Exam tomorrow on Chapter 15, 16, and 17 (Oscillations and Waves 1 &2)

What to study:

- Quiz 6
- Homework problems for Chapters 15 & 16
- Material indicated in the following review slides

- Other Specific things:
  - Sample Problems 16-2, 16-3, and 16-4
  - You should be able to follow a derivation to equation 16-30
  - Checkpoint 5 in Ch16
  - Sample problem 16-8 (you would be given eqn 16-67)
  - Ch. 16 Questions (not Problems) 1 and 11
  - Sample Problems 17-3 and 17-6
  - Chapter 17, Checkpoint 4

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15.10 Review & Summary

you should know these things

Frequency The *frequency f* of periodic, or oscillatory, motion is the number of oscillations per second. In the SI system, it is measured in hertz:

$$1\,\text{hertz} = 1\,\text{Hz} = 1\,\text{oscillation per second} = 1\,\text{s}^{-1}. \tag{15-1}$$

Period The *period T* is the time required for one complete oscillation, or **cycle**. It is related to the frequency by

$$T = \frac{1}{f}. \tag{15-2}$$

Simple Harmonic Motion In *simple harmonic motion* (SHM), the displacement *x(t)* of a particle from its equilibrium position is described by the equation

$$x = x_m \cos(\omega t + \phi) \quad \text{(displacement)}, \tag{15-3}$$

in which x_m is the **amplitude** of the displacement, the quantity $(\omega t + \phi)$ is the phase of the motion, and φ is the **phase constant**. The **angular frequency** ω is related to the period and frequency of the motion by

$$\omega = \frac{2\pi}{T} = 2\pi f \quad \text{(angular frequency)}. \tag{15-5}$$

Differentiating Eq. 15-3 leads to equations for the particle's SHM velocity and acceleration as functions of time:

$$v = -\omega x_m \sin(\omega t + \phi) \quad \text{(velocity)} \tag{15-6}$$

and

$$a = -\omega^2 x_m \cos(\omega t + \phi) \quad \text{(acceleration)}. \tag{15-7}$$

In Eq. 15-6, the positive quantity ωx_m is the **velocity amplitude** v_m of the motion. In Eq. 15-7, the positive quantity $\omega^2 x_m$ is the **acceleration amplitude** a_m of the motion.

The Linear Oscillator A particle with mass *m* that moves under the influence of a Hooke's law restoring force given by $F = -kx$ exhibits simple harmonic motion with

$$\omega = \sqrt{\frac{k}{m}} \quad \text{(angular frequency)} \tag{15-12}$$

and

$$T = 2\pi\sqrt{\frac{m}{k}} \quad \text{(period)}. \tag{15-13}$$

Such a system is called a **linear simple harmonic oscillator**.

Energy A particle in simple harmonic motion has, at any time, kinetic energy $K = \frac{1}{2}mv^2$ and potential energy $U = \frac{1}{2}kx^2$. If no friction is present, the mechanical energy $E = K + U$ remains constant even though *K* and *U* change.

Pendulums Examples of devices that undergo simple harmonic motion are the **torsion pendulum** of Fig. 15-7, the **simple pendulum** of Fig. 15-9, and the **physical pendulum** of Fig. 15-10. Their periods of oscillation for small oscillations are, respectively,

$$T = 2\pi\sqrt{I/k} \quad \text{(torsion pendulum)}, \tag{15-23}$$

you will be given these if needed

$$T = 2\pi\sqrt{L/g} \quad \text{(simple pendulum)}, \tag{15-28}$$

$$T = 2\pi\sqrt{I/mgh} \quad \text{(physical pendulum)}. \tag{15-29}$$

Simple Harmonic Motion and Uniform Circular Motion Simple harmonic motion is the projection of uniform circular motion onto the diameter of the circle in which the circular motion occurs. Figure 15-14 shows that all parameters of circular motion (position, velocity, and acceleration) project to the corresponding values for simple harmonic motion.

Damped Harmonic Motion The mechanical energy E in a real oscillating system decreases during the oscillations because external forces, such as a drag force, inhibit the oscillations and transfer mechanical energy to thermal energy. The real oscillator and its motion are then said to be **damped**. If the **damping force** is given by $\vec{F}_d = -b\vec{v}$, where $\vec{v}$ is the velocity of the oscillator and b is a **damping constant**, then the displacement of the oscillator is given by

understand what the form of this function means

$$x(t) = x_m e^{-bt/2m}\cos(\omega' t + \phi), \tag{15-42}$$

where ω′, the angular frequency of the damped oscillator, is given by

$$\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}. \tag{15-43}$$

If the damping constant is small $\left(b \ll \sqrt{km}\right)$, then $\omega' \approx \omega$, where ω is the angular frequency of the undamped oscillator. For small b, the mechanical energy E of the oscillator is given by

$$E(t) \approx \frac{1}{2}kx_m^2 e^{-bt/m}. \tag{15-44}$$

Forced Oscillations and Resonance If an external driving force with angular frequency ω_d acts on an oscillating system with *natural* angular frequency ω, the system oscillates with angular frequency ω_d. The velocity amplitude v_m of the system is greatest when

$$\omega_d = \omega, \tag{15-46}$$

a condition called **resonance**. The amplitude x_m of the system is (approximately) greatest under the same condition.

