Chapter 8: Potential Energy and Conservation of Energy

Work and **kinetic energy** are energies of ***motion***.

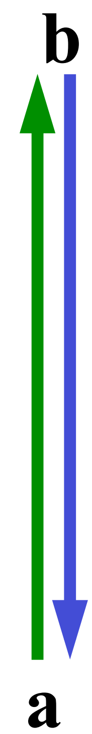
$$\Delta K = K_f - K_i = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = W_{net} = \int_{r_i}^{r_f} \vec{F}\cdot d\vec{r}$$

Consider a vertical spring oscillating with mass m attached to one end. At the extreme ends of travel the kinetic energy is zero, but something caused it to accelerate back to the equilibrium point.

We need to introduce an energy that depends on **location** or ***position***. This energy is called ***potential energy***.

Chapter 8: Potential Energy and Conservation of Energy

Work done by gravitation for a ball thrown upward that then falls back down



$$W_{ab} + W_{ba} = -\,mgd + mg\,d = 0$$

The gravitational force is said to be a **conservative force**.

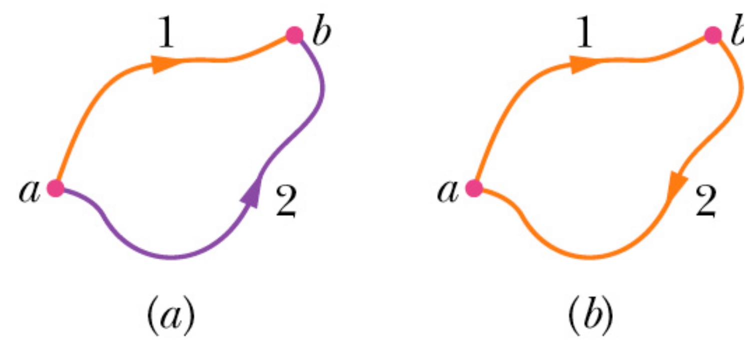
A force is a **conservative** force if the net work it does on a particle moving around every closed path is zero.

Conservative forces

$W_{ab,1} + W_{ba,2} = 0$

$W_{ab,2} + W_{ba,2} = 0$

therefore: $W_{ab,1} = W_{ab,2}$



i.e. The work done by a conservative force on a particle moving between two points does not depend on the path taken by the particle.

So, ... choose the easiest path!!

Conservative & Non-conservative forces

Conservative Forces: (path independent)

- **gravitational**
- **spring**
- **electrostatic**

Non-conservative forces: (dependent on path)

- **friction**
- **air resistance**
- **electrical resistance**

These forces will convert mechanical energy into heat and/or deformation.

Gravitation Potential Energy

Potential energy is associated with the configuration of a system in which a conservative force acts: $\Delta U = -W$

For a general conservative force F = F(x)

$$\Delta U = -W = -\int_{x_i}^{x_f} \vec{F}(x) \cdot d\hat{x}$$

Gravitational potential energy:

$$\Delta U = -\int_{y_i}^{y_f} (-mg)dy = mg\left(y_f - y_i\right)$$

assume $U_i = 0$ at $y_i = 0$ (reference point)

$U(y) = mgy$ (gravitational potential energy)

only depends on vertical position

Nature only considers changes in Potential energy important. Thus, we will ***always*** measure ΔU.

So, . . . We need to set a reference point!

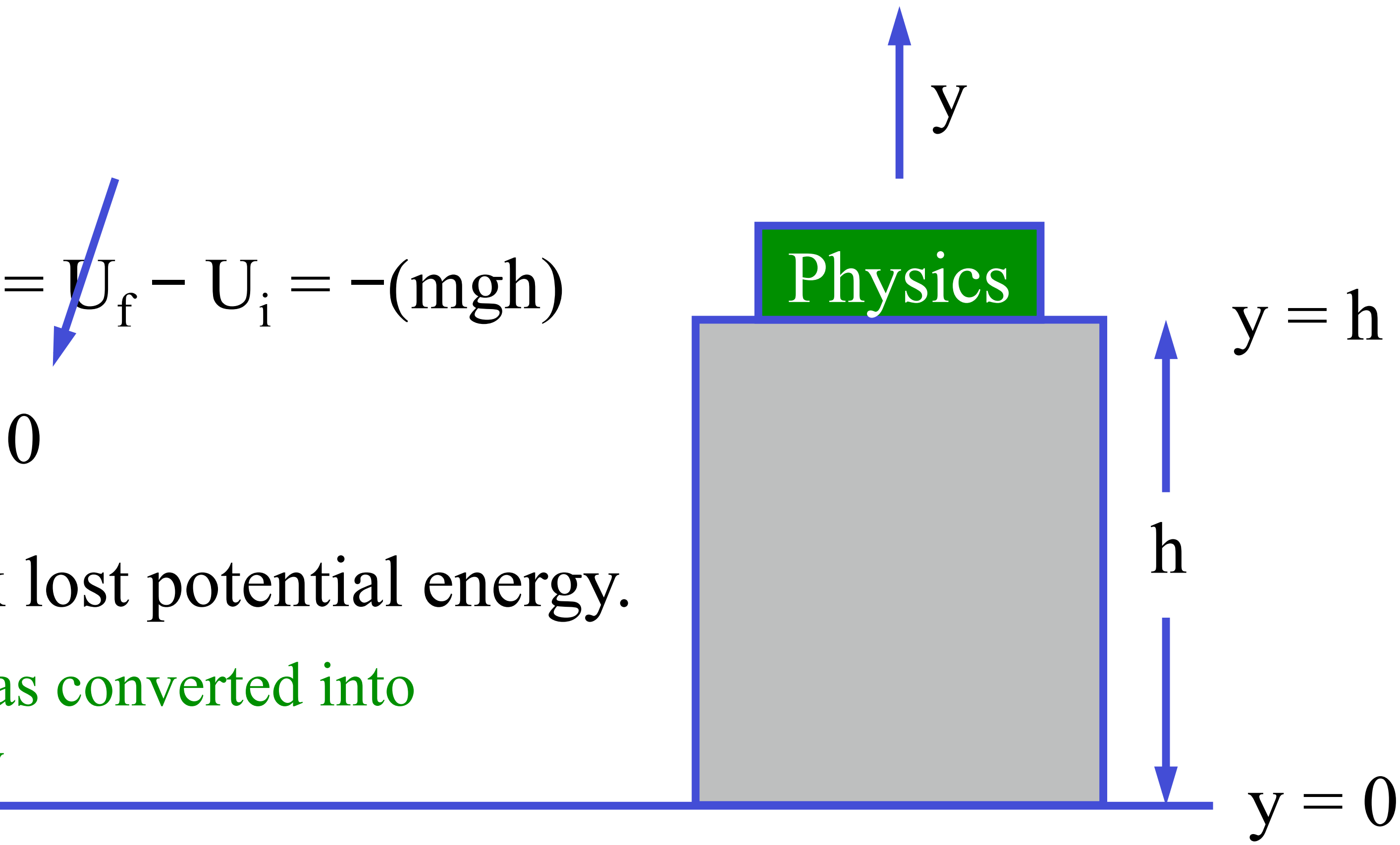
The potential at that point could be defined to be zero (i.e., $U_{\text{ref. point}} = 0$) in which case we drop the "Δ" for convenience.

BUT, it is always understood to be there!

- **Potential energy and reference point (y = 0 at ground)**

  A 0.5 kg physics book falls from a table 1.0 m to the ground. What is U and ΔU if we take reference point y = 0 (assume U = 0 at y = 0) at the ground?

$$\Delta U = \underset{0}{\cancel{U_f}} - U_i = -(mgh)$$



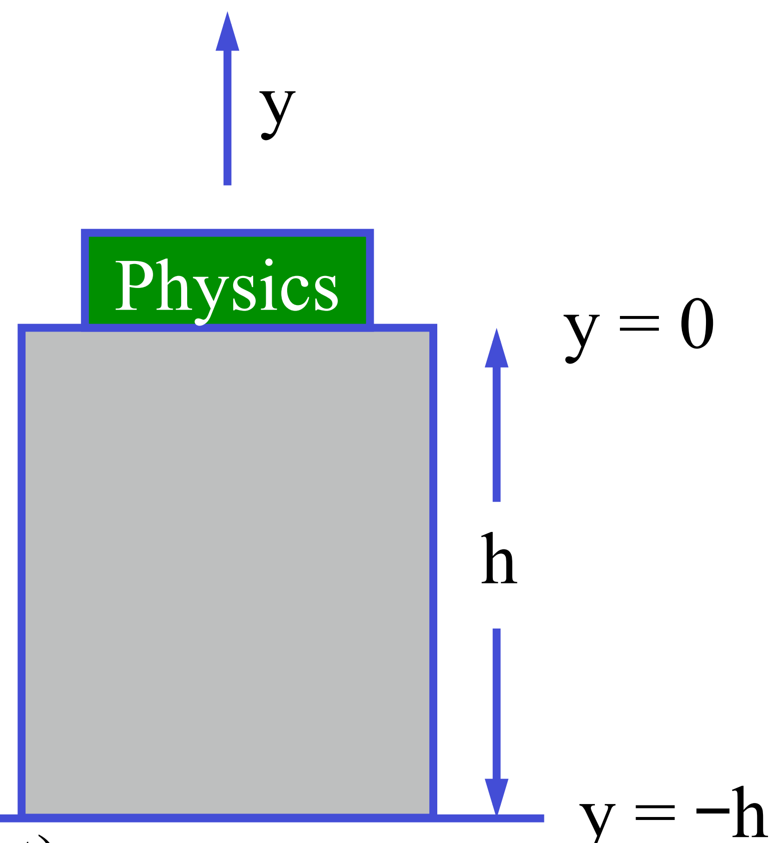
The book lost potential energy.

