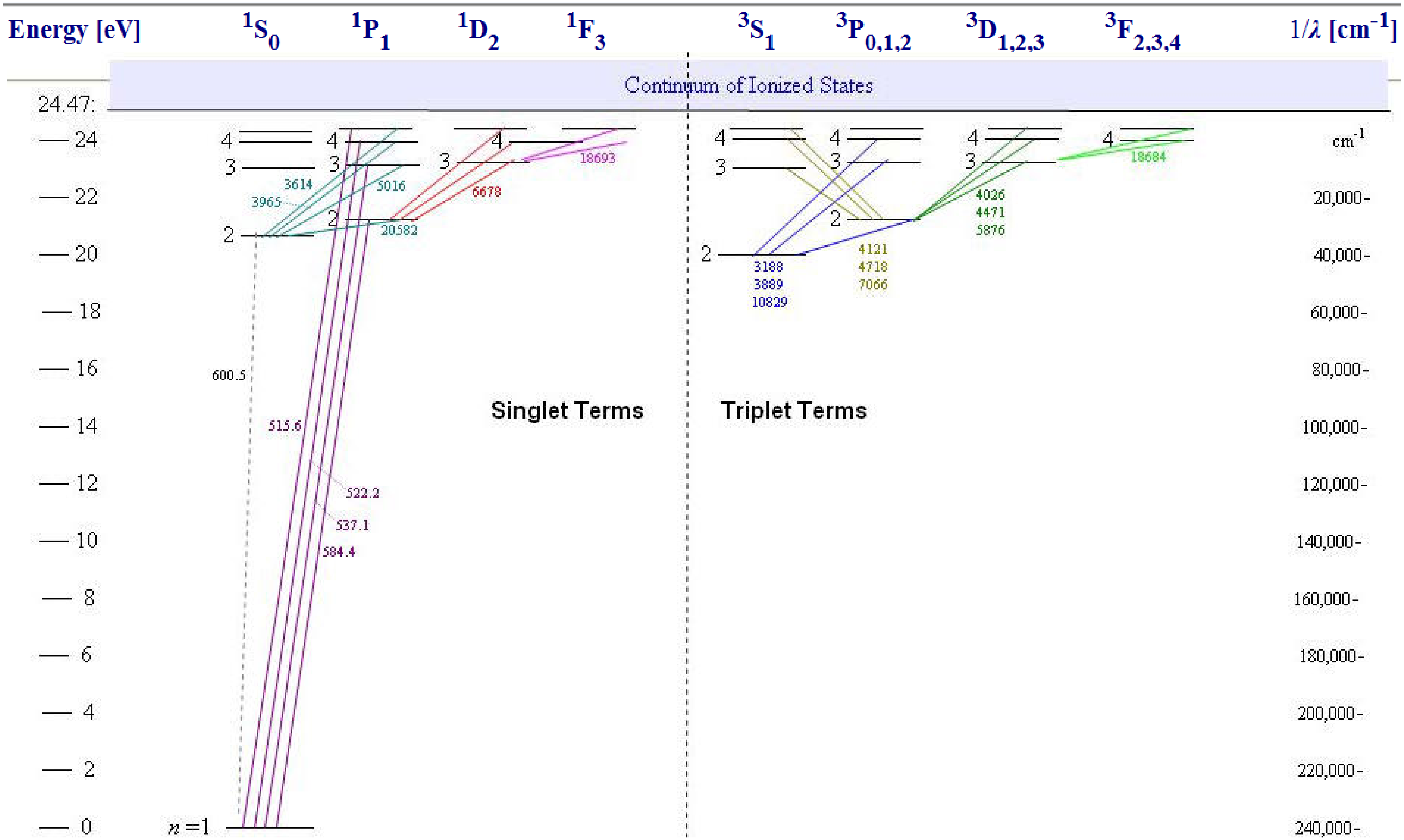
Deuterium and hydrogen α lines



Visible helium spectrum

From http://upload.wikimedia.org/wikipedia/commons/a/a4/Visible_spectrum_of_helium.jpg

Helium Grotrian diagram



From http://www.physics.byu.edu/faculty/christensen/Physics%20428/FTI/Helium%20Grotrian%20Diagram'.htm

The “wavenumbers” unit of energy

- In spectroscopy, the wavenumber $\tilde{\nu}$ for electromagnetic radiation is defined as:

$$\tilde{\nu} = \frac{1}{\lambda}$$

- Wavenumber values are commonly expressed in cm^{-1}, in which case one can convert a wavenumber value to energy in J by multiplying by hc where h is Planck’s constant and c is exprfessed in terms of cm/s (c = 3 x 10^{10} cm/s). In this case, for a photon corresponding to λ =650 nm = 6.5 x 10^{-5} cm, $\tilde{\nu}$ = 1.538 x 10^{4} cm^{-1}, so that (6.63 x 10^{-34}J-s)(3 x 10^{10} cm/s)(1.538 x 10^{4} cm^{-1}) = 3.059e-19 J.
- Converting a photon energy from Joules to eV utilizes the relation 1.602 x 10^{-19} J/eV, or 6.24220 x 10^{18} eV/J.
- Converting from a photon energy in eV to approximate wavelength in nm involves this relation:

$$E(eV) \approx \frac{1240 eV \cdot nm}{\lambda(nm)}$$