Chapter 07: Kinetic Energy and Work

Conservation of Energy is one of Nature's fundamental laws that is not violated.

Energy can take on different forms in a given system. This chapter we will discuss **work** and **kinetic energy**.

If we put energy ***into*** the system by doing **work**, this additional energy has to go somewhere. That is, the **kinetic energy** increases or as in next chapter, the **potential energy** increases.

The opposite is also true when we take energy out of a system. the **grand** total of all forms of energy in a given system ***is*** (and ***was***, and ***will be***) a constant.

Exam 2 Review

- Chapters 7, 8, 9, 10.
- A majority of the Exam (~75%) will be on Chapters 7 and 8 (problems, quizzes, and concepts)
- Chapter 9 lectures and problems
- Chapter 10 lecture

Different forms of energy

Kinetic Energy:
linear motion
rotational motion

Potential Energy:
gravitational
spring compression/tension
electrostatic/magnetostatic
chemical, nuclear, etc....

Mechanical Energy is the sum of **Kinetic energy** + **Potential energy**. (**reversible** process)

Friction will convert **mechanical energy** to **heat**. Basically, this (conversion of mechanical energy to heat energy) is a **non-reversible** process.

Chapter 07: Kinetic Energy and Work

Kinetic Energy is the energy associated with the **motion** of an object.

$$\boxed{K = \frac{1}{2} m v^2}$$

m: mass and v: speed

SI unit of energy: 1 joule = 1 J = 1 $kg.m^2/s^2$

Work

Work is **energy** transferred to or from an object by means of a force acting on the object.

Formal definition: $$\boxed{W = \int \vec{F} \cdot d\vec{s}}$$

Special case: Work done by a ***constant*** force:

$$W = (\underline{F \cos\theta})\, d = F\, d \cos\theta$$

Component of $\vec{F}$ in direction of $\vec{d}$

Work done on an object moving with constant velocity?

constant velocity => acceleration = 0 => force = 0

=> work = 0

Consider 1-D motion.

$$W = \int_{x_i}^{x_f} F\,dx = \int_{x_i}^{x_f} (m\,a)\,dx = \int_{x_i}^{x_f} m\left(\frac{dv}{dt}\right)dx$$

$$= \int_{x_i}^{x_f} m\left(\frac{dv}{dx}\cancelto{v}{\frac{dx}{dt}}\right)dx = \int_{x_i}^{x_f} mv\left(\frac{dv}{dx}\right)dx$$

$$= \int_{v_i}^{v_f} mv\,dv = \frac{1}{2}mv^2\Big|_{v_i}^{v_f} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 \quad \boxed{= K_f - K_i}$$

(Integral over displacement becomes integral over velocity)

So, **kinetic energy** is mathematically connected to **work**!!

Work-Kinetic Energy Theorem

The change in the kinetic energy of a particle is equal the net work done on the particle.

$$\Delta K = K_f - K_i = W_{net} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

.... or in other words,

$$\boxed{\underline{\textbf{Final kinetic energy}} = \underline{\textbf{Initial kinetic energy}} + \underline{\textbf{net Work}}}$$

$$K_f = \frac{1}{2}mv_f^2 = K_i + W_{net} = \frac{1}{2}mv_i^2 + W_{net}$$

W_{net} is work done by all forces

Work done by a **constant** force:

$$W = F\, d \cos \text{Ø} = \vec{F} \cdot \vec{d}$$

Consequences: (ø if the angle between $\vec{F}$ and $\vec{d}$)

when $\text{Ø} < 90^o$, W is positive

when $\text{Ø} > 90^o$, W is negative

when $\text{Ø} = 90^o$, $W = 0$

when F or d is zero, $W = 0$

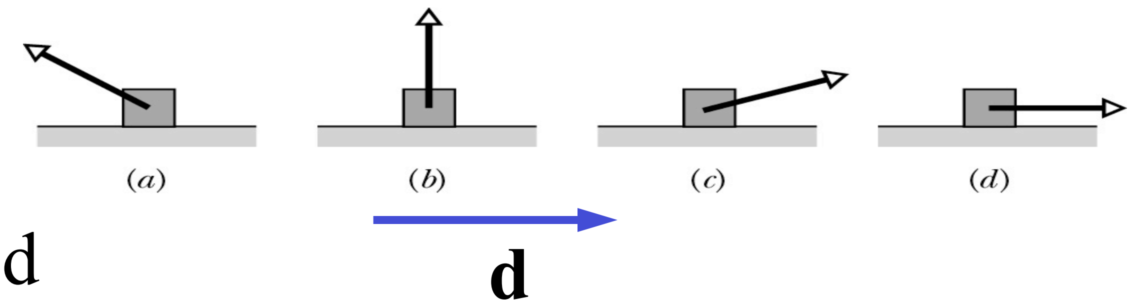
Work ***done on the object by the force***:

–**Positive work**: object receives energy

–**Negative work**: object loses energy

Force displacement question

In all of the four situations shown below, the box is sliding to the right a distance of d. The magnitudes of the forces (black arrows) are identical. Rank the work done during the displacement, from most positive to most negative.



1. a, b, c, d
2. d, c, b, a
3. b, a, c, d
4. all result in the same work
5. none of the above

- Net work done by several forces

$$\Delta W = \vec{F}_{net} \cdot \vec{d} = (\vec{F}_1 + \vec{F}_2 + \vec{F}_3) \cdot \vec{d}$$

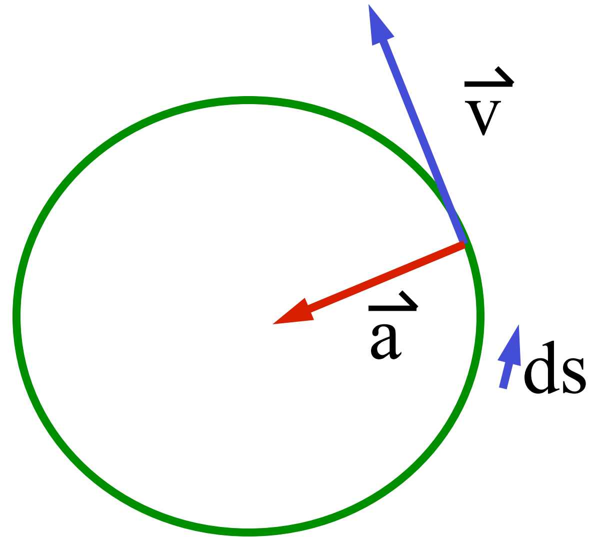
$$= \vec{F}_1 \cdot \vec{d} + \vec{F}_2 \cdot \vec{d} + \vec{F}_3 \cdot \vec{d} = W_1 + W_2 + W_3$$

Remember that the above equations hold **ONLY** for ***constant*** forces. In general, you must integrate the force(s) over displacement!

Question: What about circular motion?

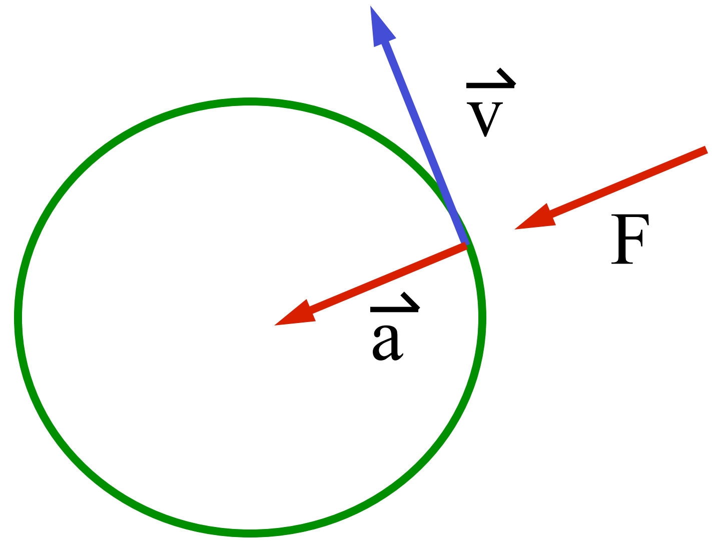
How much work was done and when?

$$\boxed{W = \int \vec{F} \cdot d\vec{s}}$$



Question: What about circular motion?

How much work was done and when?



Work was done to start the motion and then **NO** work is required to keep the object in circular motion! Centripetal force is perpendicular to the velocity and thus the displacement.

(F and d vectors are perpendicular, cos $90^o = 0$, dot product is zero)

A particle moves along the x-axis. Does the kinetic energy of the particle increase, decrease , or remain the same if the particle's velocity changes?

(a) from –3 m/s to –2 m/s?

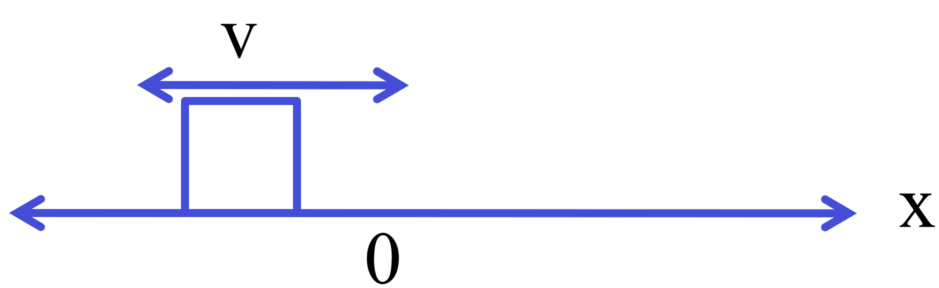
$|v_f| < |v_i|$ means $K_f < K_i$, so work is negative

(b) from –2 m/s to 2 m/s?

−2 m/s → 0 m/s: work negative

0 m/s → +2 m/s: work positive

Together they add up to net zero work.



