

Isoelectronic trends of line strength data in the Li and Be isoelectronic sequences*

E Träbert^{1,2} and L J Curtis³

¹ Fakultät für Physik und Astronomie, Ruhr-Universität Bochum, D-44780 Bochum, Germany

² High Temperature and Astrophysics Division, University of California Lawrence Livermore National Laboratory, Livermore, CA 94550, USA

³ Department of Physics and Astronomy, The University of Toledo, OH 43606, USA

E-mail: traebert@ep3.rub.de and ljc@physics.utoledo.edu

Received 24 February 2006

Accepted for publication 26 June 2006

Published 15 August 2006

Online at stacks.iop.org/PhysScr/74/C46

Abstract

The decays of the $2p$ $J = 1/2$ and $J = 3/2$ levels of Li-like ions and of the $2s2p$ $^1,3P_1^o$ levels of Be-like ions can be used as simple-atom test beds for lifetime measurements and for the development of accurate calculations of the transition rates. We have summarized and filtered the experimental data in order to obtain consistent data sets and isoelectronic trends that can be compared to theoretical predictions. The graphical presentation of line strength data enables direct comparison and evaluation of the merit of data along extended isoelectronic sequences. From this, the precision that is necessary in future meaningful experiments can be deduced.

PACS numbers: 32.70.Cs, 31.50.+w, 31.15.Gy

(Some figures in this article are in colour only in the electronic version.)

1. Introduction

When developing quantum mechanics in the 1920s and trying to understand the structure of atoms and ions with several electrons, Hylleraas [1] realized that ions of similar electron number must be rather similar in structure, with the nuclear charge Z as the governing parameter. He developed expressions for various atomic entities (excitation energies E , oscillator strengths f , line strengths S , etc) that could be represented by series expansions in powers of Z . The leading terms of such expansions are characteristic for specific properties, for example, the Z^2 dependence of the electrostatic energy differences of levels with different principal quantum numbers n (as in the Bohr formula), or the Z^4 dependence of electric dipole transition rates between such levels. For transitions within a given atomic shell, the energy intervals increase proportionally to Z , and the transition rates of fully allowed E1 transitions increase also linearly with Z (take as an example the $2s$ – $2p$ transitions in Li-like ions). Spin-changing transitions within the same shell, as in the case of the $2s^2$ 1S_0 – $2s2p$ $^3P_1^o$ intercombination transition in Be-like ions, increase in transition rate as Z^7 , because the spin–orbit (multiplet mixing) matrix element

introduces its own strong Z -dependence on top of that of the E1-allowed transition. These basic trends are modified by electron–electron interaction effects at low Z , and by relativistic effects at high Z (see [2]).

There are various ways to linearize isoelectronic trends in favour of intercomparison along the isoelectronic sequence or among theory and experiment. One of the traditional ways is the presentation of the oscillator strength f as a function of $1/Z$. This results in almost a straight line for transitions that are little affected by relativity (and far enough from the beginning of an isoelectronic sequence so that the central charge of the screened nucleus dominates over electron–electron interactions). In order to reduce the influence of relativistic effects (for example, the fine structure splitting growing with Z^4) on $\Delta n = 0$ transitions, we elect a presentation of the line strength S instead. Transition probability A_{ki} , oscillator strength f_{ik} , and line strength S can be easily converted into each other. The lifetime τ of a level k with transitions to lower levels i is $\tau_k = 1/\Sigma(A_{ki})$. Transition probability and oscillator strength are linked as $g_k A_{ki} = 6.67 \times 10^{13} g_i f_{ik}/(\lambda(\text{nm}))^2$. For E1 transitions the expression $f_{ik} = 1/g_i \cdot 30.3756 S_{ik}/\lambda$ (nm) holds.

Nowadays, ions of all charge states of all naturally abundant elements can be produced and investigated in

* Dedicated to the memory of Professor P H Heckmann (14 June 2005).

the laboratory. (Theory is ahead, of course: already in the 1930s, when Edlén was producing ions of ever higher charge states, Hylleraas contemplated atoms of infinitely high nuclear charge, or what today is called the high- Z limit [3].) However, it is not presently feasible to measure all lifetimes of all levels of all ions. There are practical limitations even to lifetime measurements of all of the low-lying, most easily excited levels of all ions, i.e. levels that give rise to the most prominent lines in many spectra and that are used in a variety of ways for elemental analysis, plasma diagnostics, or astrophysics. The limitations originate from the working principles of the few techniques that permit such lifetime measurements at all. We will briefly describe the techniques and then present graphical overviews of lifetime data on fundamental transitions in Li- and Be-like ions. As of September 2005, the NIST bibliographic effort on transition probabilities listed 363 papers on ions in the Li sequence and 364 papers on ions in the Be sequence, up to some cut-off date in 2002. These high numbers underline the interest in these fundamental isoelectronic sequences, of which we can cover only the cornerstone of the strongest ground-state transitions. By far the majority of the titles covered in the bibliography refer to theoretical studies and experimental reports are few and far between. Our emphasis is on the identification of a reliable experimental data set that can be used to eventually judge the vast number of calculated results.

This emphasis differs from that of most early systematic work (for example, [4]) in which isoelectronic trends from calculations with the then very limited computer power were combined with early experimental data that, in particular for low-charge state ions, often scattered abominably and thus left one having to guess whether experiment and calculation actually described the same system. That situation called for improved calculations and more and better experimental data. Calculations that dealt with a single ion species only were rather useless for systematic efforts; correspondingly, experiments that were evaluated without taking the general atomic structure information (cascade repopulation) into account often contributed to confusion about systematics. In due course, experimenters and theoreticians have learned to do better.