Actually, it was converted into kinetic energy

- **Potential energy and reference point (y= 0 at table)**

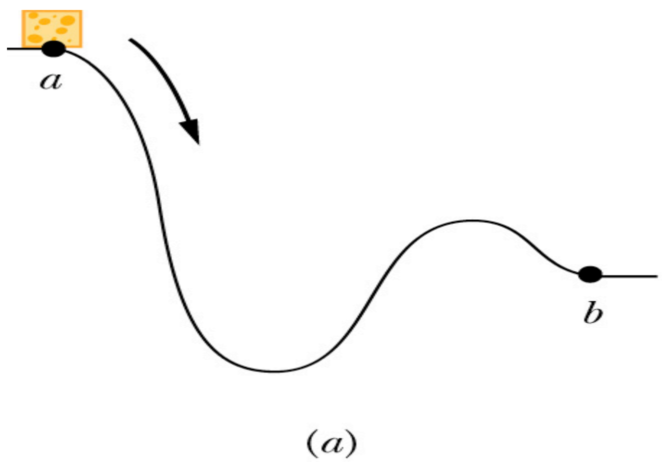
  A 0.5 kg physics book falls from a table 1.0 m to the ground. What is U and ΔU if we take reference point y = 0 (assume U = 0 at y = 0) at the table?

$$\Delta U = U_f - \cancelto{0}{U_i} = mg(-h)$$



The book lost the <u>same</u> potential energy.
(independent of y axis assignment)

Sample problem 8-1: A block of cheese, m = 2 kg, slides along frictionless track from a to b, the cheese traveled a total distance of 2 m along the track, and a net vertical distance of 0.8 m. How much is W_g?

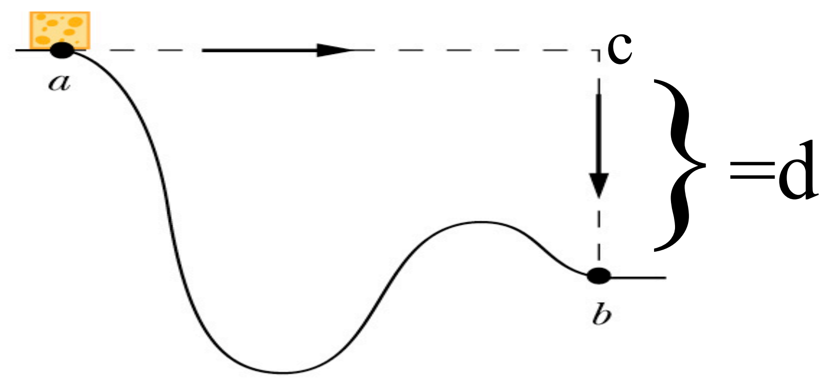


$$W_{net} = \int_a^b \vec{F}_g \cdot d\vec{r}$$

But, ... We don't know the angle between F and dr

Easier (or another) way?

Sample problem 8-1: A block of cheese, m = 2 kg, slides along frictionless track from a to b, the cheese traveled a total distance of 2 m along the track, and a net vertical distance of 0.8 m. How much is W_g?



(b)

Split the problem into two parts (two paths) since gravity is conservative

$$W_{net} = \underbrace{\int_a^c \vec{F}_g \cdot d\hat{x}}_{0} + \underbrace{\int_c^b \vec{F}_g \cdot d\hat{y}}_{mg(b-c)\,=\,mgd}$$

Elastic Potential Energy

Spring force is also a conservative force

$$F = -kx$$

$$\Delta U = -W = \int_{x_i}^{x_f} (-kx)dx$$

$$U_f - U_i = ½\, kx_f^2 - ½\, kx_i^2$$

Choose the free end of the relaxed spring as the reference point:

that is: $U_i = 0$ at $x_i = 0$

$\mathbf{U = ½\, kx^2}$ **Elastic potential energy**

Conservation of Mechanical Energy

- Mechanical energy

  $E_{mec} = K + U$

- For an isolated system (no external forces), if there are only conservative forces causing energy transfer within the system….

We know: $\Delta K = W$ (work-kinetic energy theorem)

Also: $\Delta U = -W$ (definition of potential energy)

Therefore: $\Delta K + \Delta U = 0$ ➔ $(K_f - K_i) + (U_f - U_i) = 0$

therefore $\mathbf{K_1 + U_1 = K_2 + U_2}$ (States 1 and 2 of system)

$\mathbf{E_{mec,1} = E_{mec,2}}$ the mechanical energy is conserved

Conservation of mechanical energy

For an object moved by a spring in the presence of a gravitational force.

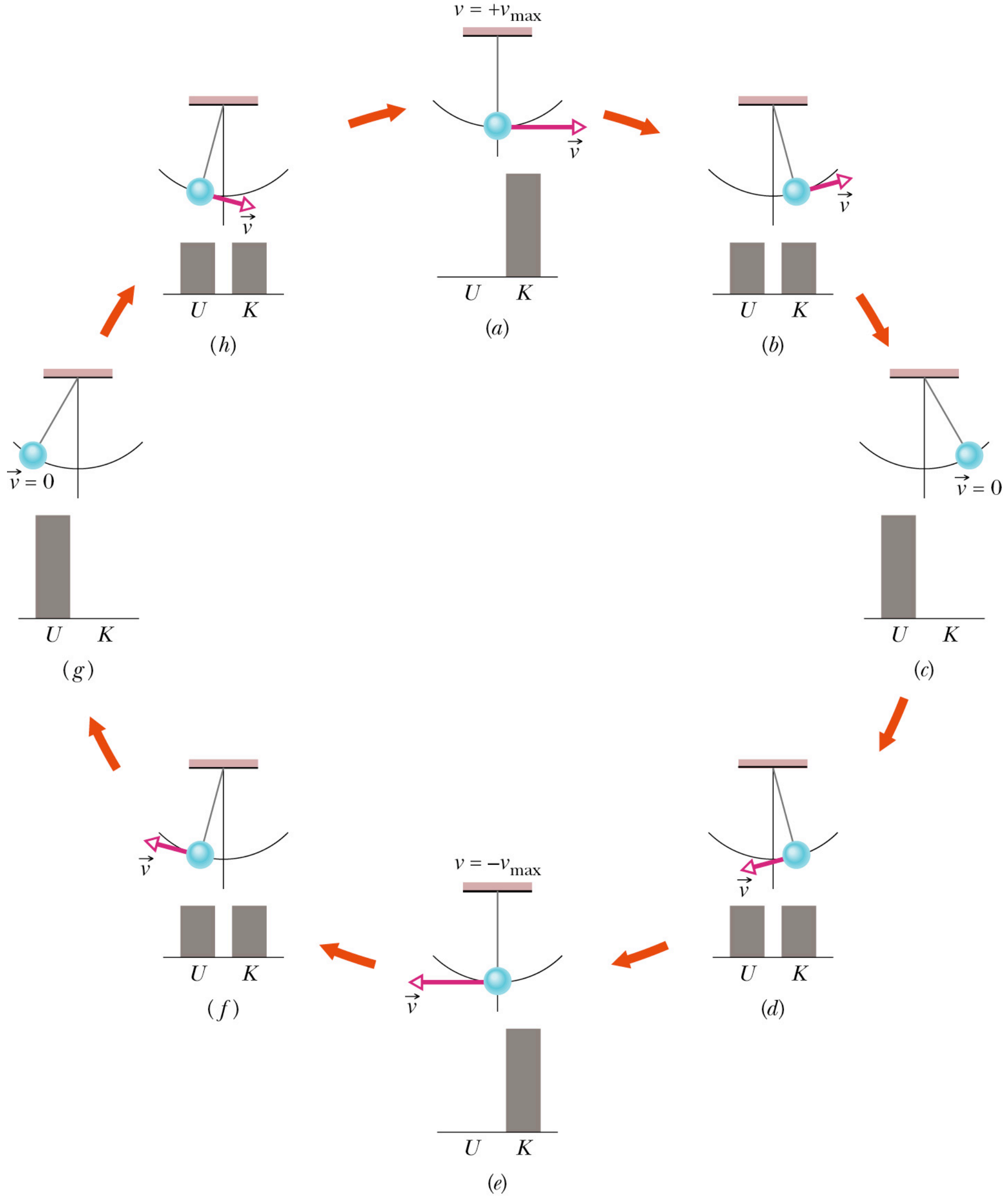
This is an **isolated** system with only conservative forces ($F = mg$, $F = -kx$) acting inside the system

$$E_{mec,1} = E_{mec,2}$$

$$K_1 + U_1 = K_2 + U_2$$

$$\tfrac{1}{2} mv_1^2 + mgy_1 + \tfrac{1}{2} kx_1^2 = \tfrac{1}{2} mv_2^2 + mgy_2 + \tfrac{1}{2} kx_2^2$$