Work done by Gravitation Force

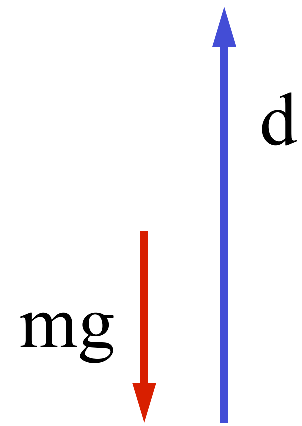
$W_g = mg\ d \cos ø$

throw a ball upwards with v_0:

– during the rise,

$W_g = mg\ d \cos 180^o = -mgd$

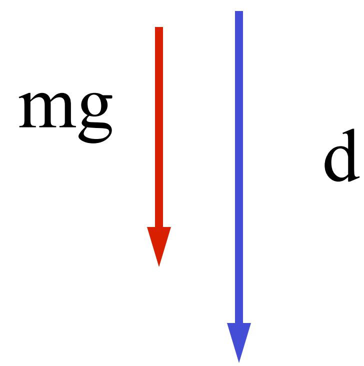
negative work $\longrightarrow$ K & v decrease



– during the fall,

$W_g = mgd \cos 0^o = mgd$

positive work $\longrightarrow$ K & v increase



<u>**Final kinetic energy**</u> = <u>**Initial kinetic energy**</u> + <u>**net Work**</u>

Work-Kinetic Energy Theorem

The change in the kinetic energy of a particle is equal the net work done on the particle.

$$W_{net} \equiv \int_i^f \vec{F} \cdot d\vec{r} = \Delta K = K_f - K_i = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

.... or in other words,

final kinetic energy = initial kinetic energy + net work

$$\boxed{K_f = \frac{1}{2}mv_f^2 = K_i + W_{net} = \frac{1}{2}mv_i^2 + W_{net}}$$

Work done by a spring force

The spring force is given by

$$\vec{F} = -k\vec{x} \quad (\textbf{Hooke's law})$$

k: spring (or force) constant

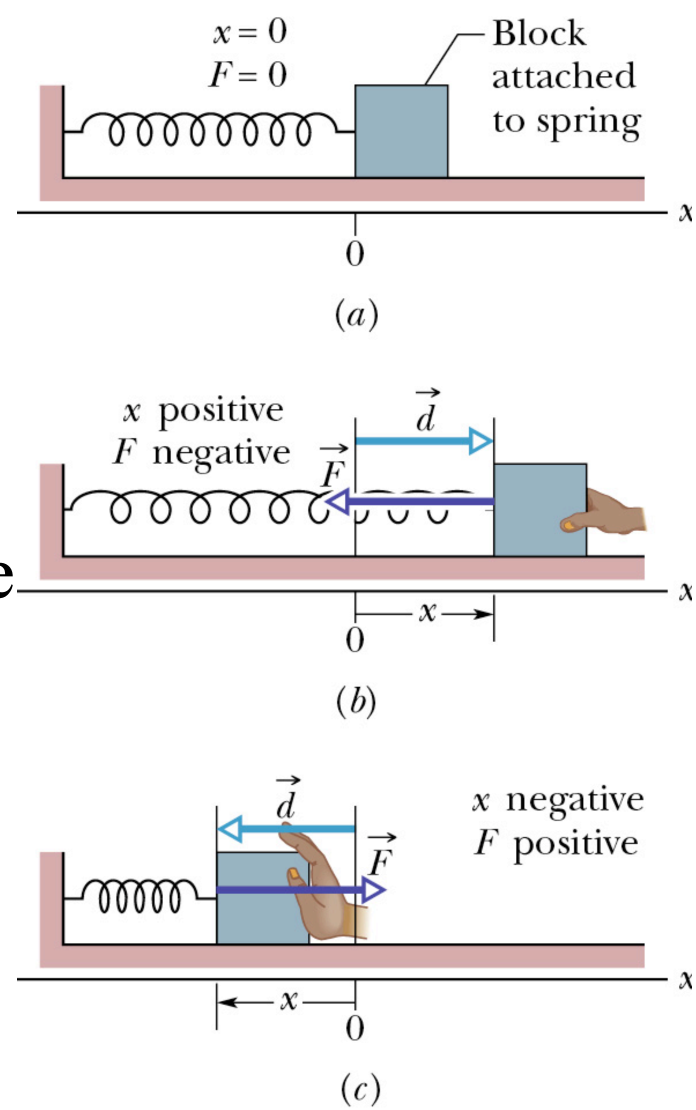
k relates to stiffness of spring;

unit for k: N/m.

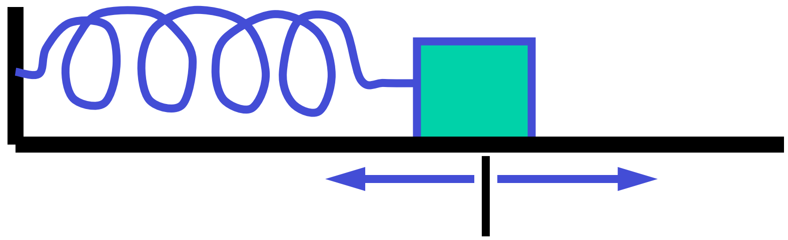
Spring force is a variable force

x = 0 at the free end of

the relaxed spring.



Spring Oscillation



1) 0 m
2) 0.05 m
3) 0.5 m
4) 10 m
5) 100 m

A 2.0 kg block is attached to a horizonal ideal spring with a spring constant of k = 200 N/m. When the spring is at its equilibrium position, the block is given a speed of 5.0 m/s. What is the maximum displacement (i.e., amplitude, A) of the spring?

$$W = \int_{x_i}^{x_f} \vec{F} \cdot d\vec{x} = \int_{x_i}^{x_f} kx(\cos 180^\circ)dx$$

Work − Kinetic Energy Theorem

$$= \int_0^A (-kx)dx = \frac{-1}{2}kA^2 = \overbrace{K_f}^{0.0} - K_i = \frac{-1}{2}mv_i^2$$

$$\Rightarrow \quad A = \sqrt{\frac{m}{k}}\, v_i = \sqrt{\frac{2.0}{200}}\,(5.0\,\mathrm{m/s}) = \sqrt{\frac{1}{100}}\,(5.0\,\mathrm{m/s}) = (0.5\,\mathrm{m/s})$$

Chapter 8: Potential Energy and Conservation of Energy

Work and **kinetic energy** are energies of ***motion***.

$$\Delta K = K_f - K_i = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = W_{net} = \int_{r_i}^{r_f} \vec{F} \cdot d\vec{r}$$

What about the energy of an object that depends on **location** or ***position***.

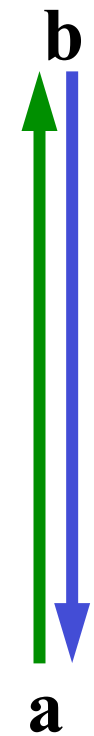
This energy is called ***potential energy***.

Chapter 8: Potential Energy and Conservation of Energy

Work done by gravitation for a ball thrown upward that then falls back down

$$W_{ab} + W_{ba} = -\,mg\,d + mg\,d = 0$$

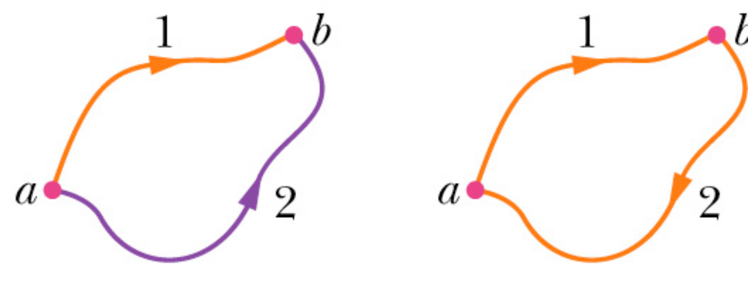
The gravitational force is said to be a **conservative force**.



A force is a **conservative** force if the net work it does on a particle moving around every closed path is zero.

Conservative forces

$W_{ab,1} + W_{ba,2} = 0$
$W_{ab,2} + W_{ba,2} = 0$
therefore: $W_{ab,1} = W_{ab,2}$



(a) (b)

i.e. The work done by a conservative force on a particle moving between two points does not depend on the path taken by the particle.

So, ... choose the easiest path!!

Conservative & Non-conservative forces

Conservative Forces: **(path independent)**

- **gravitational**
- **spring**
- **electrostatic**

Non-conservative forces: **(dependent on path)**

- **friction**
- **air resistance**
- **electrical resistance**

These forces will convert mechanical energy into heat and/or deformation.

Gravitation Potential Energy

Potential energy is associated with the configuration of a system in which a conservative force acts: $\Delta U = -W$

For a general conservative force F = F(x)

$$\Delta U = -W = -\int_{x_i}^{x_f} \vec{F}(x) \cdot d\hat{x}$$

Gravitational potential energy:

$$\Delta U = -\int_{y_i}^{y_f} (-mg)dy = mg(y_f - y_i)$$

assume $U_i = 0$ at $y_i = 0$ (reference point)

$U(y) = mgy$ (gravitational potential energy)

only depends on vertical position

Nature only considers changes in Potential energy important. Thus, we will ***always*** measure ΔU.

So, . . . We need to set a reference point!

The potential at that point could be defined to be zero (i.e., $U_{ref.\ point} = 0$) in which case we drop the "Δ" for convenience.

BUT, it is always understood to be there!

