Chapter 11

1. The velocity of the car is a constant

\[ \vec{v} = + (80 \text{ km/h})(1000 \text{ m/km})(1 \text{ h}/3600 \text{ s}) \hat{i} = (+22 \text{ m/s})\hat{i}, \]

and the radius of the wheel is \( r = 0.66/2 = 0.33 \text{ m}. \)

(a) In the car’s reference frame (where the lady perceives herself to be at rest) the road is moving towards the rear at \( \vec{v}_{\text{road}} = -v = -22 \text{ m/s}, \) and the motion of the tire is purely rotational. In this frame, the center of the tire is “fixed” so \( v_{\text{center}} = 0. \)

(b) Since the tire’s motion is only rotational (not translational) in this frame, Eq. 10-18 gives \( \vec{v}_{\text{top}} = (+22 \text{ m/s})\hat{i}. \)

(c) The bottom-most point of the tire is (momentarily) in firm contact with the road (not skidding) and has the same velocity as the road: \( \vec{v}_{\text{bottom}} = (-22 \text{ m/s})\hat{i}. \) This also follows from Eq. 10-18.

(d) This frame of reference is not accelerating, so “fixed” points within it have zero acceleration; thus, \( a_{\text{center}} = 0. \)

(e) Not only is the motion purely rotational in this frame, but we also have \( \omega = \text{constant}, \) which means the only acceleration for points on the rim is radial (centripetal). Therefore, the magnitude of the acceleration is

\[ a_{\text{top}} = \frac{v^2}{r} = \frac{(22 \text{ m/s})^2}{0.33 \text{ m}} = 1.5 \times 10^3 \text{ m/s}^2. \]

(f) The magnitude of the acceleration is the same as in part (d): \( a_{\text{bottom}} = 1.5 \times 10^3 \text{ m/s}^2. \)

(g) Now we examine the situation in the road’s frame of reference (where the road is “fixed” and it is the car that appears to be moving). The center of the tire undergoes purely translational motion while points at the rim undergo a combination of translational and rotational motions. The velocity of the center of the tire is \( \vec{v} = (+22 \text{ m/s})\hat{i}. \)

(h) In part (b), we found \( \vec{v}_{\text{top,car}} = +v \) and we use Eq. 4-39:

\[ \vec{v}_{\text{top,ground}} = \vec{v}_{\text{top,car}} + \vec{v}_{\text{car,ground}} = v \hat{i} + v \hat{i} = 2v \hat{i}. \]
which yields $2v = +44 \text{ m/s}$. This is consistent with Fig. 11-3(c).

(i) We can proceed as in part (h) or simply recall that the bottom-most point is in firm contact with the (zero-velocity) road. Either way – the answer is zero.

(j) The translational motion of the center is constant; it does not accelerate.

(k) Since we are transforming between constant-velocity frames of reference, the accelerations are unaffected. The answer is as it was in part (e): $1.5 \times 10^3 \text{ m/s}^2$.

(l) As explained in part (k), $a = 1.5 \times 10^3 \text{ m/s}^2$.

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3. Let $M$ be the mass of the car (presumably including the mass of the wheels) and $v$ be its speed. Let $I$ be the rotational inertia of one wheel and $\omega$ be the angular speed of each wheel. The kinetic energy of rotation is

$$K_{\text{rot}} = 4\left(\frac{1}{2}I\omega^2\right),$$

where the factor 4 appears because there are four wheels. The total kinetic energy is given by $K = \frac{1}{2}Mv^2 + 4\left(\frac{1}{2}I\omega^2\right)$. The fraction of the total energy that is due to rotation is

$$\text{fraction} = \frac{K_{\text{rot}}}{K} = \frac{4I\omega^2}{Mv^2 + 4I\omega^2}.$$}

For a uniform disk (relative to its center of mass) $I = \frac{1}{2}mR^2$ (Table 10-2(c)). Since the wheels roll without sliding $\omega = v/R$ (Eq. 11-2). Thus the numerator of our fraction is

$$4I\omega^2 = 4\left(\frac{1}{2}mR^2\right)\left(\frac{v}{R}\right)^2 = 2mv^2$$

and the fraction itself becomes

$$\text{fraction} = \frac{2mv^2}{Mv^2 + 2mv^2} = \frac{2m}{M + 2m} = \frac{2(10)}{1000} = \frac{1}{50} = 0.020.$$}

The wheel radius cancels from the equations and is not needed in the computation.

8. Using the floor as the reference position for computing potential energy, mechanical energy conservation leads to
\[ U_{\text{release}} = K_{\text{top}} + U_{\text{top}} \Rightarrow \quad mgh = \frac{1}{2} m v_{\text{com}}^2 + \frac{1}{2} I \omega^2 + mg(2R). \]

Substituting \( I = \frac{2}{5} mr^2 \) (Table 10-2(f)) and \( \omega = v_{\text{com}}/r \) (Eq. 11-2), we obtain

\[ mgh = \frac{1}{2} m v_{\text{com}}^2 + \frac{1}{2} \left( \frac{2}{5} mr^2 \right) \left( \frac{v_{\text{com}}}{r} \right)^2 + 2mgR \quad \Rightarrow \quad gh = \frac{7}{10} v_{\text{com}}^2 + 2gR \]

where we have canceled out mass \( m \) in that last step.