The bowling ball pendulum demonstration

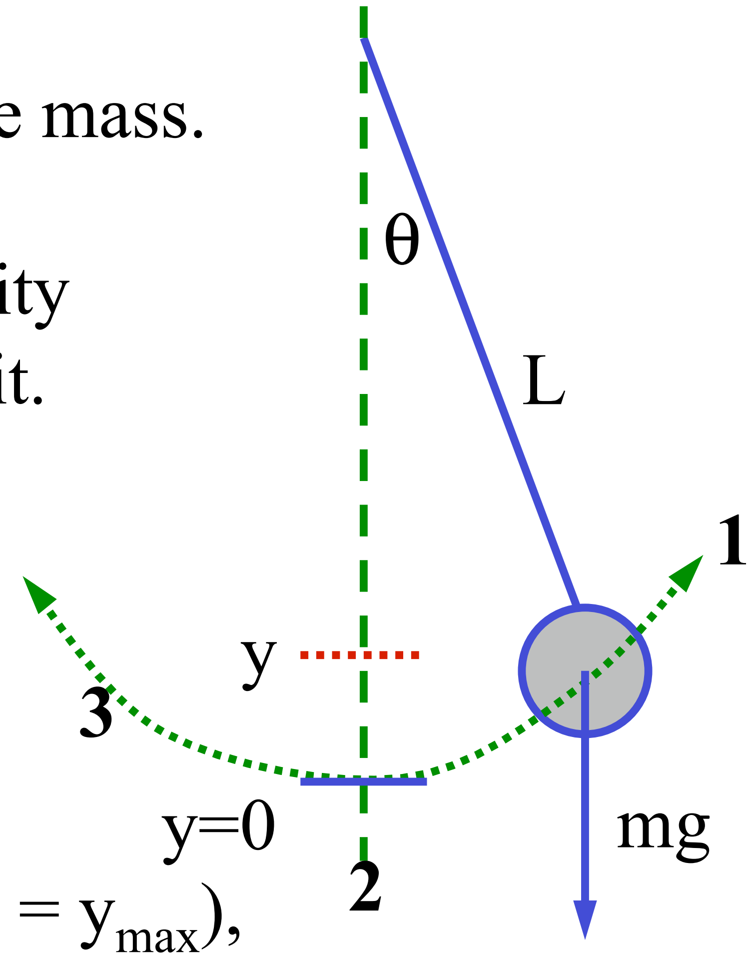


Mechanical energy is conserved ***if*** there are only conservative forces acting on the system.

Pendulum (1-D) vertically

Find the maximum speed of the mass.

Isolated system with only gravity (conservative force) acting on it.

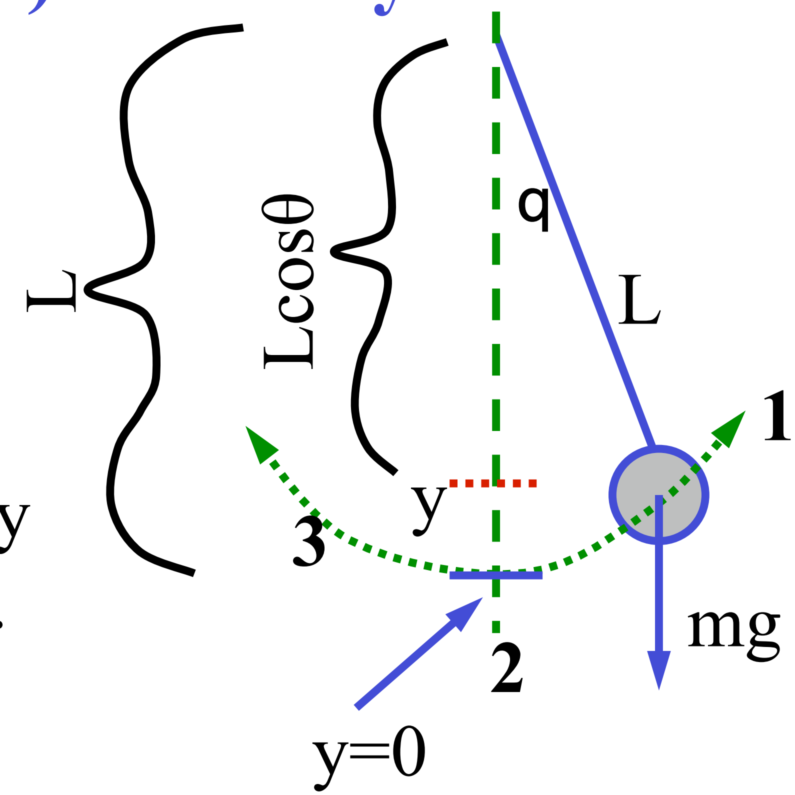


Let 1 be at maximum height ($y = y_{max}$),
Let 2 be at minimum height ($y = 0$), and
3 be somewhere between max and min.

$y = L(1-\cos\theta)$

Pendulum (1-D) vertically

Where is the speed a max?



Isolated system with only gravity (conservative force) acting on it.

$$\overset{0}{K_1} + U_1 = K_2 + U_2 = E_{total}$$

$$\underset{0}{\tfrac{1}{2} m v_1^2} + mgy_1 = \tfrac{1}{2} m v_2^2 + \underset{0}{mgy_2}$$

In general,
$y = L(1-\cos\theta)$

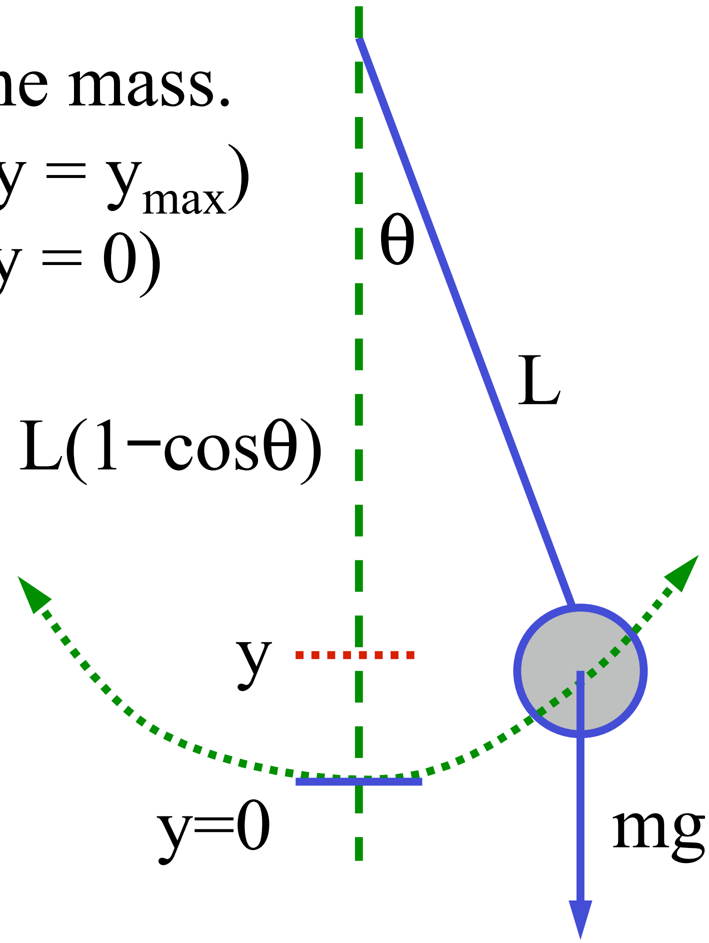
K_{max} must be when U_{min} (= 0)

Answer: **2**

Pendulum (1-D) vertically

Find the maximum speed of the mass.
Let 1 be at maximum height ($y = y_{max}$)
and 2 be at minimum height ($y = 0$)

$$E_{mech,1} = E_{mech,2} = E_{total} \qquad y = L(1-\cos\theta)$$



$$\underset{0}{K_1} + U_1 = K_2 + \underset{0}{U_2} = E_{total}$$

$$mgy_{max} = \tfrac{1}{2} mv_{max}^2 = E_{total}$$

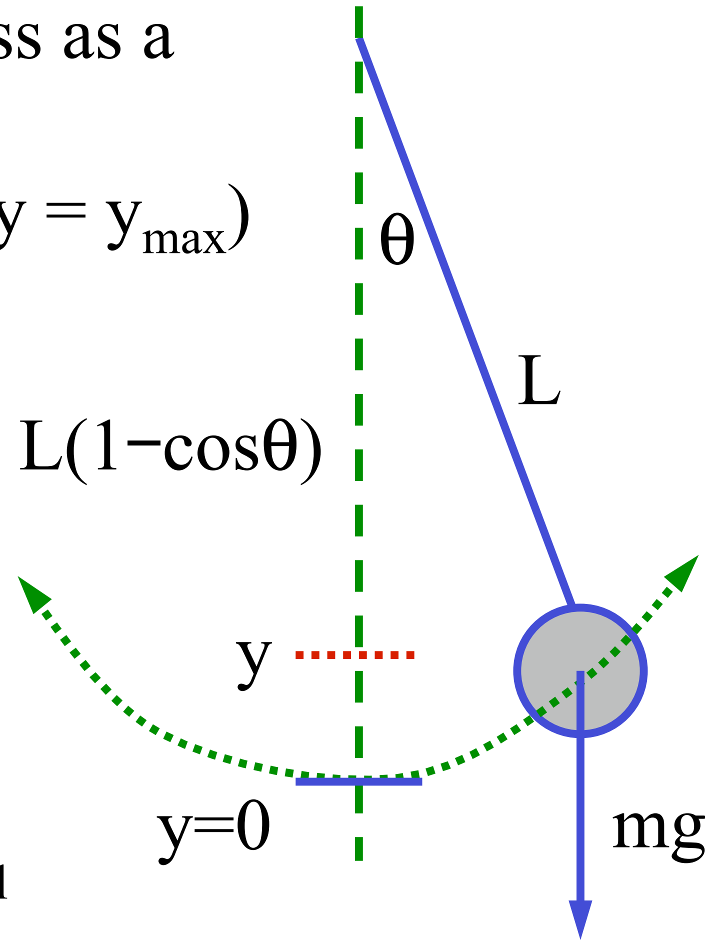
$$\Rightarrow \quad v_{max} = \sqrt{2gy_{max}} = \sqrt{2gL(1-\cos\theta_{max})}$$

Pendulum (1-D) vertically

Now find the speed of the mass as a function of angle.