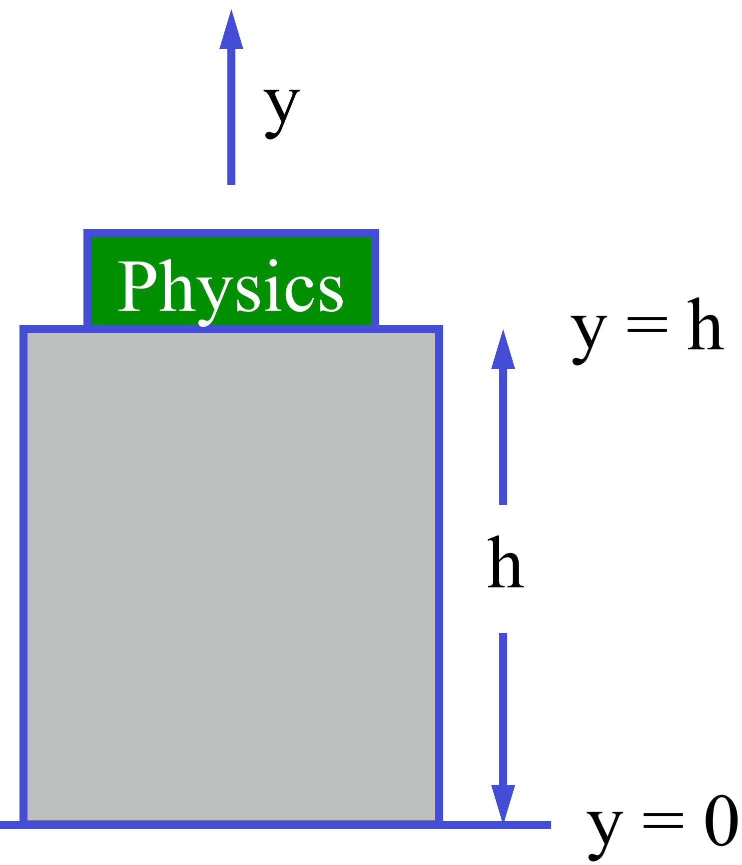
- **Potential energy and reference point (y = 0 at ground)**

  A 0.5 kg physics book falls from a table 1.0 m to the ground. What is U and ΔU if we take reference point y = 0 (assume U = 0 at y = 0) at the ground?

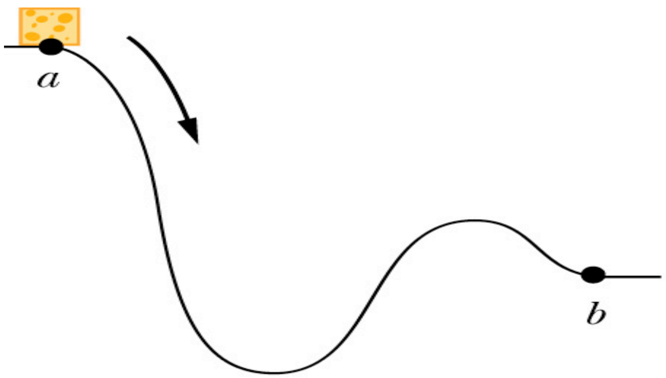
$$\Delta U = \underbrace{U_f}_{0} - U_i = -(mgh)$$

The book lost potential energy.

Actually, it was converted into kinetic energy



Sample problem 8-1: A block of cheese, m = 2 kg, slides along frictionless track from a to b, the cheese traveled a total distance of 2 m along the track, and a net vertical distance of 0.8 m. How much is W_g?



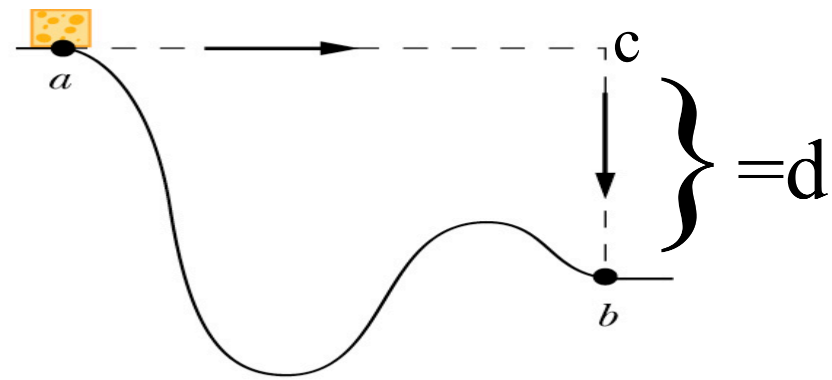
(a)

$$W_{net} = \int_a^b \vec{F}_g \cdot d\vec{r}$$

But, ... We don't know the angle between F and dr

Easier (or another) way?

Sample problem 8-1: A block of cheese, m = 2 kg, slides along frictionless track from a to b, the cheese traveled a total distance of 2 m along the track, and a net vertical distance of 0.8 m. How much is W_g?



(b)

Split the problem into two parts (two paths) since gravity is conservative

$$W_{net} = \underbrace{\int_a^c \vec{F}_g \cdot d\hat{x}}_{0} + \underbrace{\int_c^b \vec{F}_g \cdot d\hat{y}}_{mg(b-c) = mgd}$$

Elastic Potential Energy

Spring force is also a conservative force

$$F = -kx$$

$$\Delta U = -W = \int_{x_i}^{x_f} (-kx)\,dx$$

$$U_f - U_i = \tfrac{1}{2} kx_f^2 - \tfrac{1}{2} kx_i^2$$

Choose the free end of the relaxed spring as the reference point:

that is: $U_i = 0$ at $x_i = 0$

$\mathbf{U = \tfrac{1}{2} kx^2}$ **Elastic potential energy**

Conservation of Mechanical Energy

- Mechanical energy

  $E_{mec} = K + U$

- For an isolated system (no external forces), if there are only conservative forces causing energy transfer within the system….

We know: $\Delta K = W$ (work-kinetic energy theorem)

Also: $\Delta U = -W$ (definition of potential energy)

Therefore: $\Delta K + \Delta U = 0 \rightarrow (K_f - K_i) + (U_f - U_i) = 0$

therefore $\mathbf{K_1 + U_1 = K_2 + U_2}$ (States 1 and 2 of system)

$\mathbf{E_{mec,1} = E_{mec,2}}$ the mechanical energy is conserved

Conservation of mechanical energy

For an object moved by a spring in the presence of a gravitational force.

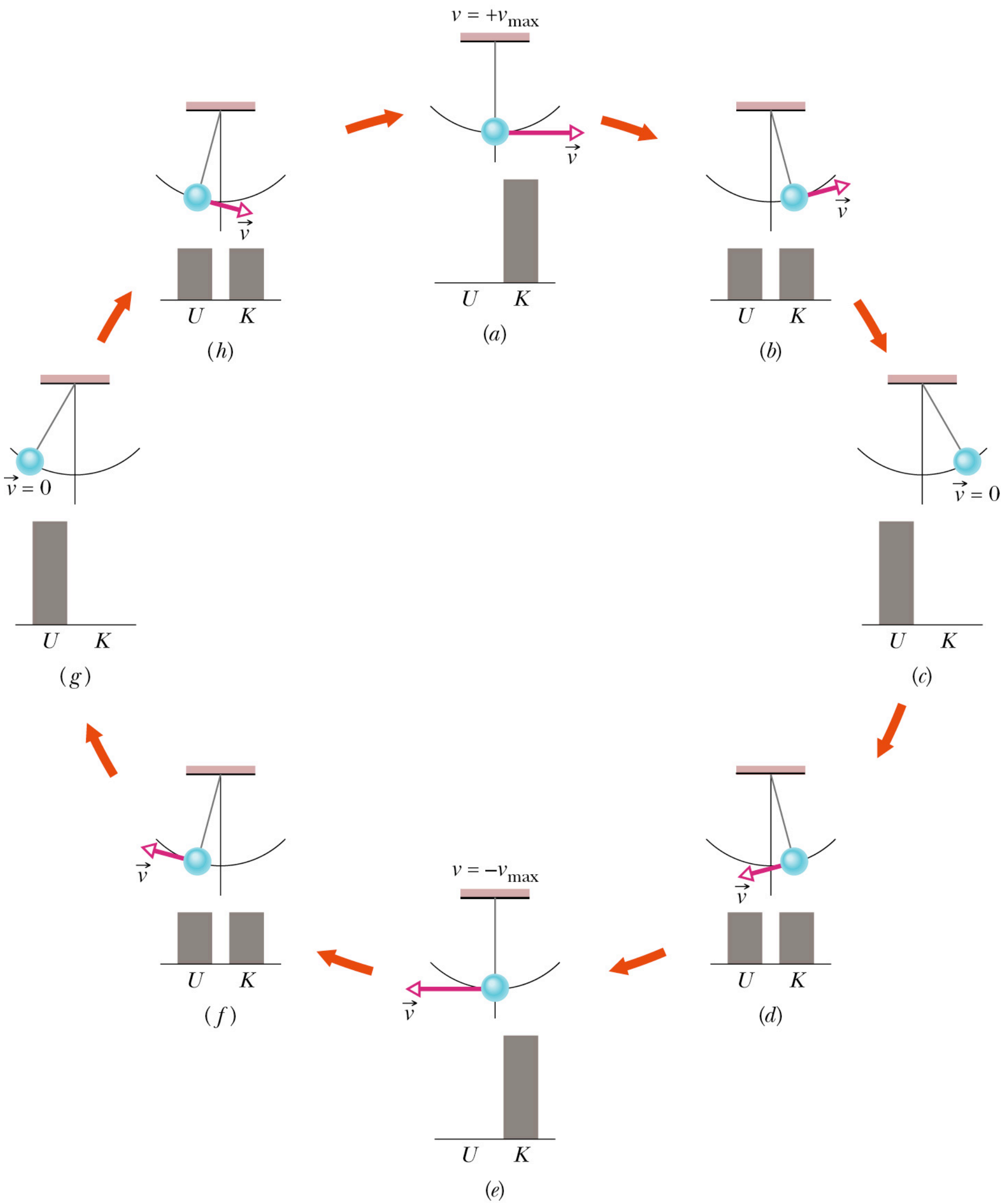
This is an **isolated** system with only conservative forces ($F = mg$, $F = -kx$) acting inside the system

$$E_{mec,1} = E_{mec,2}$$

$$K_1 + U_1 = K_2 + U_2$$

$$\tfrac{1}{2} mv_1^2 + mgy_1 + \tfrac{1}{2} kx_1^2 = \tfrac{1}{2} mv_2^2 + mgy_2 + \tfrac{1}{2} kx_2^2$$

The bowling ball pendulum demonstration



Mechanical energy is conserved ***if*** there are only conservative forces acting on the system.