Our present study provides an update of the presentations that have been put forward in various earlier papers. With typical beam-foil spectroscopic activities—which have provided the bulk of the lifetime data on multiply charged ions—fading out, the present report may be seen as a concluding review. However, since a previous such review of Li-like ions [5], the experimental data for neutral Li have received a boost of precision, and some high- Z ion values have been added. Since a recent such review of Be-like ions [6], the heavy-ion storage ring technique has been introduced (see below), and this extension of the beam-foil technique has yielded the most precise atomic lifetime numbers on Be-like ions, challenging theory at new levels of accuracy.

2. Experimental techniques

Techniques for the determination of oscillator strengths in a multitude of atoms and low charge state ions have

been reviewed by Huber and Sandeman [7]. The technique most widely employed for atomic lifetime measurements in multiply charged ions is beam-foil spectroscopy [8–12]. In beam-foil spectroscopy, a monoenergetic beam of fast ions of a preselected ion species is sent through a thin foil (most often carbon). The ions may suffer changes of their charge states, as the charge state distribution in an ion beam that emerges from a target foil depends on the ion energy, and the ion energy in beam-foil spectroscopy may be in the range from less than 100 keV total to some GeV/nucleon. A sizeable fraction of the ions will usually be excited upon leaving the foil. The ions mostly collide only with electrons in the foil target, and thus they suffer a certain energy loss and angular straggling, but continue as a fairly well collimated ion beam and with about the same kinetic energy. Owing to the common velocity, distance from the exciter foil thus corresponds to time after excitation, and a signal curve recorded as a function of distance yields the temporal history of the decay. The vacuum is high enough and the particle density in the beam very low so that the ions travel practically collision-free. The length of ion beam to be studied, however, is limited by beam collimation, angular straggling (versus detector size), detector dark rate (fighting an exponentially decreasing signal) and the size (price) of the vacuum chamber. Because μm detector or foil displacement increments are simple to achieve, the short-time limit is given by the spatial resolution of the detection system. With ion velocities of order 1 cm ns^{-1} (this corresponds to an ion beam energy of 0.5 MeV amu^{-1}), the lifetime range thus geometrically coverable is of the order of a few picoseconds to many nanoseconds.

In the first few years after the inception of beam-foil spectroscopy, a flurry of enthusiastic activities has covered many levels in many, mostly low-charge state, ions. The results were eagerly taken up in astrophysics and plasma physics, and graphical presentations were assembled to help researchers in the field keep up with the rapidly expanding database and in establishing systematics [4, 13]. This meritorious work, alas, revealed a deplorable scatter of the results for similar or isoelectronically equivalent systems, and it raised doubts about the quality of some of the data as well as of some predictions. Obviously, at the time, some of the systematic error problems in beam-foil spectroscopy (like cascade repopulation [14]) had not been addressed sufficiently well in many laboratories, poor spectral resolution had not yet been countered by refocusing techniques [15, 16] and some data analyses may have neglected the atomic structure context. With spreading insight and experience, improved detectors and a certain concentration of the field into fewer laboratories, the standards rose, and reliable results became the rule (to any reasonable rule, there are exceptions). More recently it was also found that when working far below the optimum energy to produce an ion of the desired charge state (as happened to a group who tried to observe high charge state ions under conditions that pushed the performance envelope of their accelerator), the actual spectral line observed might suffer from the blending presence of core-excited ions of lower charge states, resulting in systematically shorter apparent lifetimes [17, 18]. The data presented here appear not to have suffered from this problem.

The beam-foil technique has enabled lifetime measurements (of the atomic systems discussed here) up to $Z = 36$

(Kr) for the $2p^2P_{3/2}^0$ level of Li-like ions and the $2s2p^1P_1^0$ level of Be-like ions [19–21] (in both cases the valence electron is in a $2p_{3/2}$ state). For $2p_{1/2}$ electrons, the beam-foil measurements—facilitated by the much longer level lifetimes—reach up to the $2s2p^3P_1^0$ level of Be-like Xe ($Z = 54$) [22] and to the $2p^2P_{1/2}^0$ level of Li-like U ($Z = 92$) [23]. It should be possible to measure the intercombination decay in the Be sequence for higher values of Z than has been done, given the necessary effort to combine available spectroscopic equipment and a suitably powerful heavy-ion accelerator. Moreover, beyond adding a few more data points for a better systematic control along the sequence, the measurement accuracy in the high- Z range is not yet at the limit expected for dedicated experiments. The shorter-lived $2p_{3/2}$ levels have been pursued almost to the practical limits of spectral and time resolution in soft-x-ray spectroscopy, and extensions of the measurements on this atomic level towards higher Z are unlikely.

The experimental situation is very different at the low- Z end of the same isoelectronic sequences. A variety of techniques and devices are available for lifetime measurements on atoms and ions [24–29]. Beam-foil spectroscopy works also for low-charge state ions, and has been used for neutrals and even negative ions that emerge from a target hit by a fast beam of positive ions. However, other techniques offer higher precision and accuracy. For regular E1 transitions in neutral atoms, state-selective laser excitation (and electronic timing) is often the method of choice, be it on atoms in a gas or a vapour. Laser-excited fast atomic beams and time information obtained in the same geometric way as in beam-foil work [28, 30–32] can offer an alternative in some cases. In neutral alkalis, extremely precise resonance transition line strengths are available also from very different measurements on cold dimers [33, 34]. For very long-lived ions, ion trapping techniques have been employed to determine level lifetimes (for an overview and for points of access to the rich literature on this field, see, e.g., the recent reviews [35, 36]).

3. Calculations

Many calculations have been performed on the two isoelectronic sequences and four levels of our present interest. However, the transition rate calculations are particularly difficult for atoms and near-neutral ion species, whereas the convergence of relativistic codes is much better at high nuclear charges. As a consequence, there are calculations that target one or the other range, but none that is expected to do well for the full range of experimental data along the isoelectronic sequence. Sims *et al* [37] have found a way to determine upper and lower bounds for the oscillator strength, that is, an interval into which any calculated f value should fall if correct.