Let 1 be at maximum height ($y = y_{max}$) and 2 be at some height (y)

$$E_{mech,1} = E_{mech,2} = E_{total} \qquad y = L(1-\cos\theta)$$



$$K_1 + U_1 = K_2 + U_2 = E_{total}$$

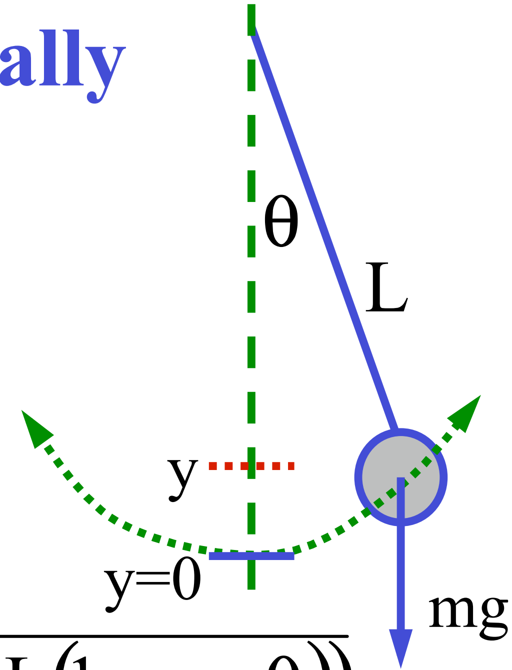
$$mgy_{max} = \tfrac{1}{2}\, mv^2 + mgy = E_{total}$$

$$v = \sqrt{2g\left(y_{max} - y\right)} = \sqrt{2g\left(L\left(1 - \cos\theta_{max}\right) - L\left(1 - \cos\theta\right)\right)}$$

$$= \sqrt{2gL\left(\cos\theta - \cos\theta_{max}\right)}$$

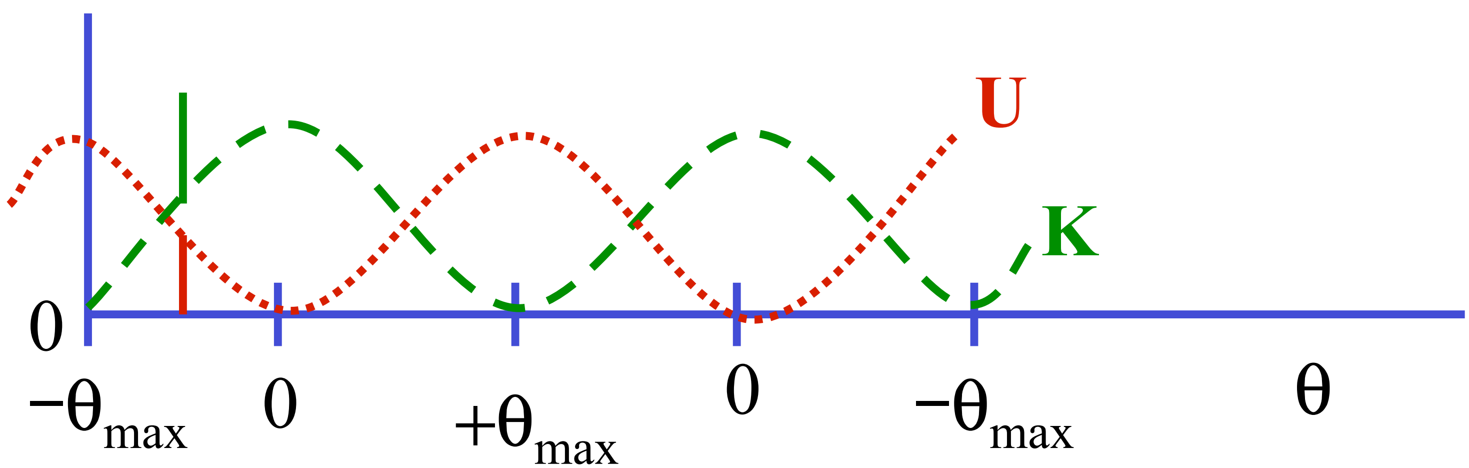
Pendulum (1-D) vertically

Find the speed of the mass as a function of angle.
Let 1 be at maximum height ($y = y_{max}$) and 2 be at some height (y)



$$v = \sqrt{2g\left(y_{max} - y\right)} = \sqrt{2g\left(L\left(1-\cos\theta_{max}\right) - L\left(1-\cos\theta\right)\right)}$$

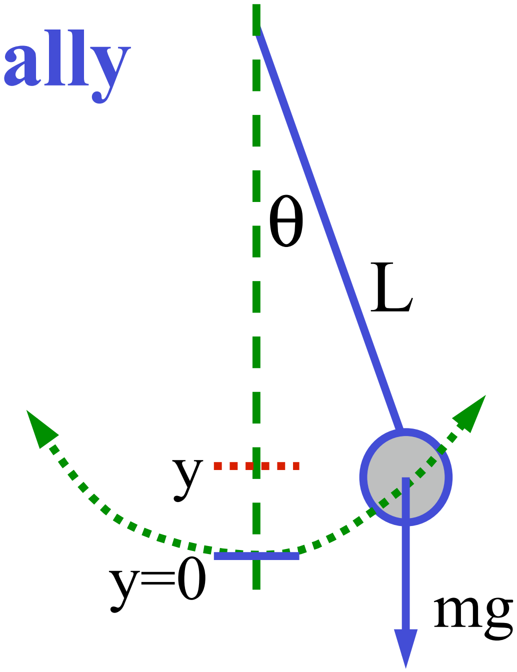
$$= \sqrt{2gL\left(\cos\theta - \cos\theta_{max}\right)}$$



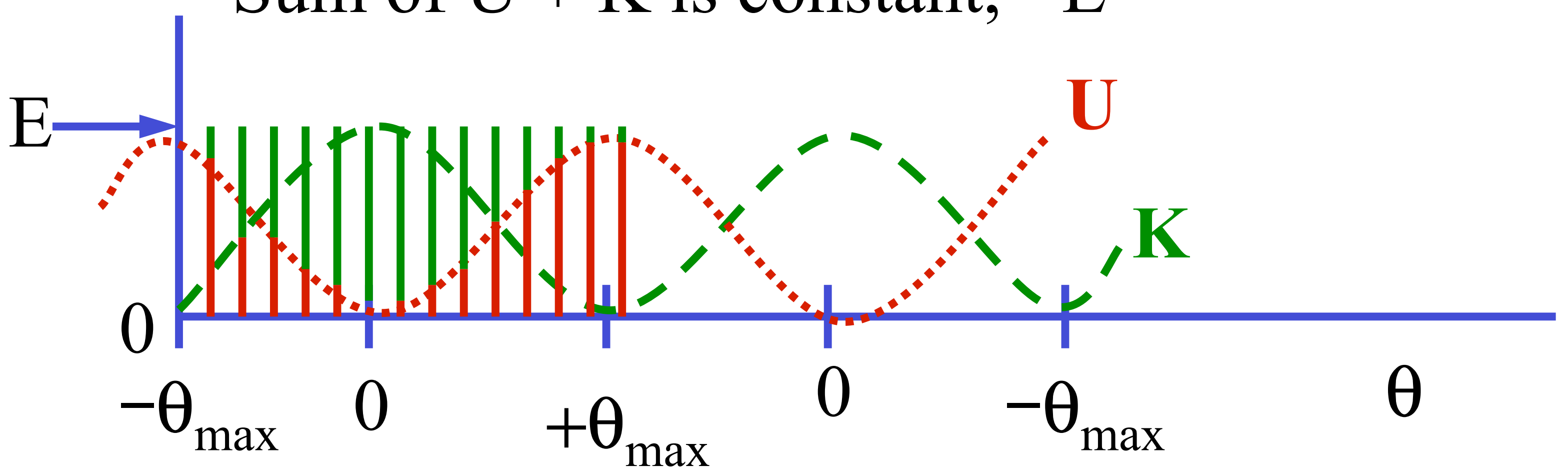
Pendulum (1-D) vertically

Find the speed of the mass as a function of angle.

Let 1 be at maximum height ($y = y_{max}$) and 2 be at some height (y)