Work done by external force

- When no friction acts within the system, the net work done by the external force equals to the change in mechanical energy

  $W_{net} = \Delta E_{mec} = \Delta K + \Delta U$

- Friction is a non-conservative force
- When a kinetic friction force acts within the system, then the thermal energy of the system changes: $\Delta E_{th} = f_k d$

  Therefore $W_{net} = \Delta E_{mec} + \Delta E_{th}$

Conservation of Energy

The total energy E of a system can change only by amounts of energy that are transferred to or from the system

$$W = \Delta E = \Delta E_{mec} + \Delta E_{th} + \Delta E_{int}$$

If there is no change in internal energy, but friction acts within the system: $W = \Delta E_{mec} + \Delta E_{th}$

If there are only conservative forces acting within the system: $W = \Delta E_{mec}$

If we take only the single object as the system

$$W = \Delta K$$

Law of Conservation of Energy

- For an isolated system ($W = 0$), the total energy E of the system cannot change

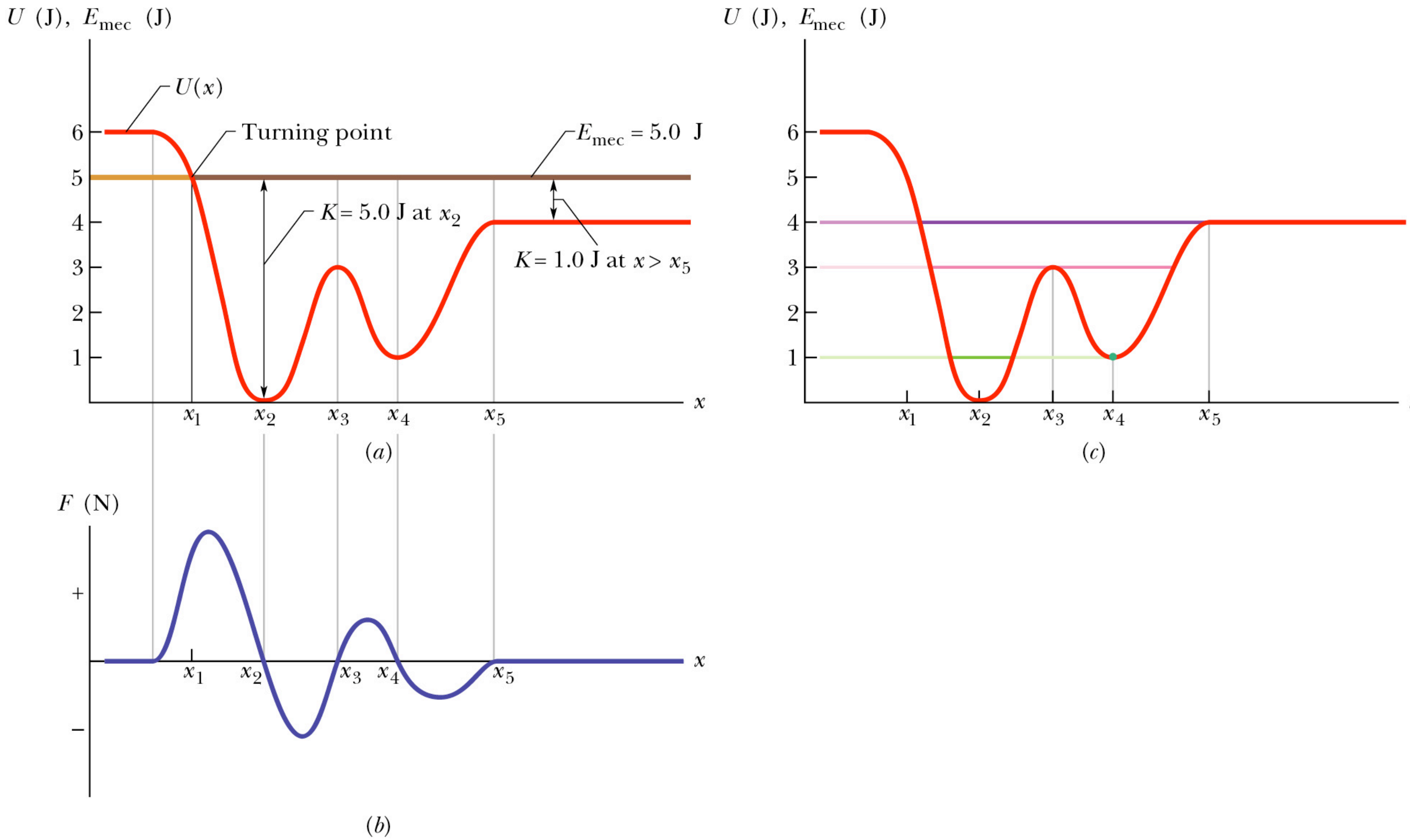
  $$\Delta E_{mec} + \Delta E_{th} + \Delta E_{int} = 0$$

- For an isolated system with only conservative forces, ΔE_{th} and ΔE_{int} are both zero. Therefore:

  $$\Delta E_{mec} = 0$$

Potential Energy Curve

Thus, what causes a force is the variation of the potential energy function, i.e., the force is the negative 3-D derivative of the potential energy!



Potential Energy Curve

We know: $\Delta U(x) = -W = -F(x)\,\Delta x$

Therefore: $F(x) = -dU(x)/dx$

Now integrate along the displacement:

$$W = \int \vec{F}\cdot d\vec{x} = -\int \frac{dU}{dx}\,dx$$

$$\int \vec{F}\cdot d\vec{x} = K_f - K_i = -\int \frac{dU}{dx}\,dx = -(U_f - U_i) = U_i - U_f$$

$$\int \vec{F}\cdot d\vec{x} = -\int \frac{dU}{dx}\,dx \quad = \quad K_f - K_i = U_i - U_f$$

Rearrange terms: $K_f + U_f = K_i + U_i$

Conservation of Mechanical Energy

Holds ***only*** for an isolated system **(no external forces)** and if only **conservative** forces are causing energy transfer between kinetic and potential energies within the system.

Mechanical energy: $E_{mec} = K + U$

We know:

$\Delta K = W$ (work-kinetic energy theorem)

$\Delta U = -W$ (definition of potential energy)

Therefore: $\Delta K + \Delta U = 0 \rightarrow (K_f - K_i) + (U_f - U_i) = 0$

Rearranging terms:

$$K_f + U_f = K_i + U_i = K_2 + U_2 = E_{mec} \quad \text{(a constant)}$$

Work done by Spring Force

Spring force is a conservative force $\vec{F} = -k\vec{x}$

Work done *by* the spring force:

$$W_s = \int_{x_i}^{x_f} \vec{F}\cdot d\vec{x} = \int_{x_i}^{x_f} (-kx\,\hat{x})\cdot d\vec{x} = -k\int_{x_i}^{x_f} x\,dx\,(\hat{x}\cdot\hat{x})$$

$$= -\left[\frac{1}{2}kx_f^2 - \frac{1}{2}kx_i^2\right] = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2$$

If $|x_f| > |x_i|$ (**further away from** equilibrium position); $W_s < 0$

My hand did **positive** work, while the spring did **negative** work so the total work on the object = 0

Work done by Spring Force

Spring force is a conservative force $\vec{F} = -k\vec{x}$

Work done *by* the spring force:

$$W_s = \int_{x_i}^{x_f} \vec{F} \cdot d\vec{x} = \int_{x_i}^{x_f} (-kx\,\hat{x}) \cdot d\vec{x} = -k \int_{x_i}^{x_f} x\,dx\,(\hat{x} \cdot \hat{x})$$

$$= -\left[\frac{1}{2}kx_f^2 - \frac{1}{2}kx_i^2\right] = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2$$

If $|x_f| < |x_i|$ (**closer to** equilibrium position); $W_s > 0$

My hand did **negative** work, while the spring did **positive** work so the total work on the object = 0

Work done by Spring Force -- Summary

Spring force is a conservative force $\vec{F} = -k\vec{x}$

Work done *by* the spring force:

$$W_s = \int_{xi}^{xf} \vec{F} \cdot d\vec{x} = \int_{xi}^{xf} (-kx\,\hat{x}) \cdot d\vec{x} = -k \int_{xi}^{xf} x\,dx\,(\hat{x} \cdot \hat{x})$$

$$= -\left[\frac{1}{2}kx_f^2 - \frac{1}{2}kx_i^2\right] = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2$$

If $|x_f| > |x_i|$ (**further away from** equilibrium position); $W_s < 0$

If $|x_f| < |x_i|$ (**closer to** equilibrium position); $W_s > 0$

Let $x_i = 0$, $x_f = x$ then $W_s = -\frac{1}{2} k x^2$

Elastic Potential Energy

Spring force is a conservative force $\vec{F} = -k\vec{x}$

$$\Delta U \equiv U_f - U_i = -W = \frac{1}{2}kx_f^2 - \frac{1}{2}kx_i^2$$

Choose the free end of the relaxed spring as the reference point: $U_i = 0$ at $x_i = 0$

$$U(x) = -W(x) = \frac{1}{2}kx^2$$

The work went into potential energy, since the speeds are zero before and after.

Law of Conservation of Energy

Count up the initial energy in ***all*** of its forms.

$$E_i = K_i + U_i + E_{thermal_i} + E_{internal_i}$$

Count up the final energy in ***all*** of its forms.

$$E_f = K_f + U_f + E_{thermal_f} + E_{internal_f}$$

These two ***must*** be equal (if nothing is added form outside the system).

$$E_i = K_i + U_i + E_{thermal_i} + E_{internal_i}$$
$$= E_f = K_f + U_f + E_{thermal_f} + E_{internal_f}$$

Chapter 9 (look at lecture notes too)
Linear Momentum

- The linear momentum of a particle is a vector defined as

$$\vec{p} = m\vec{v}$$

- Newton's second law in terms of momentum

$$\frac{d\vec{p}}{dt} = \frac{d}{dt}(m\vec{v}) = m\frac{d\vec{v}}{dt} + \vec{v}\cancel{\frac{dm}{dt}} = m\vec{a} = \vec{F}_{net}$$

Most of the time the mass doesn't change, so this term is zero. Exceptions are rockets

$$\vec{F}_{net} = \frac{d\vec{P}}{dt}$$

Linear momentum of a system of particles

$$\vec{P} = \sum_{i=1}^{n} \vec{p}_i = \sum_{i=1}^{n} m_i \vec{v}_i = M\vec{v}_{c.m.}$$

$$\vec{P} = M\,\vec{v}_{c.m.}$$

Newton's 2nd law for a system of particles

$$\frac{d\vec{P}}{dt} = M\frac{d\vec{v}_{c.m.}}{dt} = M\vec{a}_{c.m.} = \vec{F}_{net}$$

$$\vec{F}_{net} = \frac{d\vec{P}}{dt}$$

Conservation of Linear Momentum

For a system of particles, if it is both isolated (the net external force acting on the system is zero) and closed (no particles leave or enter the system)….