We elected to refrain from singling out individual calculations for their particular merit. Instead, we represent calculations by simple expressions for the line strength S in combination with experimental transition energy (wavelength) data. Following Hylleraas [1], the line strength S can be approximated by a series expansion in powers of Z . The leading term in a series expansion of the E1 line

strength is proportional to Z^{-2} , and therefore a graph of the type $Z^2S = A + B/(Z - C) + D/(Z - C)^2 + \dots$, that is as a function of $1/Z$, or of $1/(Z - C)$, is expected to be rather linear. The constant C represents some screening of the valence electron by the core. A previous study has found a value of $C = 2.17$ as helpful for linearizing plots of Z^2S for Li-like ions [5].

Such a $1/(Z - C)$ expansion up to the second term has been used by Reistad and Martinson [38] in order to obtain an isoelectronic smoothing function for oscillator strength data on the resonance transitions of the first five members of the Be isoelectronic sequence. Träbert has included the next term of the series in order to extend such an analysis farther along the sequence [39], where the higher terms become important. Curtis has expanded on this technique by isoelectronic smoothing of line strengths in intermediate coupling [40].

Near the neutral-atom beginning of isoelectronic sequences, the isoelectronic trend regularly deviates from the near-linear high- Z behaviour, because the central (nuclear) charge does not yet dominate over the electron–electron interaction. In our quest for a presentation of the data in which all data were to appear on an equal footing, irrespective of the vast range of actual lifetime values represented, we found that this low- Z nonlinear trend was much reduced by using $(Z - \sigma)^2S$ instead of Z^2S . Trial and error provided values of $\sigma = 1.75$ or 2 as practical choices for Li-like and Be-like ions, respectively. Whereas the aforementioned constant C can be interpreted in terms of charge screening—although the actual value of the constant was obtained by fits of linearization curves to isoelectronic data and does not truly describe an atomic property—there is no rigorous physical justification for our constant σ . In the given framework, σ is unimportant in comparison to Z for high values of Z , and at low Z it happens to provide a major improvement in the linearization effort for displaying the data on a suitably large scale. One can show that a parameter like our parameter σ can be used as an alternative to the aforementioned screening correction C (with a conversion between the two formulations that is possible either way), or as an additional parameter that might help with the adjustment of representative curves. We use σ to obtain flat curves that put all data at approximately the same amplitude, so that the relative errors are immediately comparable to each other. With such flat curves, one can then disregard the (physically motivated) screening constant C for our specific presentation.

Curtis has demonstrated in a number of cases how suitable parametrizations can enable straight-line analyses of data over wide ranges of isoelectronic sequences. One of the techniques employed is the mixing-angle formalism for resonance and intercombination transitions in ions with two valence electrons or electron-hole coupling [43]. If the mixing is described by a mixing angle Θ , a mixing-reduced effective line strength S_r can be introduced that represents the resonance line strength $S(\text{Res})$ by $S_r(\text{Res}) = S(\text{Res})/\cos^2\Theta$ as well as the intercombination line strength $S(\text{Int})$ by $S_r(\text{Int}) = S(\text{Int})/\sin^2\Theta$. In favourable cases, the reduced line strength data for resonance and intercombination lines can then be treated in a common analysis. We use this formalism in our presentation of data of Be-like ions. It is also helpful

to calculate the high- Z limit of entities like the reduced line strength and to include it in the analysis of an isoelectronic trend.

Because the transition energy has been removed from the Z dependence, the line strength S (as a quasi-geometric property of the atomic system) suffers much less from relativistic effects than do the transition rate or the oscillator strength. However, there is also a relativistic influence on the line strength that distorts the Z dependence at very high Z . Curtis *et al* [5] have shown how to account for this effect in the isoelectronic sequences of alkali atoms and how to introduce a suitable correction so that linear trends emerge again. We make use of the same formalism for the presentation of the Li sequence data.

4. Data and isoelectronic discussion

The data set that we include in our presentation is biased by our personal judgment. If several measurements are available for a given level in a given ion, we have selected those data that to us appear to be more reliable (in terms of both precision and accuracy), in particular by reference to a consistent isoelectronic trend. We see no need to cite all the less successful work too, which would only serve a goal of completeness, not of insight. Corresponding discussions (see, for example, [5, 21] for Li-like ions, [6, 40, 44] for Be-like ions) have previously included other selections and further references may be found there.

4.1. Li-like ions

The lifetime of the $2^2P_{1/2}^o$ level of neutral Li has been a worrisome issue for decades. (The $2^2P_{3/2}^o$ level decay shows quantum beats, that is, oscillations in the directional emission intensity, because of the coherent and non-statistical excitation of the magnetic sublevels. Consequently, this level has been avoided in experiments that aimed at precision. Therefore the experimental data on the $J = 3/2$ level decay begin only at Be^+ , where the quantum beat frequency is much higher and has little influence on the evaluation of beam-foil lifetime measurements.) Decades ago, precise beam-gas-laser experiments on Li and Na [45, 46] showed most theory to be seriously wrong on the transition probability of the resonance line. Theory was subsequently improved to a point where it declared experiment to be not quite as correct as had been assumed either. Experimental confirmation (and correction of the early data) has come from two very different quarters. One was a perfection of the beam-gas-laser technique [47] suggesting minute shortcomings of the earlier alignment of displaceable optical elements along the particle beam path. The second is based on the study of cold atoms and molecules and their interaction at long distances, from which the resonance line oscillator strength can be derived [33, 34]. The new data agree with each other and are no longer in disagreement with recent theory.