Sum of U + K is constant, =E

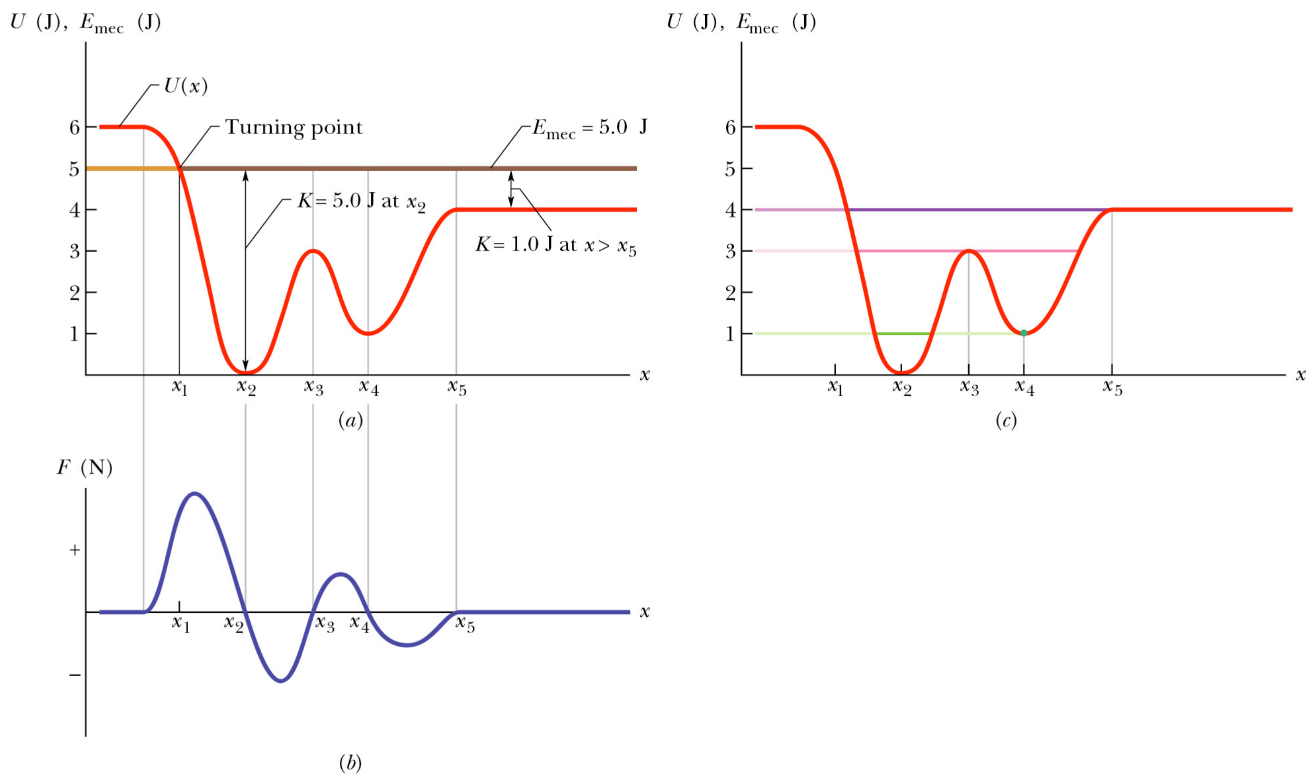


Potential Energy Curve

- We know

  $\Delta U(x) = -W = -F(x)\,\Delta x$

  Therefore $\quad F(x) = -dU(x)/dx$



Work done by external force

- When no friction acts within the system, the net work done by the external force equals to the change in mechanical energy

  $W = \Delta E_{mec} = \Delta K + \Delta U$

- Friction is a non-conservative force
- When a kinetic friction force acts within the system, then the thermal energy of the system changes: $\Delta E_{th} = f_k d$

  Therefore $W = \Delta E_{mec} + \Delta E_{th}$

Work done by external force

- When there are non-conservative forces (like friction) acting on the system, the net work done by them equals to the change in mechanic energy

$$W_{net} = \Delta E_{mec} = \Delta K + \Delta U$$

$$W_{friction} = -f_k d$$

Conservation of Energy

The total energy E of a system can change only by amounts of energy that are transferred to or from the system

$$W = \Delta E = \Delta E_{mec} + \Delta E_{th} + \Delta E_{int}$$

If there is no change in internal energy, but friction acts within the system: $W = \Delta E_{mec} + \Delta E_{th}$

If there are only conservative forces acting within the system: $W = \Delta E_{mec}$

If we take only the single object as the system

$$W = \Delta K$$

Law of Conservation of Energy

- For an isolated system (W = 0), the total energy E of the system cannot change

  $\Delta E_{mec} + \Delta E_{th} + \Delta E_{int} = 0$

- For an isolated system with only conservative forces, ΔE_{th} and ΔE_{int} are both zero. Therefore:

  $\Delta E_{mec} = 0$

Chapter 8: Potential Energy and The Conservation of Total Energy

Work and kinetic energy are energies of *motion*.

$$\Delta K = K_f - K_i = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = W_{net} = \int_{r_i}^{r_f} \vec{F}\cdot d\vec{r}$$

Potential energy is an energy that depends on *location*.

1-Dimension	3-Dimensions
$F_x = -\frac{dU(x)}{dx}$	$\vec{F}(\vec{r}) = -\nabla U(\vec{r}) = -\left[\frac{\partial U(\vec{r})}{\partial x}\hat{i} + \frac{\partial U(\vec{r})}{\partial y}\hat{j} + \frac{\partial U(\vec{r})}{\partial z}\hat{k}\right]$

Force Potential Energy Relationship

$$F(x) = -dU(x)/dx$$

Thus, the force in the x-direction is the negative derivative of the potential energy! The same holds true for y- and z-directions.

$$F_x(x,y,z) = -\frac{\partial U(x,y,z)}{\partial x}$$

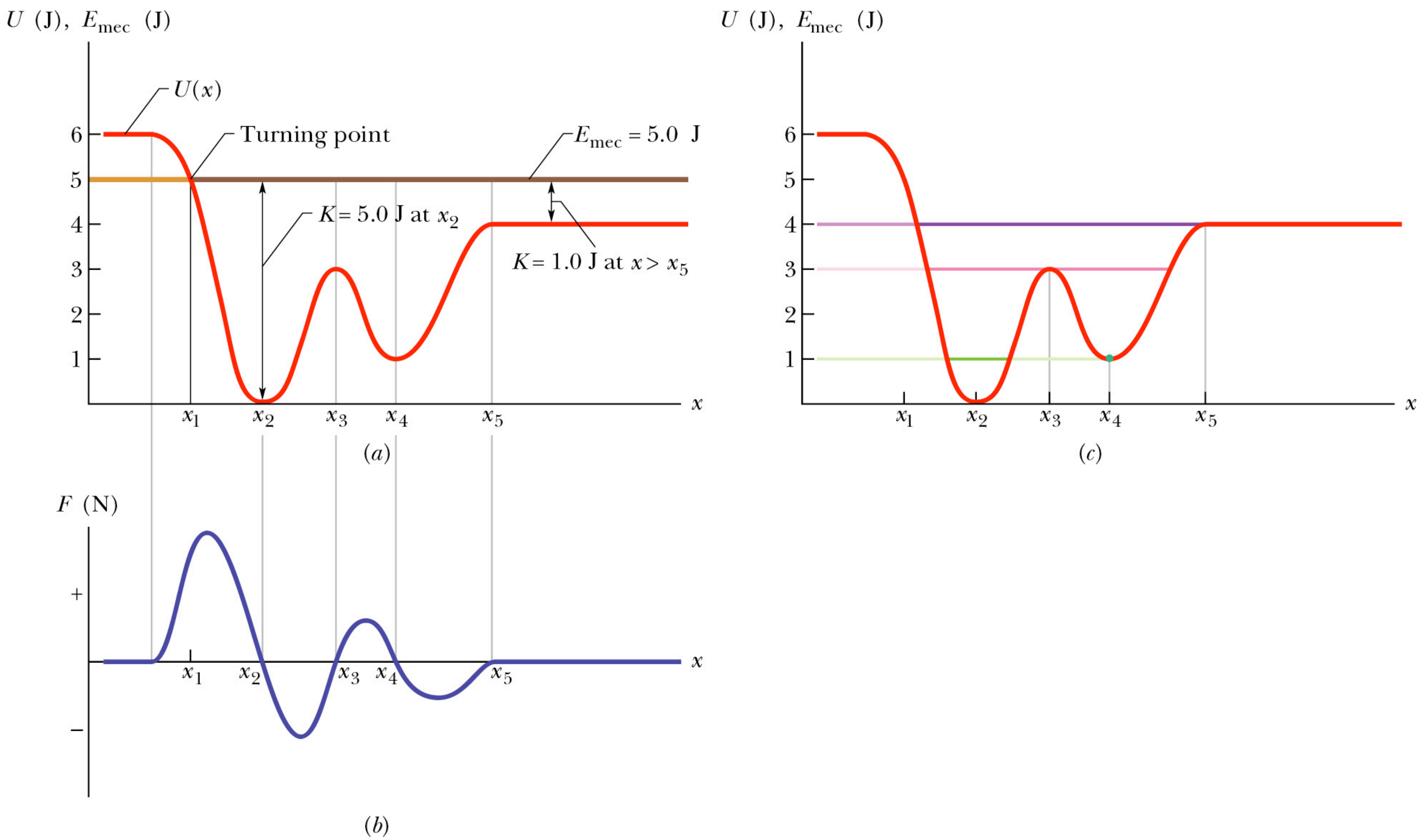
$$F_y(x,y,z) = -\frac{\partial U(x,y,z)}{\partial y}$$

$$F_z(x,y,z) = -\frac{\partial U(x,y,z)}{\partial z}$$

$$\vec{F} = \left(-\frac{\partial U(x,y,z)}{\partial x}\right)\hat{i} + \left(-\frac{\partial U(x,y,z)}{\partial y}\right)\hat{j} + \left(-\frac{\partial U(x,y,z)}{\partial z}\right)\hat{k}$$

Potential Energy Curve

Thus, what causes a force is the variation of the potential energy function, i.e., the force is the negative 3-D derivative of the potential energy!



Potential Energy Curve

We know: $\Delta U(x) = -W = -F(x)\,\Delta x$

Therefore: $F(x) = -dU(x)/dx$

Now integrate along the displacement:

$$W = \int \vec{F} \cdot d\vec{x} = -\int \frac{dU}{dx} dx$$

$$\int \vec{F} \cdot d\vec{x} = K_f - K_i = -\int \frac{dU}{dx} dx = -(U_f - U_i) = U_i - U_f$$

$$\int \vec{F} \cdot d\vec{x} = -\int \frac{dU}{dx} dx \quad = \quad K_f - K_i = U_i - U_f$$

Rearrange terms: $K_f + U_f = K_i + U_i$

Conservation of Mechanical Energy

Holds ***only*** for an isolated system **(no external forces)** and if only **conservative** forces are causing energy transfer between kinetic and potential energies within the system.