(a) To be on the verge of losing contact with the loop (at the top) means the normal force is vanishingly small. In this case, Newton’s second law along the vertical direction (+\( y \) downward) leads to

\[ mg = ma_y \Rightarrow g = \frac{v_{\text{com}}^2}{R-r} \]

where we have used Eq. 10-23 for the radial (centripetal) acceleration (of the center of mass, which at this moment is a distance \( R-r \) from the center of the loop). Plugging the result \( v_{\text{com}}^2 = g(R-r) \) into the previous expression stemming from energy considerations gives

\[ gh = \frac{7}{10} (g)(R-r) + 2gR \]

which leads to \( h = 2.7R - 0.7r = 2.7R \). With \( R = 14.0 \text{ cm} \), we have \( h = (2.7)(14.0 \text{ cm}) = 37.8 \text{ cm} \).

(b) The energy considerations shown above (now with \( h = 6R \)) can be applied to point \( Q \) (which, however, is only at a height of \( R \)) yielding the condition

\[ g(6R) = \frac{7}{10} v_{\text{com}}^2 + gR \]

which gives us \( v_{\text{com}}^2 = 50gR/7 \). Recalling previous remarks about the radial acceleration, Newton’s second law applied to the horizontal axis at \( Q \) leads to

\[ N = m \frac{v_{\text{com}}^2}{R-r} = m \frac{50gR}{7(R-r)} \]

which (for \( R \gg r \)) gives

\[ N = \frac{50mg}{7} = \frac{50(2.80 \times 10^{-4} \text{ kg})(9.80 \text{ m/s}^2)}{7} = 1.96 \times 10^{-2} \text{ N}. \]
(b) The direction is toward the center of the loop.

35. (a) We note that
\[ \vec{v} = \frac{d\vec{r}}{dt} = 8.0t \hat{i} - (2.0 + 12t)\hat{j} \]

with SI units understood. From Eq. 11-18 (for the angular momentum) and Eq. 3-30, we find the particle’s angular momentum is \( 8r^2\hat{k} \). Using Eq. 11-23 (relating its time-derivative to the (single) torque) then yields \( \vec{\tau} = (48t \hat{k}) \text{ N} \cdot \text{m} \).

(b) From our (intermediate) result in part (a), we see the angular momentum increases in proportion to \( t^2 \).

45. (a) No external torques act on the system consisting of the two wheels, so its total angular momentum is conserved. Let \( I_1 \) be the rotational inertia of the wheel that is originally spinning (at \( \omega_i \)) and \( I_2 \) be the rotational inertia of the wheel that is initially at rest. Then
\[ I_1 \omega_i = (I_1 + I_2)\omega_f \]

and
\[ \omega_f = \frac{I_1}{I_1 + I_2} \omega_i \]

where \( \omega_f \) is the common final angular velocity of the wheels. Substituting \( I_2 = 2I_1 \) and \( \omega_i = 800 \text{ rev/min} \), we obtain \( \omega_f = 267 \text{ rev/min} \).

(b) The initial kinetic energy is \( K_i = \frac{1}{2} I_1 \omega_i^2 \) and the final kinetic energy is \( K_f = \frac{1}{2}(I_1 + I_2)\omega_f^2 \). We rewrite this as
\[ K_f = \frac{1}{2} \left( I_1 + 2I_1 \right) \left( \frac{I_1 \omega_i}{I_1 + 2I_1} \right)^2 = \frac{1}{6} I \omega_i^2 \]

Therefore, the fraction lost,
\[ \frac{K_i - K_f}{K_i} \]

is
\[ 1 - \frac{K_f}{K_i} = 1 - \frac{I \omega_i^2 / 6}{I \omega_i^2 / 2} = \frac{2}{3} = 0.667. \]

77. As the wheel-axel system rolls down the inclined plane by a distance \( d \), the decrease in potential energy is \( \Delta U = mgd\sin \theta \). This must be equal to the total kinetic energy gained:
Since the axle rolls without slipping, the angular speed is given by \( \omega = v/r \), where \( r \) is the radius of the axle. The above equation then becomes

\[
mgd \sin \theta = \frac{1}{2} mv^2 + \frac{1}{2} I \omega^2.
\]

(a) With \( m = 10.0 \text{ kg}, \ d = 2.00 \text{ m}, \ r = 0.200 \text{ m}, \) and \( I = 0.600 \text{ kg}\cdot\text{m}^2, \ mr^2/I = 2/3, \) the rotational kinetic energy may be obtained as \( 98 \text{ J} = K_{\text{rot}} (5/3), \) or \( K_{\text{rot}} = 58.8 \text{ J}. \)

(b) The translational kinetic energy is \( K_{\text{trans}} = (98 - 58.8)\text{ J} = 39.2 \text{ J}. \)