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16.14 Review & Summary

Transverse and Longitudinal Waves Mechanical waves can exist only in material media and are governed by Newton's laws. **Transverse** mechanical waves, like those on a stretched string, are waves in which the particles of the medium oscillate perpendicular to the wave's direction of travel. Waves in which the particles of the medium oscillate parallel to the wave's direction of travel are **longitudinal** waves.

Sinusoidal Waves A sinusoidal wave moving in the positive direction of an *x* axis has the mathematical form

$$y(x,t) = y_m \sin(kx - \omega t), \tag{16-2}$$

where y_m is the **amplitude** of the wave, *k* is the **angular wave number**, ω is the **angular frequency**, and $kx - \omega t$ is the **phase**. The **wavelength** λ is related to *k* by

$$k = \frac{2\pi}{\lambda}. \tag{16-5}$$

The **period** *T* and **frequency** *f* of the wave are related to ω by

$$\frac{\omega}{2\pi} = f = \frac{1}{T}. \tag{16-9}$$

Finally, the **wave speed** *v* is related to these other parameters by

$$v = \frac{\omega}{k} = \frac{\lambda}{T} = \lambda f. \tag{16-13}$$

Equation of a Traveling Wave Any function of the form

$$y(x,t) = h(kx \pm \omega t). \tag{16-17}$$

can represent a **traveling wave** with a wave speed given by Eq. 16-13 and a wave shape given by the mathematical form of *h*. The plus sign denotes a wave traveling in the negative direction of the *x* axis, and the minus sign a wave traveling in the positive direction.

Wave Speed on Stretched String The speed of a wave on a stretched string is set by properties of the string. The speed on a string with tension τ and linear density μ is

$$v = \sqrt{\frac{\tau}{\mu}}. \tag{16-26}$$

Power The **average power** of, or average rate at which energy is transmitted by, a sinusoidal wave on a stretched string is given by

$$P_{avg} = \frac{1}{2}\mu v \omega^2 y_m^2. \tag{16-33}$$

Superposition of Waves When two or more waves traverse the same medium, the displacement of any particle of the medium is the sum of the displacements that the individual waves would give it.

Interference of Waves Two sinusoidal waves on the same string exhibit **interference**, adding or canceling according to the principle of superposition. If the two are traveling in the same direction and have the same amplitude y_m and frequency (hence the same wavelength) but differ in phase by a **phase constant** ϕ, the result understand the parts and what they mean, as in Fig. 16-6 and Checkpoint 4

is a single wave with this same frequency:

$$y'(x, t) = \left[2y_m \cos\frac{1}{2}\phi\right]\sin\left(kx - \omega t + \frac{1}{2}\phi\right). \tag{16-51}$$

If $\phi = 0$, the waves are exactly in phase and their interference is fully constructive; if $\phi = \pi\,\mathrm{rad}$, they are exactly out of phase and their interference is fully destructive.

Phasors A wave *y(x, t)* can be represented with a **phasor**. This is a vector that has a magnitude equal to the amplitude y_m of the wave and that rotates about an origin with an angular speed equal to the angular frequency ω of the wave. The projection of the rotating phasor on a vertical axis gives the displacement *y* of a point along the wave's travel.

Standing Waves The interference of two identical sinusoidal waves moving in opposite directions produces **standing waves**. For a string with fixed ends, the standing wave is given by

understand the parts and what they mean, as in Fig. 16-19 and Checkpoint 5

$$y'(x, t) = [2y_m \sin kx]\cos\omega t. \tag{16-60}$$

Standing waves are characterized by fixed locations of zero displacement called **nodes** and fixed locations of maximum displacement called **antinodes**.

Resonance Standing waves on a string can be set up by reflection of traveling waves from the ends of the string. If an end is fixed, it must be the position of a node. This limits the frequencies at which standing waves will occur on a given string. Each possible frequency is a **resonant frequency**, and the corresponding standing wave pattern is an **oscillation mode**. For a stretched string of length *L* with fixed ends, the resonant frequencies are

understand the parts and what they mean

$$f = \frac{v}{\lambda} = n\frac{v}{2L}, \quad \text{for } n = 1, 2, 3, \ldots. \tag{16-66}$$

The oscillation mode corresponding to $n = 1$ is called the *fundamental mode* or *the first harmonic;* the mode corresponding to $n = 2$ is the *second harmonic;* and so on.