If $\Sigma \vec{F} = 0$ then $\dfrac{d\vec{P}}{dt} = 0$

Therefore $\vec{P} = \text{constant}$ or $\vec{P}_i = \vec{P}_f$

then the total linear momentum of the system cannot change.

Conservation of linear momentum along a specific direction:

$$\text{If } \Sigma F_x = 0, \text{ then } P_{i,\,x} = P_{f,\,x}$$

$$\text{If } \Sigma F_y = 0, \text{ then } P_{i,\,y} = P_{f,\,y}$$

$$\text{If } \Sigma F_z = 0, \text{ then } P_{i,\,z} = P_{f,\,z}$$

If the component of the net external force on a closed system is zero along an axis, then the component of the linear momentum of the system along that axis cannot change.

Collisions take time!

Even something that seems instantaneous to us takes a finite amount of time to happen.

The collision of a ball with a bat collapses part of the ball.

Impulse and Change in Momentum

$$\vec{F}_{net} = \frac{d\vec{p}}{dt} \quad \Rightarrow \quad d\vec{p} = \vec{F}dt \quad \Rightarrow \quad \vec{p}_f - \vec{p}_i = \Delta\vec{p} = \int_{t_i}^{t_f} \vec{F}dt$$

Call this **change in momentum** the "**Impulse**" and give it the symbol J.

$$\vec{p}_f - \vec{p}_i = \Delta\vec{p} = \vec{J} = \int_{t_i}^{t_f} \vec{F}dt$$

Seat Belts & Airbags

Seat belts and airbags **increase** the "collision time" (i.e., the time your momentum is changing).

Greater Δt means smaller F_{avg}!

Collisions (1-D)

In absence of external forces,

Linear momentum is conserved.

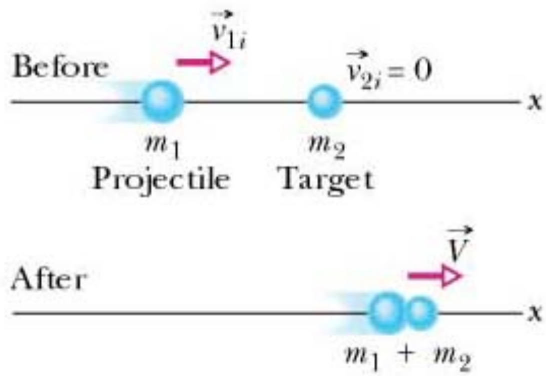
Mechanical energy may or may not be conserved.

Elastic collisions: Mechanical energy is conserved.

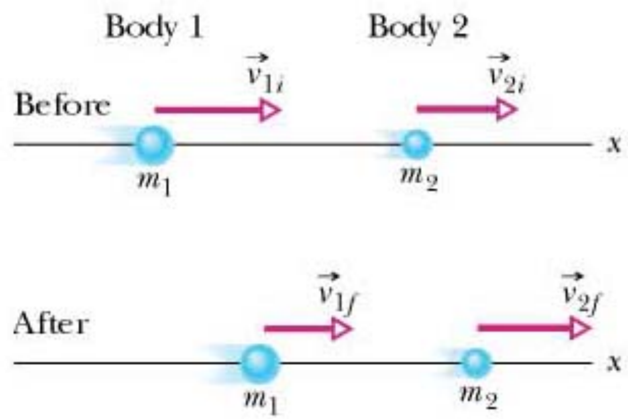
Inelastic collisions: Mechanical energy is NOT conserved.

But,
Linear momentum is conserved.

Inelastic Collisions

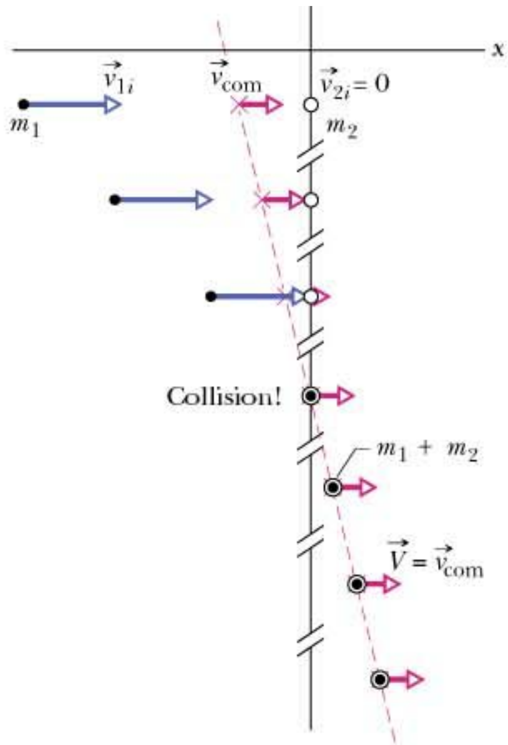


Conservation of Linear Momentum

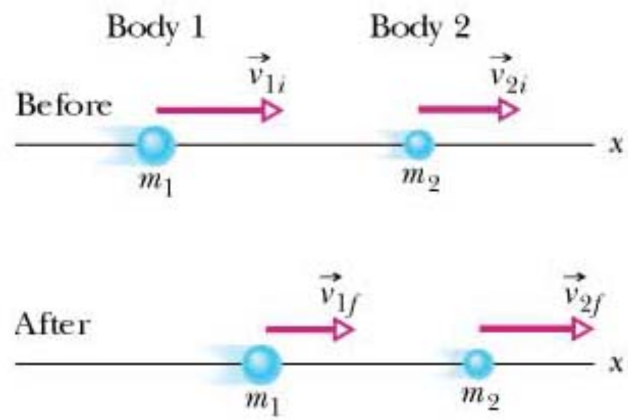


$$\vec{p}_{1,i} + \vec{p}_{2,i} = \vec{p}_i = \vec{p}_{1,f} + \vec{p}_{2,f} = \vec{p}_f$$

Center of Mass motion is constant



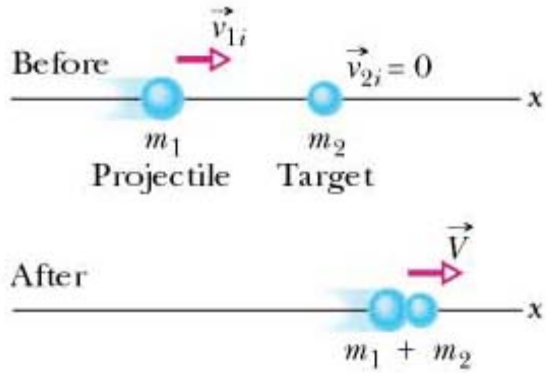
Center of Mass Motion



$$\vec{P} = \vec{p}_{1i} + \vec{p}_{2i} = M\vec{v}_{c.m.} = (m_1 + m_2)\vec{v}_{c.m.}$$

$$\Rightarrow \quad \vec{v}_{c.m.} = \frac{\vec{P}}{m_1 + m_2}$$

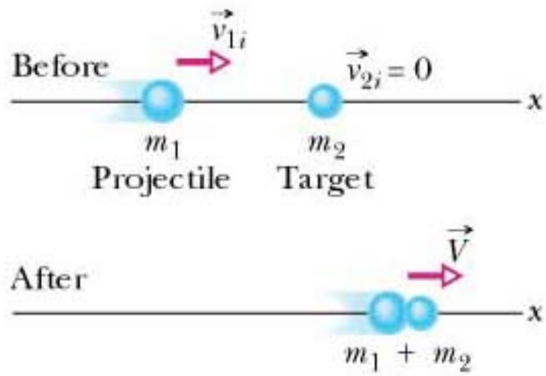
Inelastic Collisions



The mechanical energy is not conserved in inelastic collisions. $(E_i > E_f)$ or $(E_i < E_f)$

But, ... linear momentum ***is*** conserved!