Mechanical energy: $E_{mec} = K + U$

We know:

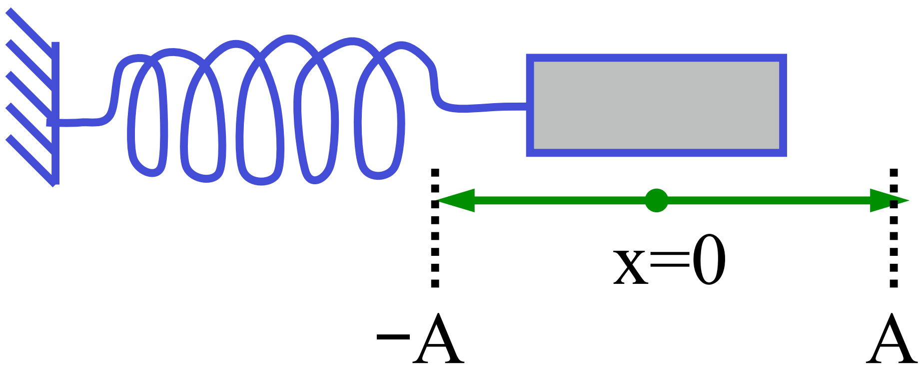
$\Delta K = W$ (work-kinetic energy theorem)

$\Delta U = -W$ (definition of potential energy)

Therefore: $\Delta K + \Delta U = 0 \Rightarrow (K_f - K_i) + (U_f - U_i) = 0$

Rearranging terms:

$$K_f + U_f = K_i + U_i = K_2 + U_2 = E_{mec} \quad \text{(a constant)}$$



Horizontal spring with mass oscillating with maximum amplitude $|x_{max}| = A$. At which displacement(s) would the kinetic energy equal the potential energy?

1) $x = 0$ 2) $x = \pm A$ 3) $x = \dfrac{\pm A}{2}$ 4) $x = \dfrac{\pm A}{\sqrt{2}}$

5) none of the above

$$\Delta U = -W = \int_{x_i}^{x_f} (-kx)\,dx$$

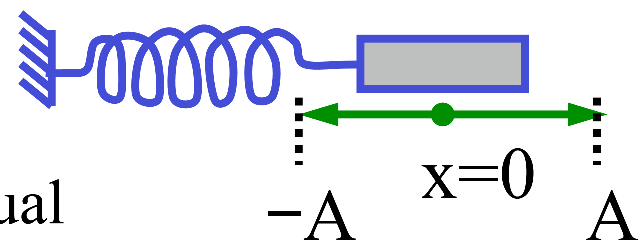
$$U_f - U_i = \tfrac{1}{2} k x_f^2 - \tfrac{1}{2} k x_i^2 \qquad U_i = 0 \text{ at } x_i = 0;\ U_{max} = \tfrac{1}{2} k x_{max}^2$$

$$E_{tot} = \frac{1}{2}kA^2 = K(x) + U(x) = 2\big(U(x)\big) = 2\left(\cancel{\frac{1}{2}}kx^2\right)$$

Since K = U

$$\Rightarrow \frac{1}{2}\cancel{k}A^2 = \cancel{k}x^2 \qquad \Rightarrow \qquad x = \frac{\pm A}{\sqrt{2}}$$

Horizontal spring with mass oscillating with maximum amplitude $|x_{max}| = A$. At which displacement(s) would the kinetic energy equal the potential energy?



1) $x = 0$

2) $x = \pm A$

3) $x = \dfrac{\pm A}{2}$

4) $x = \dfrac{\pm A}{\sqrt{2}}$

5) none of the above

Conservation of mechanical energy

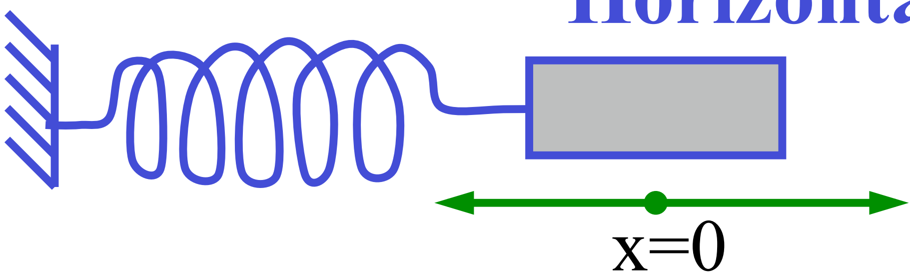
For an isolated system with only *conservative forces* (e.g., $F = mg$ and $F = -kx$) acting on the system:

$$E_{mec,1} = E_{mec,2} = E_{total} \quad \Rightarrow \quad K_1 + U_1 = K_2 + U_2 = E_{total}$$

$$\tfrac{1}{2}mv_1^2 + mgx_1 + \tfrac{1}{2}kx_1^2 = \tfrac{1}{2}mv_2^2 + mgx_2 + \tfrac{1}{2}kx_2^2 = E_{total}$$

Initial mechanical energy　　Final mechanical energy

Horizontal Spring



Isolated system with only conservative forces acting on it.

(e.g., $\vec{F} = -k\vec{x}$)

$$E_{mech,1} = E_{mech,2} = E_{total}$$

$$K_1 + U_1 = K_2 + U_2 = E_{total}$$

$$\tfrac{1}{2}\, mv_1^2 + \tfrac{1}{2}\, kx_1^2 = \tfrac{1}{2}\, mv_2^2 + \tfrac{1}{2}\, kx_2^2 = E_{total}$$

Work done by Spring Force

Spring force is a conservative force $\vec{F} = -k\vec{x}$

Work done *by* the spring force:

$$W_s = \int_{x_i}^{x_f} \vec{F} \cdot d\vec{x} = \int_{x_i}^{x_f} (-kx\,\hat{x}) \cdot d\vec{x} = -k \int_{x_i}^{x_f} x\,dx\,(\hat{x} \cdot \hat{x})$$

$$= -\left[\frac{1}{2}kx_f^2 - \frac{1}{2}kx_i^2\right] = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2$$

If $|x_f| > |x_i|$ (**further away from** equilibrium position); $W_s < 0$

My hand did **positive** work, while the spring did **negative** work so the total work on the object = 0

Work done by Spring Force

Spring force is a conservative force $\vec{F} = -k\vec{x}$

Work done *by* the spring force:

$$W_s = \int_{xi}^{xf} \vec{F} \cdot d\vec{x} = \int_{xi}^{xf} (-kx\,\hat{x}) \cdot d\vec{x} = -k \int_{xi}^{xf} x\,dx\,(\hat{x} \cdot \hat{x})$$

$$= -\left[\frac{1}{2}kx_f^2 - \frac{1}{2}kx_i^2\right] = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2$$

If $|\,x_f\,| < |\,x_i\,|$ (**closer to** equilibrium position); $W_s > 0$

My hand did **negative** work, while the spring did **positive** work so the total work on the object = 0

Work done by Spring Force -- Summary

Spring force is a conservative force $\vec{F} = -k\vec{x}$

Work done *by* the spring force:

$$W_s = \int_{x_i}^{x_f} \vec{F} \cdot d\vec{x} = \int_{x_i}^{x_f} (-kx\,\hat{x}) \cdot d\vec{x} = -k \int_{x_i}^{x_f} x\,dx\,(\hat{x} \cdot \hat{x})$$

$$= -\left[\frac{1}{2}kx_f^2 - \frac{1}{2}kx_i^2\right] = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2$$

If $|x_f| > |x_i|$ (**further away from** equilibrium position); $W_s < 0$

If $|x_f| < |x_i|$ (**closer to** equilibrium position); $W_s > 0$

Let $x_i = 0$, $x_f = x$ then $W_s = -\frac{1}{2} k x^2$

Elastic Potential Energy

Spring force is a conservative force $\vec{F} = -k\vec{x}$

$$\Delta U \equiv U_f - U_i = -W = \frac{1}{2}kx_f^2 - \frac{1}{2}kx_i^2$$

Choose the free end of the relaxed spring as the reference point: $U_i = 0$ at $x_i = 0$

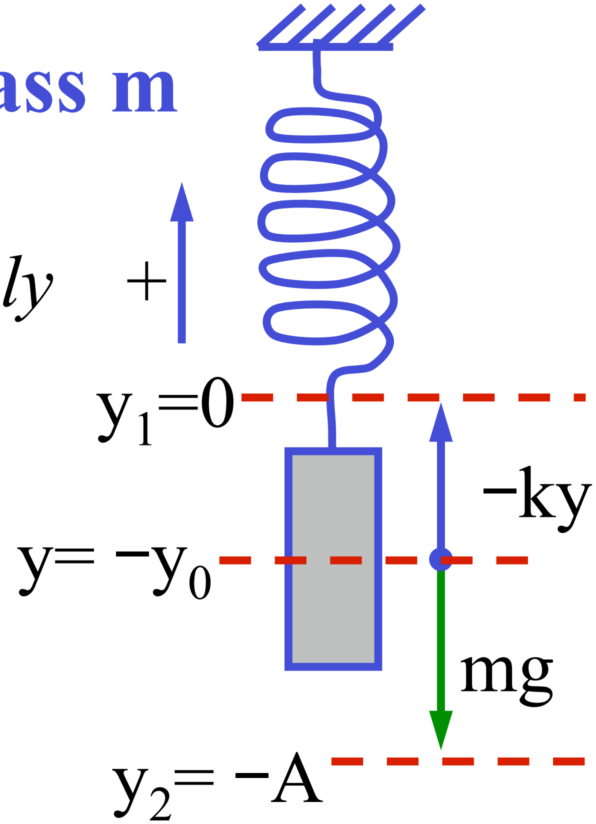
$$U(x) = -W(x) = \frac{1}{2}kx^2$$

The work went into potential energy, since the speeds are zero before and after.

Vertical Spring with mass m

Consider that mass m was held so the spring was relaxed (y_1=0) and then *slowly* let down to the equilibrium position.