17.11 Review & Summary

Sound Waves Sound waves are longitudinal mechanical waves that can travel through solids, liquids, or gases. The speed v of a sound wave in a medium having **bulk modulus** B and density ρ is

$$\boxed{v = \sqrt{\frac{B}{\rho}} \text{(speed of sound)}\ .} \tag{17-3}$$

In air at 20°C, the speed of sound is 343 m/s.

A sound wave causes a longitudinal displacement s of a mass element in a medium as given by

$$s = s_m \cos(kx - \omega t), \tag{17-13}$$

where s_m is the **displacement amplitude** (maximum displacement) from equilibrium, $k = 2\pi / \lambda$, and $\omega = 2\pi f$, λ, and f being the wavelength and frequency, respectively, of the sound wave.

The sound wave also causes a pressure change Δp of the medium from the equilibrium pressure:

$$\Delta p = \Delta p_m \sin(kx - \omega t), \tag{17.14}$$

where the **pressure amplitude** is

$$\Delta p_m = (v\rho\omega)s_m\ . \tag{17-15}$$

Interference The interference of two sound waves with identical wavelengths passing through a common point depends on their phase difference ϕ there. If the sound waves were emitted in phase and are traveling in approximately the same direction, ϕ is given by

$$\phi = \frac{\Delta L}{\lambda} 2\pi, \tag{17-21}$$

where ΔL is their **path length difference** (the difference in the distances traveled by the waves to reach the common point). Fully constructive interference occurs when ϕ is an integer multiple of 2π,

$$\phi = m(2\pi),\ \text{for} \quad m = 0, 1, 2, \ldots, \tag{17-22}$$

and, equivalently, when ΔL is related to wavelength λ by

$$\frac{\Delta L}{\lambda} = 0, 1, 2, \ldots \tag{17-23}$$

Fully destructive interference occurs when ϕ is an odd multiple of π,

$$\phi = (2m+1)\pi, \quad \text{for} \quad m = 0, 1, 2, \ldots, \tag{17-24}$$

and, equivalently, when ΔL is related to λ by

$$\frac{\Delta L}{\lambda} = 0.5, 1.5, 2.5, \ldots . \tag{17-25}$$

Sound Intensity The **intensity** I of a sound wave at a surface is the average rate per unit area at which energy is transferred by the wave through or onto the surface:

$$I = \frac{P}{A}, \tag{17-26}$$

where P is the time rate of energy transfer (power) of the sound wave and A is the area of the surface intercepting the sound. The intensity I is related to the displacement amplitude s_m of the sound wave by

$$I = \tfrac{1}{2}\rho v \omega^2 s_m^2 . \tag{17-27}$$

The intensity at a distance r from a point source that emits sound waves of power P_s is

$$I = \frac{P_s}{4\pi r^2} . \tag{17-28}$$

Sound Level in Decibels The *sound level* β in *decibels* (dB) is defined as

$$\beta = (10\,\mathrm{dB})\log\frac{I}{I_0}, \tag{17-29}$$

where $I_0\left(= 10^{-12}\,\mathrm{W/m^2}\right)$ is a reference intensity level to which all intensities are compared. For every factor-of-10 increase in intensity, 10 dB is added to the sound level.

Standing Wave Patterns in Pipes Standing sound wave patterns can be set up in pipes. A pipe open at both ends will resonate at frequencies

$$f = \frac{v}{\lambda} = \frac{nv}{2L}, n = 1, 2, 3, \ldots, \tag{17-39}$$

where v is the speed of sound in the air in the pipe. For a pipe closed at one end and open at the other, the resonant frequencies are

$$f = \frac{v}{\lambda} = \frac{nv}{4L}, n = 1, 3, 5, \ldots . \tag{17-41}$$

Beats *Beats* arise when two waves having slightly different frequencies, f_1 and f_2, are detected together. The beat frequency is

$$f_{\text{beat}} = f_1 - f_2 . \tag{17-46}$$

The Doppler Effect The *Doppler effect* is a change in the observed frequency of a wave when the source or the detector moves relative to the transmitting medium (such as air). For sound the observed frequency f' is given in terms of the source frequency f by

$$f' = f\frac{v \pm v_D}{v \pm v_S}\,(\text{general Doppler effect}), \tag{17-47}$$

where v_D is the speed of the detector relative to the medium, v_S is that of the source, and v is the speed of sound in the medium. The signs are chosen such that f' tends to be *greater* for motion toward and *less* for motion away.

Shock Wave If the speed of a source relative to the medium exceeds the speed of sound in the medium, the

Doppler equation no longer applies. In such a case, shock waves result. The half-angle θ of the Mach cone is given by

$$\sin\theta = \frac{v}{v_S} \text{ (Mach cone angle)} . \tag{17-57}$$