Inelastic Collisions



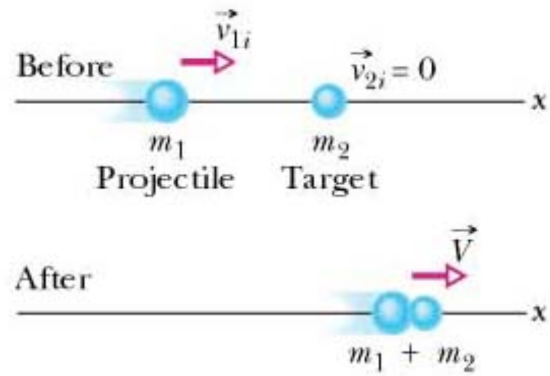
Perfectly inelastic collision: The two masses stick together

$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} \quad = \quad m_1\vec{v}_{1f} + m_2\vec{v}_{2f} = (m_1 + m_2)\vec{V}_f$$

0

$$\vec{V}_f \quad = \quad \frac{m_1}{m_1 + m_2}\vec{v}_{1i} \qquad (= \vec{v}_{c.m.})$$

Inelastic Collisions



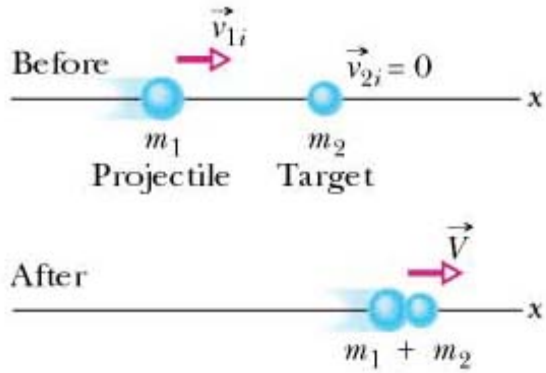
How much mechanical energy was lost in the collision?

$$\frac{1}{2}(m_1 + m_2)\vec{V}_f^2 = \frac{1}{2}(m_1 + m_2)\left(\frac{m_1}{m_1 + m_2}\right)^2 v_{1i}^2 = \frac{1}{2}\left(\frac{m_1^2}{m_1 + m_2}\right)v_{1i}^2 \quad \Rightarrow$$

$$E_{lost} = E_i - E_f = \frac{1}{2}m_1 v_{1i}^2 - \frac{1}{2}\left(\frac{m_1^2}{m_1 + m_2}\right)v_{1i}^2 = \frac{1}{2}\left(\frac{m_1 m_2}{m_1 + m_2}\right)v_{1i}^2$$

$$\boxed{E_{lost} = \frac{1}{2}\left(\frac{m_1 m_2}{m_1 + m_2}\right)v_{1i}^2}$$

Inelastic Collisions



Was the mechanical energy:

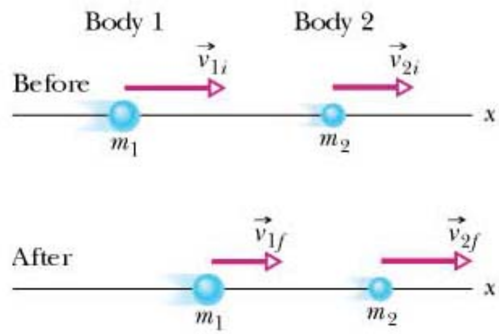
conserved $(E_i = E_f)$;

lost $(E_i > E_f)$; or

gained $(E_i < E_f)$;

in the collision?

Elastic Collisions



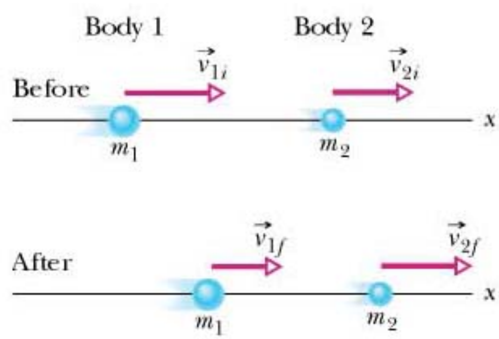
Perfectly elastic collision: Linear momentum is conserved

$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} \quad = \quad m_1\vec{v}_{1f} + m_2\vec{v}_{2f} \quad \left(= M\vec{v}_{c.m.}\right)$$

Perfectly elastic collision: Mechanical energy is conserved

$$E_i = \frac{1}{2}m_1\vec{v}_{1i}^2 + \frac{1}{2}m_2\vec{v}_{2i}^2 \quad = \quad \frac{1}{2}m_1\vec{v}_{1f}^2 + \frac{1}{2}m_2\vec{v}_{2f}^2 = E_f$$

Elastic Collisions



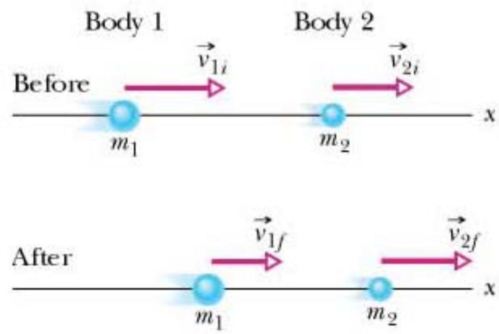
Perfectly elastic collision: Mechanical energy is conserved

$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f} \quad \left(= M\vec{v}_{c.m.}\right)$$

$$\vec{v}_{1f} = \frac{m_1 - m_2}{m_1 + m_2}\vec{v}_{1i} + \frac{2m_2}{m_1 + m_2}\vec{v}_{2i}$$

$$\vec{v}_{2f} = \frac{2m_1}{m_1 + m_2}\vec{v}_{1i} + \frac{m_2 - m_1}{m_1 + m_2}\vec{v}_{2i}$$

Elastic Collisions



Perfectly elastic collision: Mechanical energy is conserved

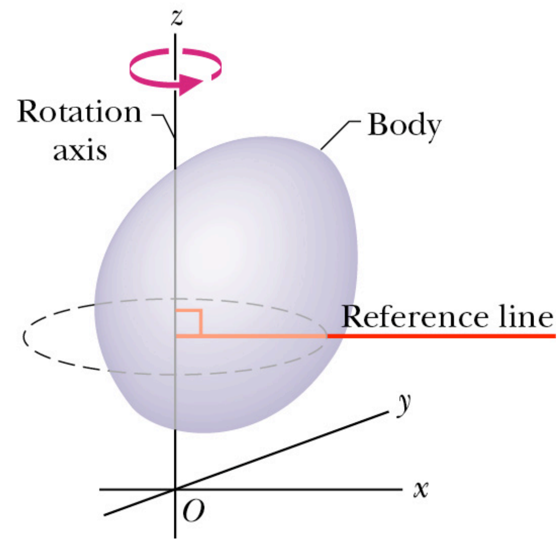
$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} \quad = \quad m_1\vec{v}_{1f} + m_2\vec{v}_{2f} \quad \left(= M\vec{v}_{c.m.}\right)$$

$$E_i = \frac{1}{2}m_1\vec{v}_{1i}^2 + \frac{1}{2}m_2\vec{v}_{2i}^2 \quad = \quad \frac{1}{2}m_1\vec{v}_{1f}^2 + \frac{1}{2}m_2\vec{v}_{2f}^2 = E_f$$

Chapter 10: Rotational variables

We will focus on the rotation of a rigid body about a fixed axis

- **Reference line**: pick a point, draw a line perpendicular to the rotation axis



- **Angular position**

  zero angular position

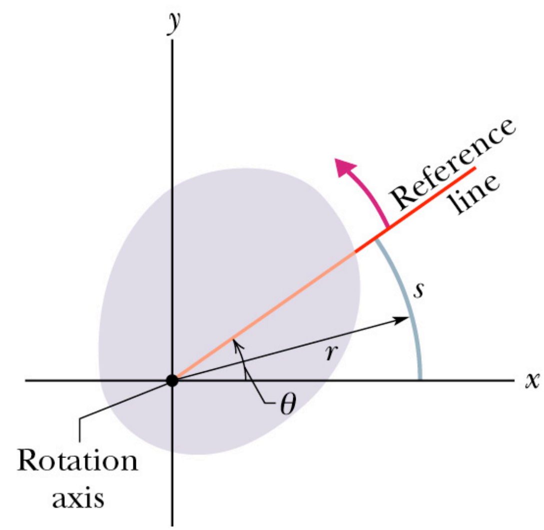
  angular position: $\theta = s/r$

  s: length of the arc, r: radius

  Unit of θ: radians (rad)

  1 rev = $360^o = 2\pi r/r = 2\pi$ rad

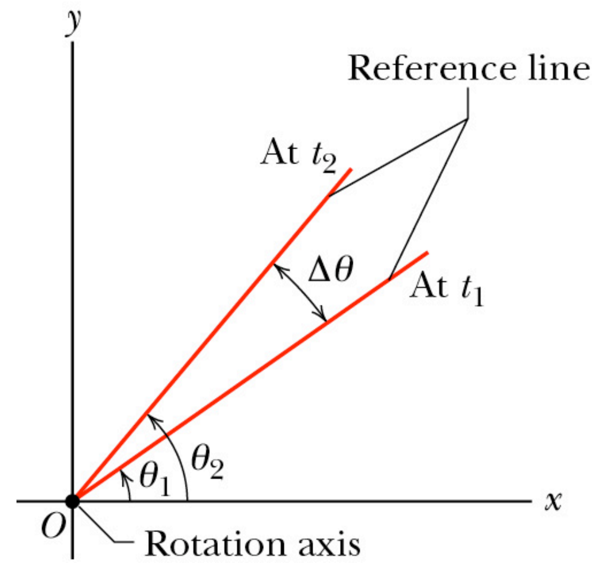
  1 rad = $57.3^o = 0.159$ rev



- **Angular displacement**

  $\Delta\theta = \theta_2 - \theta_1$

  direction: "clockwise is negative"



- **Angular velocity**

average: $\omega_{avg} = \Delta\theta/\Delta t$ instantaneous: $\omega = d\theta/dt$

unit: rad/s, rev/s

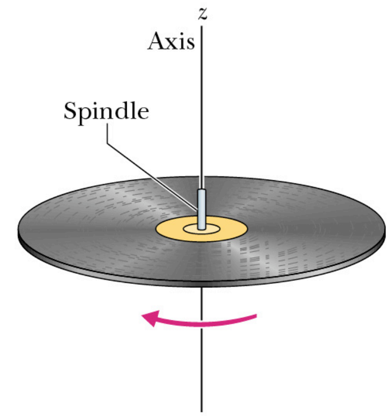
magnitude of angular velocity = angular speed

- **Angular acceleration**

average: $\alpha_{avg} = \Delta\omega/\Delta t$

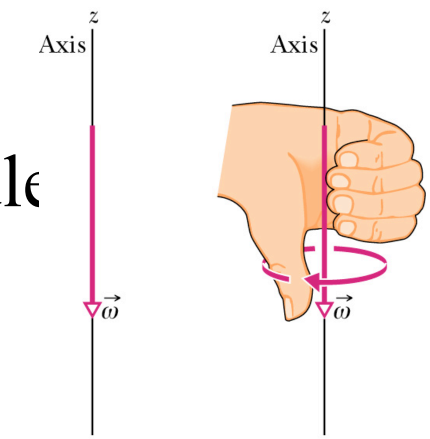
instantaneous: $\alpha = d\omega/dt$ unit: rad/s^2