Find this equilibrium position.



<u>Equilibrium position</u> ($F_g + F_s = 0$):

$F_g = -mg \qquad F_s = -ky_0$

(now the spring is *not relaxed!)*

=> $y_0 = -mg/k$ *(substitute for y_o)*

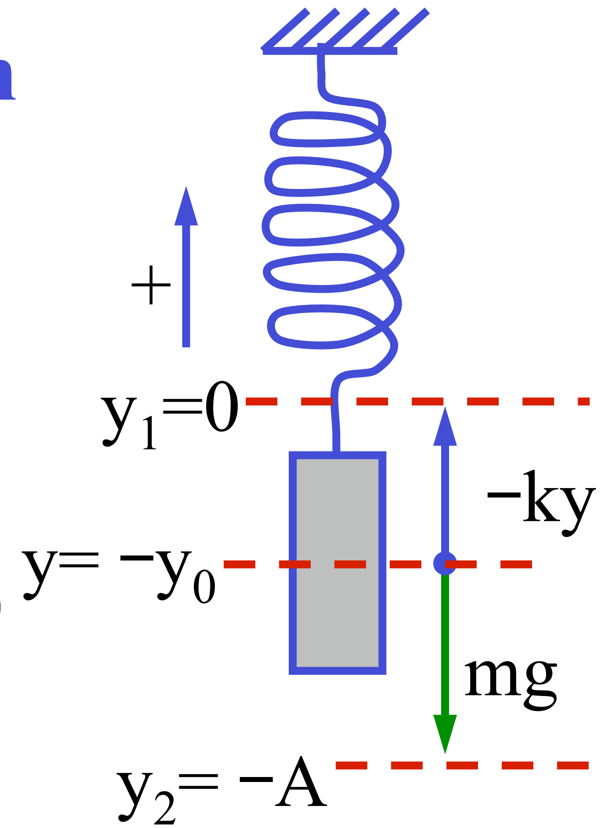
$K_e = 0, \qquad U_{g,e} = mgy_0 = mg(-mg/k) = -(mg)^2/k,$

$U_{s,e} = ½\, ky_0^2 = ½\, k\,(-mg/k)^2 = ½\,(mg)^2/k$

Vertical Spring with mass m

Now consider that the mass m was dropped from $y_1=0$

Find maximum amplitude A.



<u>Initial position ($y_1 = 0$)</u>:

$K_1 = 0$, $U_{g,1} = 0$, $U_{s,1} = \frac{1}{2}ky^2 = 0$

Choose = 0 Since $y_1 = 0$

$\Rightarrow E_{mech,1} = 0$

<u>Final position ($y_2 = A$)</u> :

$K_2 = 0$, $U_{g,2} = mgy_2 = mg(-A) = -mgA$,

$U_{s,2} = \frac{1}{2}ky_2^2 = \frac{1}{2}kA^2$

$$E_{mech,1} = E_{mech,2} = 0 = U_{g,2} + U_{s,2} = -mgA + \frac{1}{2}kA^2$$

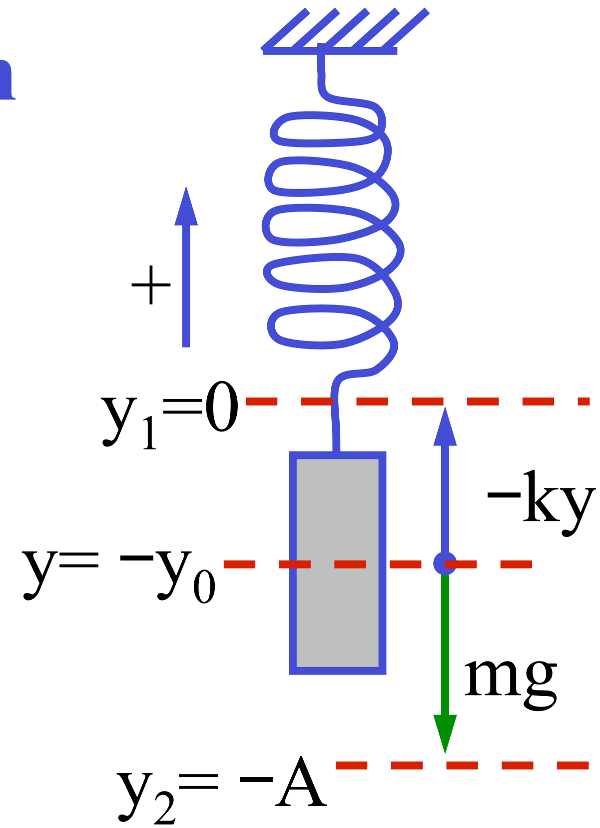
$$\Rightarrow A = 2mg/k \qquad (= 2y_0)$$

Vertical Spring with mass m

Now consider that the mass m was dropped from $y_1=0$

Where does the max speed occur?

Maximum speed occurs when the potential energy is a minimum.



<u>For an Arbitrary position</u>:

$E_{mech}= 0 = K + U_g+U_s =½\ mv^2 + mgy + ½\ ky^2$

$dU/dy = d(mgy + ½\ ky^2)/dy = 0$

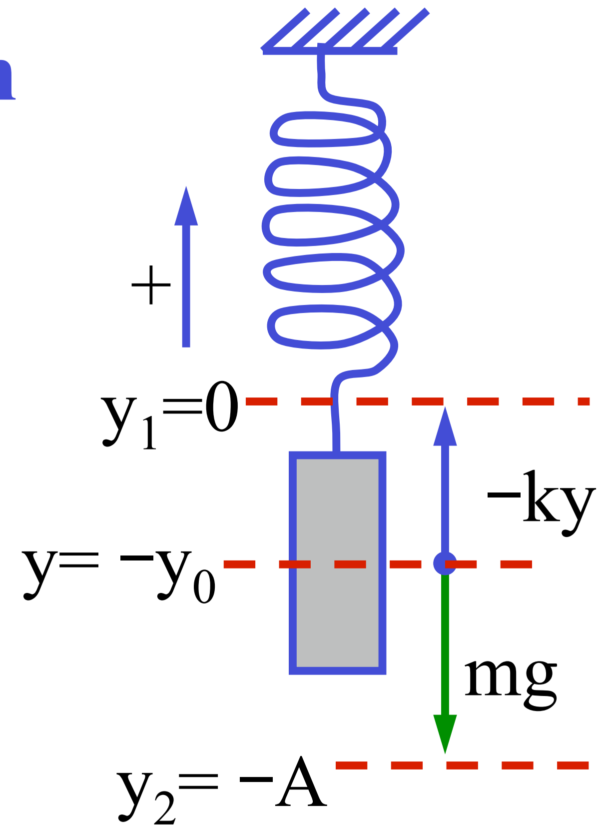
=> $mg + ky = 0$ => $y = -mg/k$ which is y_o (from 2 slides ago)

=> maximum speed is at equilibrium point (as expected)

Vertical Spring with mass m

Now consider that the mass m was dropped from $y_1=0$

What is the max speed?



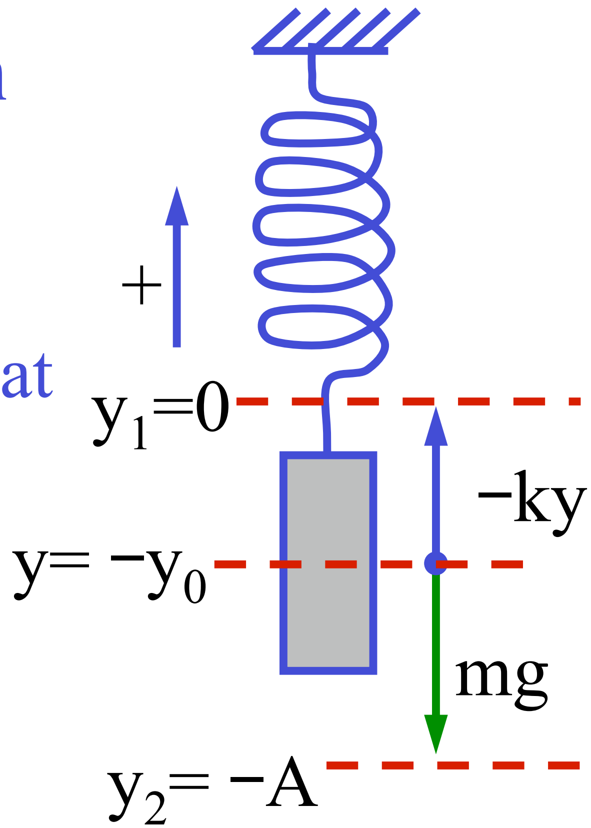
<u>Maximum speed position</u>:

$$0 = K_{max} + U_g + U_s$$
$$= \tfrac{1}{2} mv^2 + mg(-mg/k) + \tfrac{1}{2} k(-mg/k)^2$$
$$= \tfrac{1}{2} mv^2 - (mg)^2/k + \tfrac{1}{2} k(mg/k)^2$$
$$= \tfrac{1}{2} mv^2 - \tfrac{1}{2} (mg)^2/k \quad \Rightarrow \quad v^2 = mg^2/k$$

Vertical Spring with mass m

Now consider that the mass m was dropped from $y_1=0$

Find the mechanical energy of the mass at the equilibrium point.