For Be ($Z = 4$) and B ($Z = 5$), we have not found any published lifetime data on the $2^2P_{1/2}^o$ level, nor any for Li-like Na ($Z = 11$). This, no doubt, is due to ion source problems in the early days. The expected lifetime differences between $J = 1/2$ and $J = 3/2$ levels are much smaller than

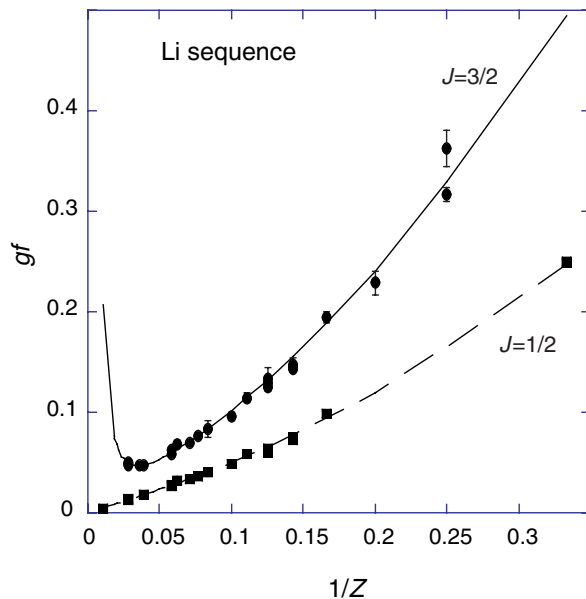


Figure 1. ‘Traditional’ presentation of the weighted oscillator strength gf as a function of $1/Z$ for the $2s-2p_{1/2,3/2}$ transitions in the Li sequence. This type of graph shows the basic proportionality of gf to $1/Z$ in the nonrelativistic case. At high Z , the finestructure splitting, growing with Z^4 , has a distorting influence on the gf value of the $J = 3/2$ level decay. The $J = 1/2$ level decay has a gf trend that is much more linear towards high Z , but the absolute values are so small that the graph becomes difficult to evaluate visually. The full and broken lines, respectively, represent calculations in a semiempirical Coulomb approximation with Hartree-Slater core [48]. References: Li [34, 47], Be [64, 65], B [65], C [66], N [67–69], O [70, 71, 73], F [67], Ne [72], Mg [17], Al [74], Si [75], S [76], Cl [77, 78], Fe [59], Ni [79], Kr [19, 21, 49], U [23].

the uncertainties in the early measurements, which therefore elected to evaluate the $J = 3/2$ level decays (twice the signal rate because of the statistical weight factor) only. Beyond $Z = 17$ (Cl) the data turn sparse—a clear indication of which charge states could be reached at many accelerator laboratories at the time, and which ions could be produced in a few places only. The Li-like ion of Na could have been measured at the Bochum tandem accelerator, one of the few places with a sufficiently intense Na ion beam, but it was not recognized as being of possible interest in time, before atomic physics activities were shut down in that laboratory.

A presentation of gf as a function of $1/Z$ results in a fairly straight line for the $2p_{1/2}$ decay (figure 1). The fine structure intervals, however, increase with Z^4 , and therefore the $2p_{3/2}$ level moves away from $2p_{1/2}$, making it impossible to maintain a straight-line analysis of the $2p_{3/2}$ level lifetime, decay rate, or oscillator strength along the isoelectronic sequence. This is a vivid illustration of Kim’s [2] claim that in high- Z ions, relativity plays a dominant role. What we choose for the presentation (figure 2) instead is $(Z - 1.75)^2 S$, an entity that is rather constant along the isoelectronic sequence. We also applied the relativistic corrections as described by Curtis *et al* [5], without which the highest Z data (for U ($Z = 92$) [23]) would disagree with the isoelectronic trend by about $1/3$ of the absolute value. The $2p_{1/2}$ level lifetime data (on Li-like ions from Li to U) and the $2p_{3/2}$ level lifetime data (for Be to Kr) are presented jointly in figure 2.

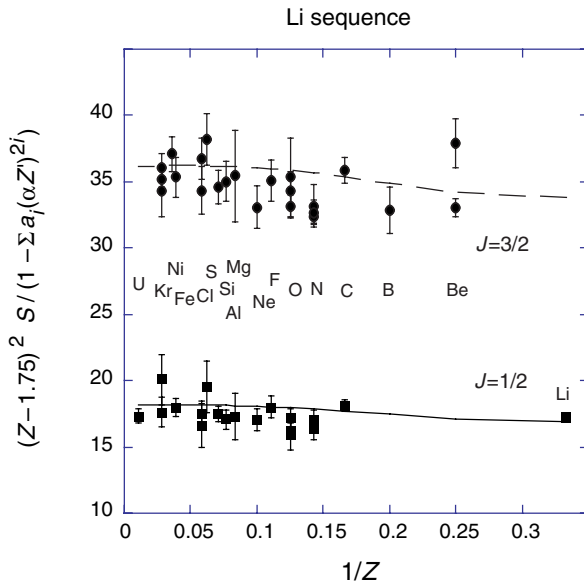


Figure 2. $(Z - 1.75)^2 S$ (with a relativistic correction) as a function of $1/Z$ for the $2s-2p_{1/2,3/2}$ transitions in the Li sequence. The relativistic correction has been explained by Curtis *et al* in [5]. It is minute for the lighter elements, but amounts to about $1/3$ in U ($Z = 92$). The scaling of Z by an additive constant of -1.75 is for display purposes only. The full and broken lines, respectively, represent calculations in a semiempirical Coulomb approximation with Hartree–Slater core [48]. References are the same as in figure 1.