- Angular velocity and angular acceleration are vectors.
- For rotation along a fixed axis, we need not consider vectors. We can just use "+" and "−" sign to represent the direction of ω and a.
  "**clockwise is negative**"



(a)

- Direction of ω is given by right hand rule



(b) (c)

Rotation with constant angular acceleration

•The equations for constant angular acceleration are similar to those for constant linear acceleration

replace θ for x, ω for v, and α for a,

		missing
$v = v_0 + at \longrightarrow$	$\omega = \omega_0 + \alpha t$	$\theta - \theta_0$
$x - x_0 = v_0 t + \frac{1}{2} at^2 \longrightarrow$	$\theta - \theta_0 = \omega_0 t + \frac{1}{2} \alpha t^2$	ω
$v^2 = v_0^2 + 2a(x - x_0) \longrightarrow$	$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$	t

and two more equations

$x - x_0 = \frac{1}{2}(v_0 + v)t \longrightarrow$	$\theta - \theta_0 = \frac{1}{2}(\omega_0 - \omega)t$	α
$x - x_0 = vt - \frac{1}{2} at^2 \longrightarrow$	$\theta - \theta_0 = \omega t - \frac{1}{2} \alpha t^2$	ω_0

Relating the linear and angular variables

The **linear** and **angular** quantities are related by radius r

- **The position** **θ *in* radians!**
  - distance $s = \theta\, r$
- **The speed**

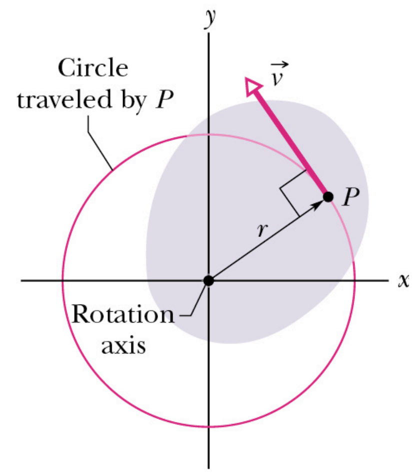
$$ds/dt = d(\theta\, r)/dt = (d\theta/dt)r$$

$$v = \omega\, r$$

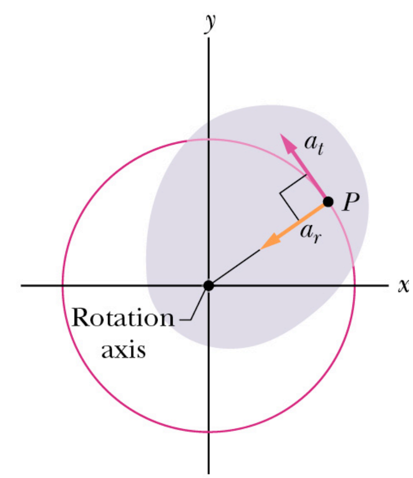
- **Time for one revolution**

$$T = 2\theta r/v = 2\pi/\omega$$

Note: θ and ω *must* be in radian measure



(a)



(b)

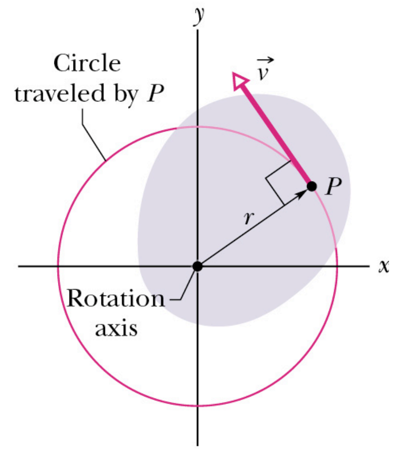
Acceleration

$dv/dt = d(\omega r)/dt = (d\omega/dt)r$

- **tangential component**

$a_t = \alpha r \qquad (\ \alpha = d\omega/dt\ )$

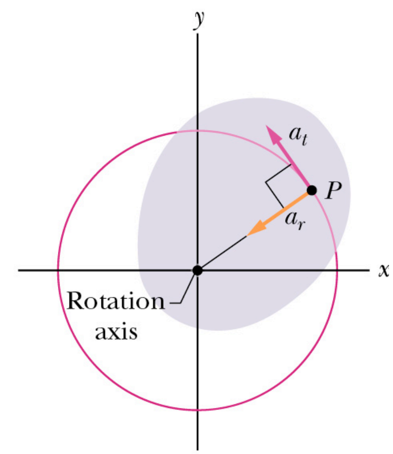
a must be in radian measure



(a)

- **radial component**

$a_r = v^2/r = (\omega r)^2/r = \ \omega^2 r$



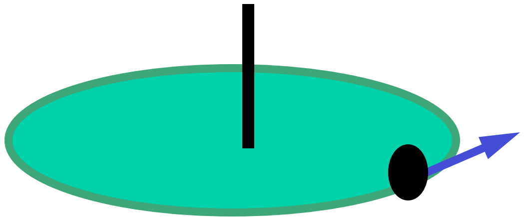
(b)

Note: a_r is present whenever angular velocity is not zero (i.e. when there is rotation), a_t is present whenever angular acceleration is not zero (i.e. the angular velocity is not constant)

<u>Cp11-3:</u> A cockroach rides the rim of a rotating merry-go-around. If the angular speed of this system (merry-go-around+ cockroach) is constant, does the cockroach have

(a) radial acceleration?

(b) tangential acceleration?



If the angular speed is decreasing, does the cockroach have

(c) radial acceleration ?

(d) tangential acceleration ?

Kinetic Energy of Rotation

- Consider a rigid body rotating around a fixed axis as a collection of particles with different linear speed, the total kinetic energy is

  $K = \Sigma \frac{1}{2} m_i v_i^2 = \Sigma \frac{1}{2} m_i (\omega r_i)^2 = \frac{1}{2} (\Sigma m_i r_i^2) \omega^2$

- Define **rotational inertia (moment of inertia)** to be

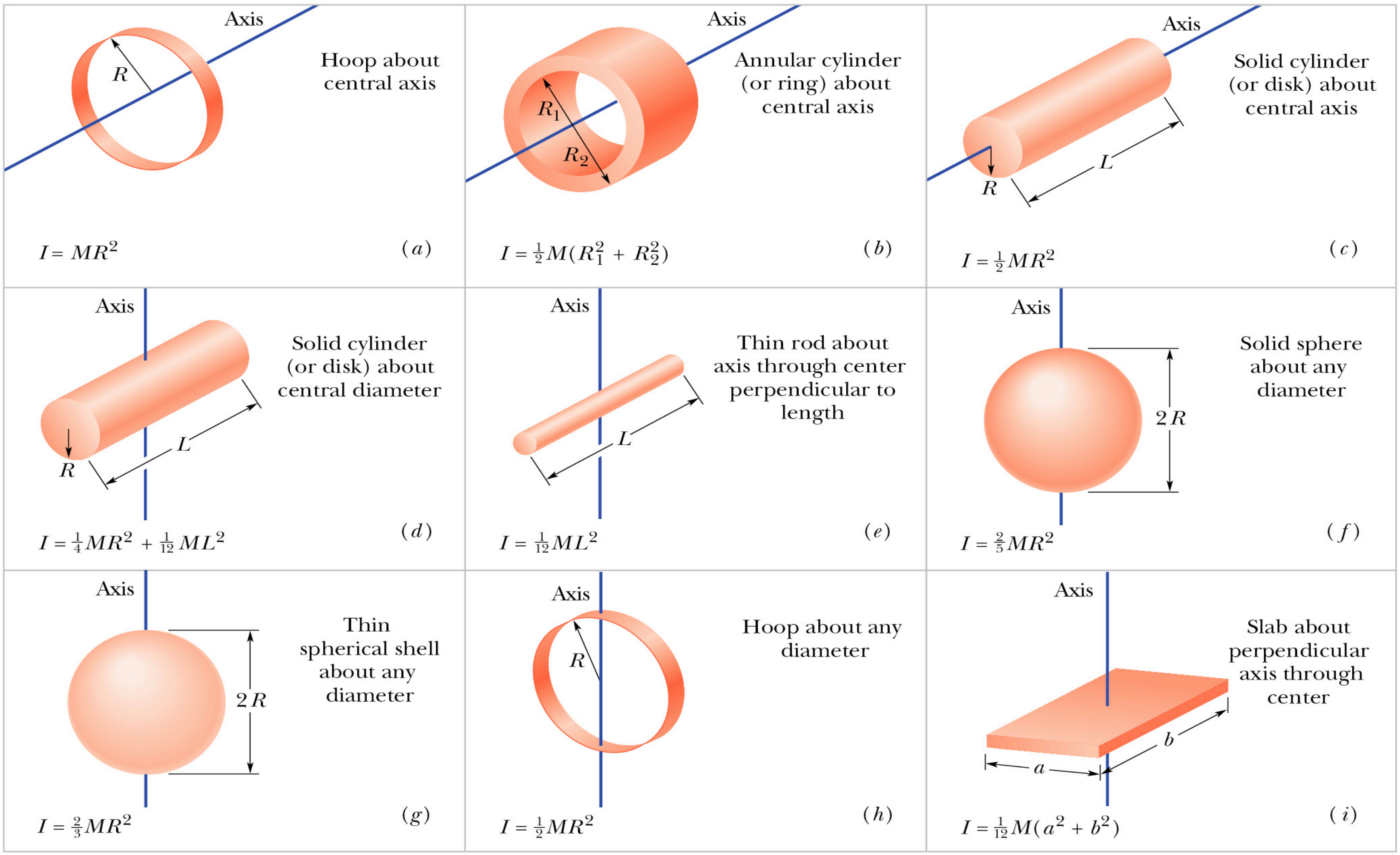
  $$I = \Sigma m_i r_i^2$$

  r_i : the perpendicular distance between m_i and the given rotation axis

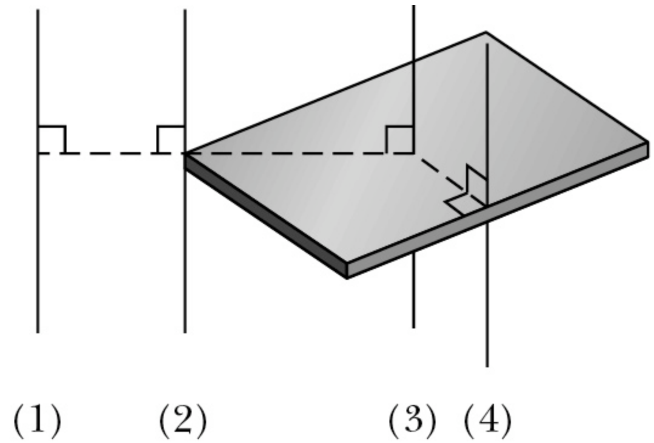
- Then $K = \frac{1}{2} I \omega^2$

  Compare to the linear case: $K = \frac{1}{2} m v^2$

- Rotational inertia involves not only the mass but also the distribution of mass for continuous masses
- Calculating the rotational inertia $I = \int r^2 dm$