Kinetic energy at $y = -y_0$:

$$K_{y0} = \tfrac{1}{2}\,(mg)^2/k$$

Potential energy at $y = -\,y_0$:

$$(U_g+U_s)|_{y=-y0} = mg(-y_0) + \tfrac{1}{2}\,k(-y_0)^2 = -mgy_0 + \tfrac{1}{2}k(y_0)^2$$

$$= -mg(mg/k) + \tfrac{1}{2}k(mg/k)^2 = -\tfrac{1}{2}(mg)^2/k$$

Mechanical energy at $y = -\,y_0$:

$$E_{mech} = K_{y0} + (U_g+U_s)|_{y=-y0} = \tfrac{1}{2}\,(mg)^2/k - \tfrac{1}{2}(mg)^2/k = 0$$

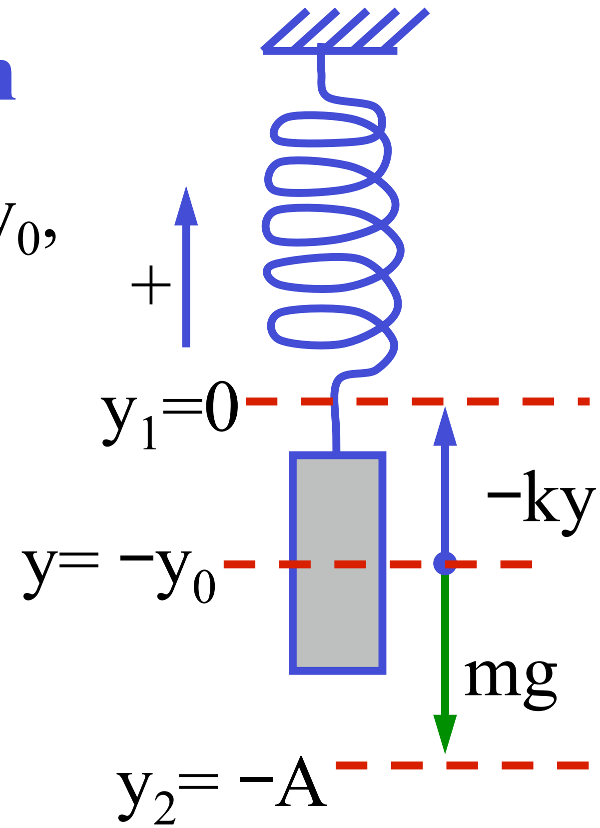
Vertical Spring with mass m

When we slowly let the mass relax to $-y_0$, the mechanical energy is:

$$E_{mech} = K_{-y0} + (U_g + U_s)|_{y=-y0}$$

$$0 \qquad -(mg)^2/k \qquad \tfrac{1}{2}(mg)^2/k$$

$$\Rightarrow \quad E_{mech} = -\tfrac{1}{2}(mg)^2/k$$



But when we let the mass drop, the mechanical energy at $y = y_0$ is $E_{mech} = 0$

Where did the missing energy ($\tfrac{1}{2}(mg)^2/k$) go?

Answer: Our hand provided a non-conservative force on m!

Work done by external force

When no friction acts within the system, the net work done by an external force from outside the system equals the change in mechanical energy of the system

$$W_{net} = \Delta E_{mec} = \Delta K + \Delta U$$

Friction is a **non-**conservative force that opposes motion

Work done by friction is: $W_{friction} = -f_k d$

When a kinetic friction force acts within the system, then the thermal energy of the system changes:

$$\Delta E_{th} = f_k d$$

Therefore $W = \Delta E_{mec} + \Delta E_{th}$

Conservation of Total Energy

The total energy E of a system can change only by an amount of energy that is transferred to or from the system.

$$W = \Delta E = \Delta E_{mec} + \Delta E_{th} + \Delta E_{int}$$

Here, ΔE_{int} are energy changes due to other non-conservative internal forces.

If there is no internal energy change, but friction acts within the system: $W = \Delta E_{mec} + \Delta E_{th}$

If there are only conservative forces acting within the system: $W = \Delta E_{mec}$

Isolated Systems

For an isolated system (no work done on the system, W = 0), the total energy E of the system cannot change

$$\Delta E_{mec} + \Delta E_{th} + \Delta E_{int} = 0$$

For an isolated system with only conservative forces, ΔE_{th} and ΔE_{int} are both zero.

Therefore: $\Delta E_{mec} = 0$

Law of Conservation of Total Energy in an Isolated System

$$E_i = K_i + U_i + E_{thermal_i} + E_{internal_i}$$

$$=$$

$$E_f = K_f + U_f + E_{thermal_f} + E_{internal_f}$$

Rearrange terms.

$$(E_f - E_i) =$$

$$(K_f - K_i) + (U_f - U_i) + (E_{thermal_f} - E_{thermal_i}) + (E_{internal_f} - E_{internal_i}) = 0$$

$$\boxed{\begin{aligned} \Delta E &= (\Delta K + \Delta U) + \Delta E_{thermal} + \Delta E_{internal} \\ &= \Delta E_{mechanical} + \Delta E_{thermal} + \Delta E_{internal} = 0 \end{aligned}}$$

Law of Conservation of Energy

Count up the initial energy in ***all*** of its forms.

$$E_i = K_i + U_i + E_{thermal_i} + E_{internal_i}$$

Count up the final energy in ***all*** of its forms.

$$E_f = K_f + U_f + E_{thermal_f} + E_{internal_f}$$

These two ***must*** be equal (if nothing is added form outside the system).

$$E_i = K_i + U_i + E_{thermal_i} + E_{internal_i}$$

$$= E_f = K_f + U_f + E_{thermal_f} + E_{internal_f}$$

Sample Problem 8-6

A wooden crate of $m = 14kg$ is pushed along a horizontal floor with a constant force of $|F| = 40N$ for a total distance of $d = 0.5m$, during which the crate's speed decreased from $v_o = 0.60$ m/s to $v = 0.20$m/s.

A) Find the work done by F.

$$W = Fd\cos\theta = (40N)(0.50m)\cos 0^o = 20J$$

B) Find the increase in thermal energy.

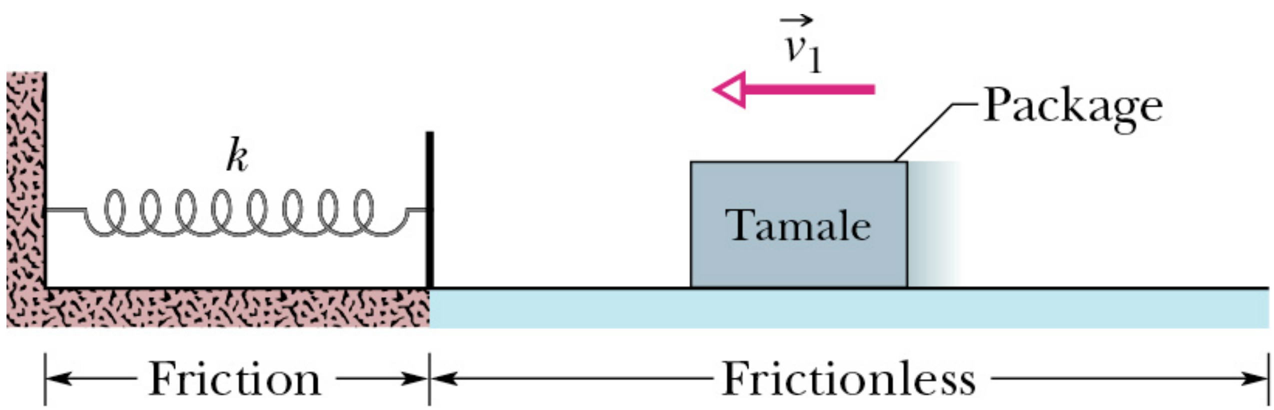
(Consider the work done on the block/spring system)

$$W = \Delta E_{mec} + \Delta E_{thermal} = 20J = \tfrac{1}{2}mv^2 - \tfrac{1}{2}mv_o^{\,2} + \Delta E_{thermal}$$

$$\Rightarrow \; \Delta E_{thermal} = W - \Delta E_{mec} = 20J - (\tfrac{1}{2}mv^2 - \tfrac{1}{2}mv_o^{\,2}) = 22.2J$$

Sample Problem 8-7

In the figure, a 2.0kg package slides along a floor with speed v_1 = 4.0m/s. It then runs into and compresses a spring, until the package momentarily stops. Its path to the initially relaxed spring is frictionless, but as it compresses the spring, a kinetic friction force from the floor, of magnitude 15 N , acts on it. The spring constant is 10,000 N/m. By what distance d is the spring compressed when the package stops?



The net change in energy in the system must equal zero.

$$\Delta E_{\text{mechanical}} + \Delta E_{\text{thermal}} = 0$$

$$\Rightarrow \left(E_{\text{mec},2} - E_{\text{mec},1}\right) + \Delta E_{\text{thermal}} = 0$$

Initial mechanical energy, (The spring is relaxed)

$$E_{\text{mec},1} = K_1 + U_1 = \tfrac{1}{2}mv_1^2 + U_1 = \tfrac{1}{2}mv_1^2 + 0 = \tfrac{1}{2}mv_1^2$$

Final mechanical energy, (The mass is stopped)

$$E_{\text{mec},2} = K_2 + U_2 = \tfrac{1}{2}mv_2^2 + U_2 = 0 + \tfrac{1}{2}kd_2^2 = \tfrac{1}{2}kd^2$$

The change in mechanical energy must equal the energy converted to thermal energy.

$$\Delta E_{\text{thermal}} = f_k d$$

$$\left(E_{\text{mec},2} - E_{\text{mec},1}\right) + \Delta E_{\text{thermal}} = \left(\tfrac{1}{2}kd^2 - \tfrac{1}{2}mv_1^2\right) + f_k d = 0$$

Thus, a quadratic equation in d with:

m = 2.0 kg, v_1 = 4.0 m/s, f_k = 15 N, and k = 10,000 N/m

$$\tfrac{1}{2}kd^2 + f_k d - \tfrac{1}{2}mv_1^2 = 0$$

$$\Rightarrow d = 0.055\text{m}$$