Because of relativistic effects, the data for the heaviest ions in the sequence are of particular importance. For the $2p_{1/2}$ level, the data point of greatest interest is the one for U ($Z = 92$). Without relativistic corrections, the S_r trend would not meet the measured data, but with the corrections as implied in our graph, the trend (as represented by calculations in a semiempirical Coulomb approximation with Hartree–Slater core [48]) reaches the measured value close to the experimental uncertainty. For the $2p_{3/2}$ level, the heaviest Li-like ion measured is that of Kr ($Z = 36$), with four published reports [19–21, 49] (the two papers [20, 21] refer to the same experiment). It is interesting to see that the study by Kukla *et al* arrives at a lifetime result (50.7 ± 0.9 ps) that carries a smaller error bar than the result (49.5 ± 1.5 ps) given by Träbert *et al*; the isoelectronic trend favours the latter value. It happens that Kukla *et al* performed a multi-exponential fit only, whereas Träbert *et al* did the same (with a result not far from that obtained by Kukla *et al*), but also employed a cascade model fit. The latter yielded a slightly, but significantly shorter lifetime that appears to be taking better care of systematic errors.

Without overstating the value of the particular calculations mentioned above, the experimental data for both fine structure levels lie on average slightly below this particular prediction. Such a trend would be expected from a multi-exponential fit (in contrast to a cascade model fit) in the presence of the typical yrast cascade pattern. An ANDC experiment [14] could avoid such a systematic error, but it would have to measure the $3d$ cascades to the $2p$ levels. (If one records the decay curves of transitions feeding a level, the shape of these curves—hence the ANDC acronym for

arbitrarily normalized direct cascades—permits extraction of the proper lifetime value of the ‘primary’ level from the shape of the subsequent decay curve.) However, those $2p-3d$ transitions fall into a range of much shorter wavelengths than those of the $2s-2p$ transitions. Judging from figure 2, the experimental lifetime measurements on atomic Li, Li-like C and Li-like U (and to a lesser degree some others) are the most meaningful ones in this isoelectronic sequence. For Be^+ , none of the available data are compatible with the isoelectronic trend. For the other elements, there are more data available than included in our graphs; we have rejected various measurements that are incompatible with the isoelectronic trends. References to the ‘surviving’ data are given in the figure captions. Most of the data bear error bars in the range of 5 to 10%. Considering the scatter of the data around our reference calculation, a practical estimate of the correct values would be within about 3% of this calculation. Further experimental data that do not reach a comparable accuracy would seem to be of little use.

4.2. Be-like ions

Be-like ions are the simplest atomic systems with two valence electrons in the same shell. The transition probability for the singlet resonance line $2s^2\ ^1S_0-2s2p\ ^1P_1^o$ is supposedly simple to calculate (as in a hydrogenlike ion), but suffers from systematic error in the typical experiments. The probability for the $2s^2\ ^1S_0-2s2p\ ^3P_1^o$ intercombination transition is many orders of magnitude lower. This causes problems in the calculations that have to deal accurately with small entities (the mixing matrix element in the Breit–Pauli approximation [41], or the leftover of the cancellation of large and small Dirac wave functions). The low transition probability is associated with a long level lifetime (in the millisecond range for the first ions of the isoelectronic sequence), which causes severe problems in regular beam–foil lifetime measurements (that work best in the picosecond to nanosecond range). However, the introduction of storage ring techniques [42] has turned the situation around, and some lifetime measurements on intercombination transitions in low charge Be-like ions are now among the most precise atomic lifetime measurements ever done (see below). Heavy-ion storage rings can be imagined as beam–foil experiments in which the fast ion beam is being curved into a loop, so that it passes the same detector over and over again. Excitation by ion–foil (or ion–gas) interaction has to take place before the ions are injected into the ring; laser excitation is possible of the already stored ions. The ions may be stored for seconds, minutes, or even hours, depending on their electronic structure (and thus the collision cross-section), the ion energy and the (extremely high) vacuum in the ring. The technique works without any mechanically moving parts; the timing of the atomic lifetime measurements is electronic. More detailed overviews over this and other ion storage techniques can be found in recent reviews [35, 36]. For the intercombination transition rate measurements in Be-like ions, the heavy-ion storage ring has turned out to be the technique of choice. For the even longer lived electric-dipole forbidden transitions (which are outside of our present focus), electron beam ion traps have been used—references are amply given in the same reviews.

The transition probabilities of intercombination and resonance transitions between given electron configurations are linked by multiplet mixing matrix elements. The mixing can also be expressed by a mixing angle [43], and a reduced line strength S_r [5, 50] that then represents both resonance and intercombination transitions. This procedure enlarges the data sample to be intercompared and provides yet another check for systematic error.

4.2.1. Resonance line lifetime data Be–Kr. The $2s2p^1P_1^0$ level ($2p_{3/2}$ electron) in Be-like ions is short lived; it can be observed easily by beam–foil techniques, and its decay rate can be calculated with high precision [51]. However, there are two $2p^2$ levels, 1S_0 and 1D_2 , that feed this level, and the lifetime of the former level is rather close to that of the level of interest. The $2p^2^1D_2$ cascade (with a lifetime that is a factor of five (in low- Z ions) to three (in higher- Z ions) longer than that of the resonance level) can easily be separated in a multi-exponential fit to the resonance level decay curve, but the growing-in cascade from the $2p^2^1S_0$ level cannot. Neglect of this cascade is an obvious reason for the many beam–foil lifetime results that have fallen far from prediction. Several techniques have been developed to take this cascade into account. Measurements of the cascading transition decay curves can be combined with the decay curve of primary interest in the ANDC algorithm [14]; this procedure has been demonstrated in detail by Engström [52, 53]. One alternative is a multi-exponential fit in which the separately measured cascade lifetimes are introduced as fixed parameters [54], a procedure that is being helped if the finite spatial resolution (window function) of the observing instrument is taken into account and thus the relative amplitudes of the decay components can be converted to relative level populations that have to be physically sensible. A third procedure employs a model of level populations and decay branches [55]. However, only the ANDC technique can be expected to yield results with reliably small error bars.