Parallel-Axis theorem



- If we know the rotational inertia of a body about any axis that passes through its **center-of-mass**, we can find its rotational inertia about any other axis **parallel** to that axis with the **parallel axis theorem**

$$I = I_{c.m.} + M h^2$$

h: the perpendicular distance between the two axes

Torque

- The ability of a force F to rotate a body depends not only on its magnitude, but also on its direction and where it is applied.
- **Torque** is a quantity to measure this ability

  Torque is a **VECTOR**

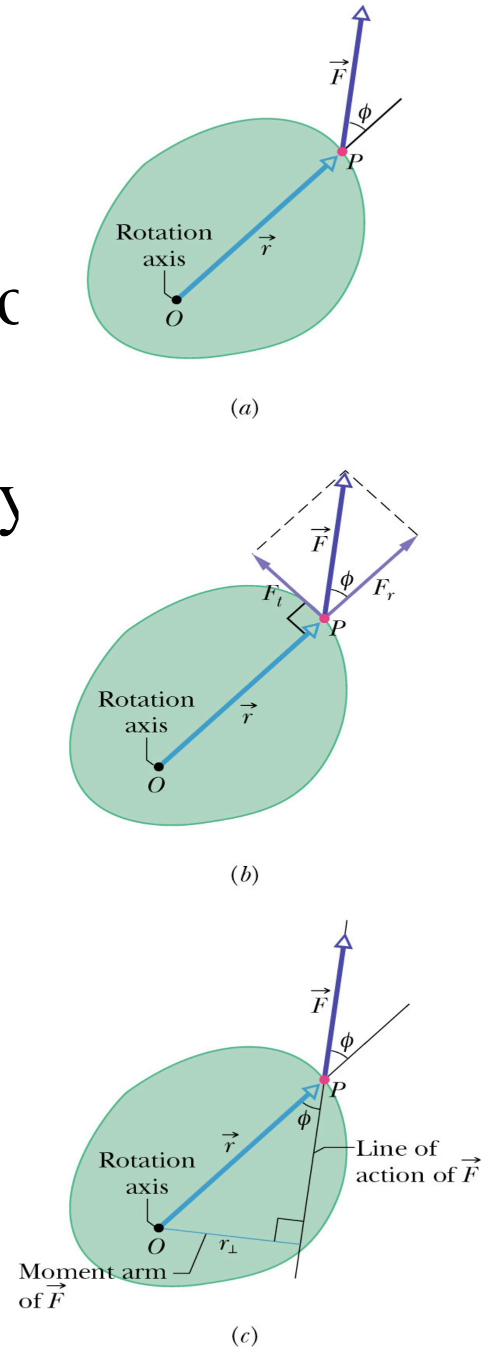
  $$|\tau| = r\,F\,\sin\Phi$$

  F is applied at point P. $\vec{\tau} = \vec{r} \times \vec{F}$

  r : distance from P to the rotation axis.

  units of τ: N · m

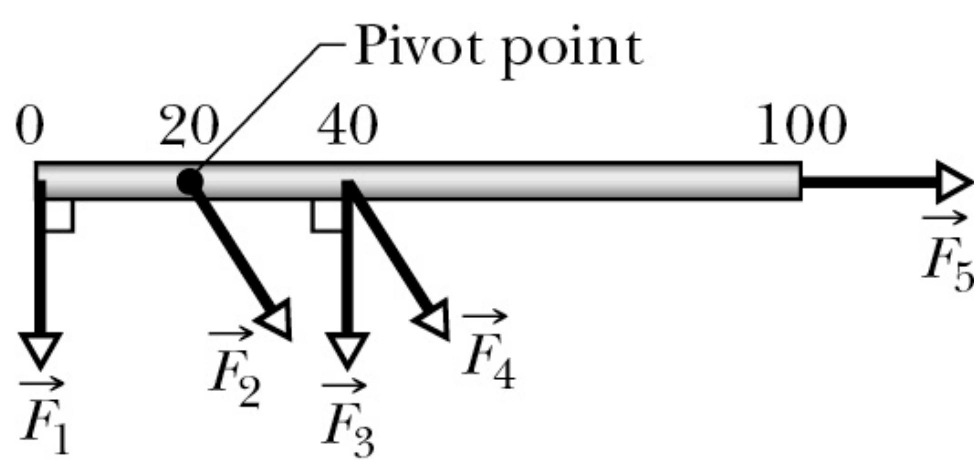
  direction: "**clockwise (CW) is negative**" because the angle is decreasing



Sample Problem

The figure shows an overhead view of a meter stick that can pivot about the dot at the position marked 20 (for 20cm). All five horizontal forces on the stick have the same magnitude. Rank those forces according to the magnitude of the torque that they produce, greatest first.

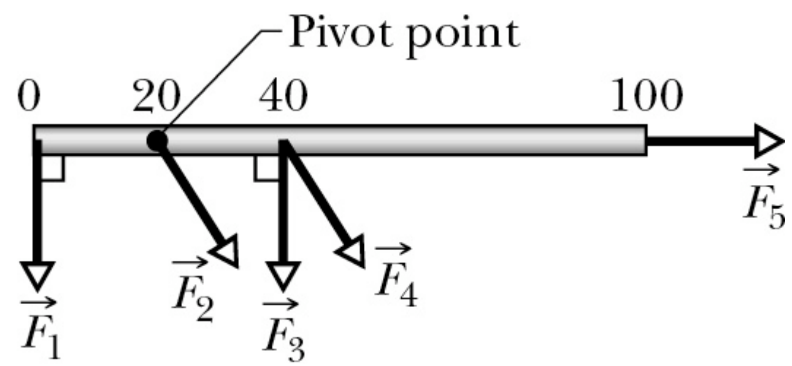
$|F_1|=|F_2|=|F_3|=|F_4|=|F_5|=F$



Sample Problem

Force producing the greatest (most positive) torque:

1. F_1
2. F_2
3. F_3
4. F_4
5. F_5
6. F_1 and F_3
7. need more information



$|F_1|=|F_2|=|F_3|=|F_4|=|F_5|=F$

Checkpoint 10-6

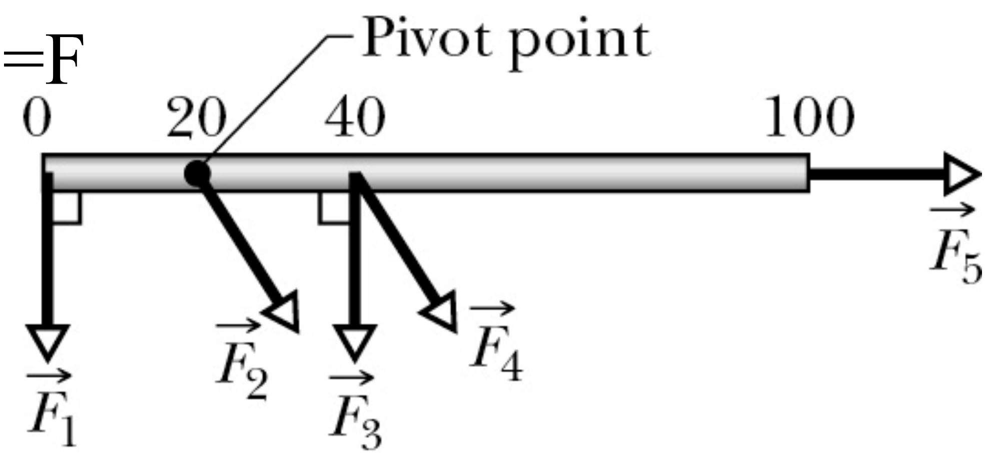
The figure shows an overhead view of a meter stick that can pivot about the dot at the position marked 20 (for 20 cm). All five horizontal forces on the stick have the same magnitude. Rank those forces according to the magnitude of the torque that they produce, greatest first.

$|F_1|=|F_2|=|F_3|=|F_4|=|F_5|=F$

$\vec{\tau} = \vec{r} \times \vec{F}$

Magnitude:

$\tau = r\,F \sin\theta$



F_1: $\tau_1 = r_1\,F_1 \sin\theta_1 = (20)F\sin(90^o) = 20F$ (CCW)

$$\vec{\tau}_1 = \vec{r}_1 \times \vec{F}_1 = +20F$$

translational motion	Quantity	Rotational motion
x	Position	θ
Δx	Displacement	$\Delta\theta$
$v = dx/dt$	Velocity	$\omega = d\theta/dt$
$a = dv/dt$	Acceleration	
m	Mass Inertia	I
$F = ma$	Newton's second law	$\vec{\tau} = \vec{r} \times \vec{F}$
$W = \int_{x_i}^{x_f} \vec{F} \cdot d\vec{x}$	Work	$W = \int_{\theta_i}^{\theta_f} \tau \, d\theta$
$K = ½ mv^2$	Kinetic energy	$K = ½ I\omega^2$
$P = \vec{F} \cdot \vec{v}$	Power (constant F or τ)	$P = \tau\omega$