In the Be isoelectronic sequence, all three procedures have yielded results that agree reasonably well with each other and with most calculations. Thus it was possible to interpolate the resonance transition line strength along the isoelectronic sequence by a simple Hylleraas type expression [38, 39] with a scatter of the order of 1%, which is much better than the individual measurement errors of typically 10%. Figure 3 shows that the scaled line strength data of the resonance transition are even compatible with a constant value to within a small fraction of the error estimates. Disregarding the cascades leads to lifetime values that usually are some 30% off the mark. Astonishingly, quite a number of experimental lifetime results have been obtained (and published!) without taking the cascades into account, and that has happened for many years after the above techniques had been demonstrated.

For the heaviest ions studied in this sequence, a new source of lifetime measurement error arises from the spectral blend of the $2s^2^1S_0-2s2p^1P_1^0$ transition with one of the $2s2p^3P^0-2p^2^3P$ transitions that moreover happens to lie close in lifetime. There is no clear procedure to counter this detrimental effect; however, one can estimate the amplitude of the blending decay curve from the intensities of the other lines

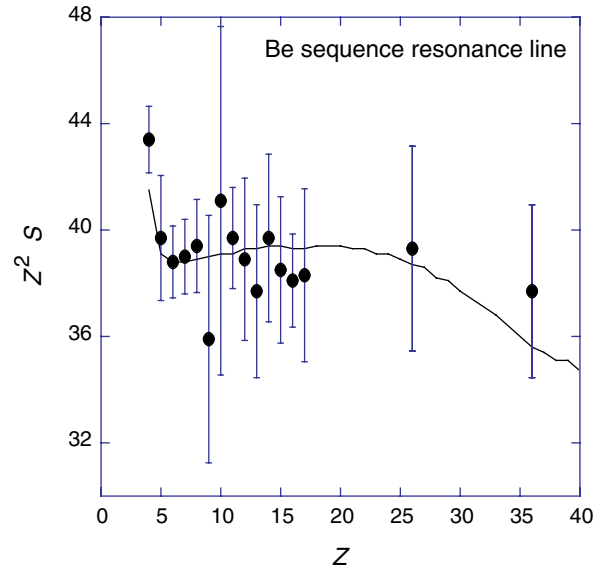


Figure 3. $(Z-2)^2 S$ for the $2s^2^1S_0-2s2p^1P_1^0$ resonance transition in Be-like ions. The full line refers to the isoelectronic smoothing function derived by Curtis and Ellis [6]. Within the experimental (5 to 10%) error bars (and with the exception of the neutral Be atom ($Z=4$)), the displayed entity is compatible with a Z -independent value of 37.5. References to the data are listed in the caption of figure 4.

of the $2s2p^3P^0-2p^2^3P$ transition multiplet and thus estimate its influence on the decay curve of interest.

Following the example by Curtis and Ellis [6], we select for our graphical presentation mostly data that have been gained with taking the cascade situation into account, by any of the above techniques. This is not just a matter of prejudice; these data points scatter very little around clearly discernible isoelectronic trend curves and thus they form a consistent set. However, the data in figure 3 scatter much less than expected in a regular error distribution. This shows how conservative the individual error estimates must have been. The sample average suggests that the typical 10% uncertainties assumed in the individual measurements are rather conservative estimates that could be lowered by a factor of two or three.

4.2.2. Intercombination transition lifetime data B–Xe. The $2s2p^3P_1^0$ level ($2p_{1/2}$ electron) decay in the light Be-like ions requires a completely different experiment, because it involves a spin change that causes a level lifetime that is not in the nanosecond range, but in the millisecond range. The corresponding decay curves in beam–foil spectroscopy are much too drawn out, for many kilometres, and are way beyond practical means of measurement in a classical beam–foil geometry. The solution lies in using a heavy-ion storage ring that bends around and passes the excited ions in front of the detector over and over again. This technique has yielded accurate lifetime values for the first four ions of the Be isoelectronic sequence [56–58].

The short lifetimes of the singlet resonance levels and the much longer lifetimes of the corresponding triplet levels are challenging for a joint display of the data. Because of the special role of the spin–orbit mixing, any

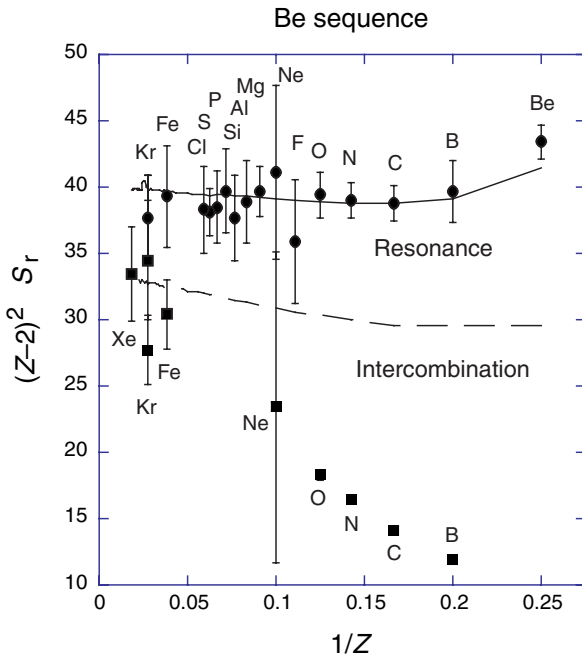


Figure 4. $(Z - 2)^2 S_r$ as a function of $1/Z$ for the $2s^2 1S_0-2s2p 1,3P_1^o$ resonance and intercombination transitions in the Be sequence. S_r is the reduced line strength as obtained by the mixing angle formalism proposed by Curtis [43]. The expression for the singlet transition (resonance) line strength (full line) is in practically perfect agreement with the experimental data (full circles). (The wiggles near the high- Z end of the theory curves are thought to reflect numerical shortcomings.) The intercombination transition line strength (broken line), however, has a trend that differs notably from that of the experimental data (full squares). This is particularly noteworthy at low Z , where the four data points from storage ring experiments have error bars that do not exceed the symbol size and indicate a consistent trend. References: Be [73, 80], B [56, 80, 81], C [57, 82], N [58, 83], O [58, 83], F [84], Ne [61, 72], Na [85], Mg [54], Al [54], Si [54], P [54], S [54, 86], Cl [86], Fe [60, 87], Kr [19, 21, 49], Xe [22].

straightforward simple scaling works either at low Z (where A_{ki} is proportional to Z^6 or Z^7), or at high Z , but not for the full range (see examples in [51]). We have employed an earlier scaling procedure, based on the mixing angle formalism [6] instead. Figure 4 combines the reduced line strength data on resonance and intercombination transitions. This presentation indicates perfect agreement of the resonance transition rate prediction with the experimental data. The intercombination transition rate data apparently meet the predicted high- Z limit, but have a trend that differs from this prediction at low Z . In this range the storage ring data are mutually consistent and very precise, so that the problem possibly lies in a higher degree of complexity of the wave functions than the mixing angle formalism reflects. The wave function details also are a likely cause for the differences among the various calculations of the transition rate. No calculation is available yet that would cover the full range of Z for which experimental data on the intercombination transition rate (and thus the line strength) are available.

Because of the role of the intercombination line in astrophysical observations, the Be-like ions of C, N and O have been studied by a multitude of calculations. The evolution of calculational techniques and the availability of

computing power have contributed to the majority of the more recent calculations yielding rather similar lifetime values which are compatible with the experimental findings. Some of the recent calculations quote uncertainties (typically in the range of a few per cent) based on convergence criteria. In hindsight it is interesting to see that several early calculations (some 20 years ago) had results that turned out to agree with the recent ones, but at the time, uncertainties usually were not specified, or were assumed to be of the order of some 20% (see discussion in [58]).

Beyond oxygen ($Z = 8$), the $2s2p 3P_1^o$ level lifetime is too short for the storage ring technique, but also much too long for typical beam-foil measurements which commence at $Z = 26$ (Fe) [59, 60] and continue with Kr ($Z = 36$) [20, 21, 49] and Xe ($Z = 54$) [22]. Within the gap, a single plasma physics estimate is available for Ne ($Z = 10$) [61], with a 50% uncertainty. It has been suggested to employ photon-ion coincidences on a slow beam of ions [62], perhaps from a recoil ion source or from a back-biased electron cyclotron resonance ion source (ECRIS), in order to obtain better data in this range (at least for Ne and Ar). In order to do the experiment within a typical vacuum chamber of about 1 m length, the beam energy would have to be of order 1 keV. For Kr, the results of two beam-foil measurements of the intercombination transition rate barely overlap by their error bars. When visually extrapolating the low- Z data, the trend points to a Kr lifetime value near this very narrow range of overlap. More data at high Z , and also for ions beyond $Z = 54$ (Xe), with an uncertainty of less than 5%, would also be of merit.

5. Conclusions

We present lifetime data on principal transitions in the atoms and ions of the Li and Be isoelectronic sequences graphically. Without going into detail, there are two observations to be made from such an exercise. Firstly, we have disregarded data that do not come close to the systematic trends that are both expected from basic considerations and that, indeed, emerge from the full body of the data. Those deviant data, especially on the resonance line in Be-like ions, were considered faulty because of insufficient sophistication of the data analysis. Basic atomic structure is known well enough to reject lifetime data that do not take obvious cascades into account. Secondly, the data sample for the resonance line in Be-like ions thus purged shows less scatter around the smoothing curve than would be expected from data with purely statistical uncertainties. Such a sub-statistical distribution indicates that the experimental error estimates, which include systematic errors, may have been more cautious than necessary. Where individual lifetime data carry error bars of 10% to, say, 3%, smoothing functions can represent the sample average with an uncertainty of the order of, say, 2%, which should be good enough for almost any application.

Theory, however, claims that the resonance transition rate in Be-like ions can be calculated with an accuracy of better than one part in a thousand [51]. Yet there is no calculation that consistently covers the full isoelectronic sequence and most calculations on given ions differ by more than the aforementioned small margin. While experiments that reach

the 0.1% level of accuracy on the resonance line of Be-like ions are not in sight, experiments that measure the rate to 2% and better would still be of some merit.

For the intercombination transition in Be-like ions, theory is much poorer. The calculation that claims the highest accuracy (about 0.7%) deals with a single ion species (C^{2+}) [63] and is slightly at odds with the experimental findings [57]. There are several other calculations for the same ion that claim about 1% uncertainty, but most calculations in the literature have been much less precise, and again there is none that covers the whole isoelectronic sequence. Better experimental data are wanted from $Z = 10$ upward. These will need the development of new techniques for lifetime measurements in the microsecond range (Ne–Ar), and accelerator time at one of the large machines for Xe and beyond. Deplorably, the spectroscopic setup that has been used at the Unilac accelerator at GSI Darmstadt for spectroscopic studies of such ions has been decommissioned.

Even 40 years after the inception of beam–foil spectroscopy, there have been no reliable lifetime measurements on many Li-like ions in the working range of the typical accelerators available in this time. However, the data collected and the analysis of their isoelectronic trends are sufficient to bridge the gaps. New measurements of high accuracy would nevertheless be welcome in order to improve on the systematics.

Acknowledgments

ET acknowledges travel support from the German Research Association (DFG). The work at the University of California Lawrence Livermore National Laboratory was performed under the auspices of the US Department of Energy under contract no. W-7405-Eng-48.

References

- [1] Hylleraas E 1929 *Z. Phys.* **54** 347
Hylleraas E 1930 *Z. Phys.* **65** 209
- [2] Kim Y-K 1997 *Phys. Scr.* T **73** 19
- [3] Martinson I M 2006 PEARL 2005 *J. Phys. Conf. Ser.* at press
- [4] Smith M W and Wiese W L 1971 *Astrophys. J. Suppl. Ser.* **23** 103
- [5] Curtis L J, Ellis D G and Martinson I 1995 *Phys. Rev. A* **51** 251
- [6] Curtis L J and Ellis D G 1996 *J. Phys. B: At. Mol. Opt. Phys.* **29** 645
- [7] Huber M C E and Sandeman R J 1986 *Rep. Prog. Phys.* **49** 397
- [8] Kay L 1963 *Phys. Lett.* **5** 36
- [9] Bashkin S and Meinel A B 1964 *Astrophys. J.* **139** 413
- [10] Bashkin S 1965 *Science* **148** 1047
- [11] Pinnington E H and Träbert E 2005 Spectroscopy of ions using fast beams and ion traps *Atomic, Molecular, and Optical Physics Reference Book* 2nd edn, ed G W F Drake (Berlin: Springer) p 265
- [12] Träbert E 1997 Radiative-lifetime measurements on highly-charged ions *Accelerator-based Atomic Physics—Techniques and Applications* ed S M Shafroth and J C Austin (Washington: AIP Press) p 567
- [13] Livingston A E 1974 *PhD Thesis* The University of Alberta, Edmonton, AB, Canada
- [14] Curtis L J, Berry H G and Bromander J 1971 *Phys. Lett. A* **34** 169
- [15] Leavitt J A, Robson and Stoner J O 1973 *Nucl. Instrum. Methods B* **110** 423
- [16] Bergkvist K-E 1976 *J. Opt. Soc. Am.* **66** 837
- [17] Zou Y, Hutton R, Hult S, Martinson I, Ando K, Kambara T, Oyama H and Awaya Y 1999 *Phys. Rev. A* **60** 982
- [18] Hutton R 1997 *Phys. Scr.* T **73** 25
- [19] Träbert E, Doerfert J, Granzow J, Büttner R, Brauckhoff J, Nicolai M, Schartner K-H, Folkmann F and Mokler P H 1995 *Phys. Lett. A* **202** 91
- [20] Jasper E, Vasilyev A, Kukla K W, Vogel Vogt C M, Livingston A E, Berry H G, Cheng S, Curtis L J and Dunford R W 1999 *Phys. Scr.* T **80** 466
- [21] Kukla K W, Livingston A E, Vogel Vogt C M, Berry H G, Dunford R W, Curtis L J and Cheng S 2005 *Can. J. Phys.* **83** 1127
- [22] Möller G, Träbert E, Lodwig V, Wagner C, Heckmann P H, Blanke J H, Livingston A E and Mokler P H 1989 *Z. Phys. D* **11** 333
- [23] Schweppe J *et al* 1991 *Phys. Rev. Lett.* **66** 1434
- [24] Curtis L J and Martinson I 1980 *Comment. At. Mol. Phys.* **10** 1
- [25] Ellis D G and Martinson I 1985 *Comment. At. Mol. Phys.* **16** 21
- [26] Ellis D G, Martinson I and Träbert E 1989 *Comment. At. Mol. Phys.* **22** 241
- [27] Curtis L J and Martinson I 1990 *Comment. At. Mol. Phys.* **24** 213
- [28] Curtis L J 1996 Precision oscillator strength and lifetime measurements *Atomic, Molecular, and Optical Physics Reference Book* ed G W F Drake (New York: AIP Press) p 206
- [29] Curtis L J and Martinson I 1999 Lifetimes of excited states in highly charged ions *Atomic Physics with Heavy Ions* ed H F Beyer and V P Shevelko (Heidelberg: Springer) p 197
- [30] Lawler J E, Bergeson S D and Wamsley R C 1993 *Phys. Scr.* T **47** 29
- [31] Volz U and Schmoranzler H 1996 *Phys. Scr.* T **65** 48
- [32] Rafac R J and Tanner C E 1998 *Phys. Rev. A* **58** 1087
- [33] McAlexander W I, Abraham E R I, Ritchie N W M, Williams C J, Stoof H T C and Hulet R G 1995 *Phys. Rev. A* **51** R871
- [34] McAlexander W I, Abraham E R I and Hulet R G 1996 *Phys. Rev. A* **54** R5
- [35] Träbert E 2000 *Phys. Scr.* **61** 237
- [36] Träbert E 2002 *Can. J. Phys.* **80** 1481
- [37] Sims J S, Hagstrom S A and Rumble J R Jr 1976 *Phys. Rev. A* **13** 242
- [38] Reistad N and Martinson I 1986 *Phys. Rev. A* **34** 2632
- [39] Träbert E 1988 *Z. Phys. D* **9** 143
- [40] Curtis L J 1991 *Phys. Scr.* **43** 137
- [41] Bethe H A and Salpeter E E 1957 *Quantum Mechanics of One- and Two-Electron Atoms* (Berlin: Springer)
- [42] Wolf A 1999 Heavy-ion storage rings *Atomic Physics with Heavy Ions* ed H F Beyer and V P Shevelko (Heidelberg: Springer) p 3
- [43] Curtis L J 1992 *J. Opt. Soc. Am.* **9** 5
- [44] Curtis L J *et al* 1995 *Phys. Rev. A* **51** 4575
- [45] Gaupp A, Kuske P and Andrä H J 1982 *Phys. Rev. A* **26** 3351
- [46] Schulze-Hagenest D 1979 *PhD Thesis* University of Kaiserslautern, Germany
- [47] Schmitt A, Volz U and Schmoranzler H 1998 *ICAMDATA Gaithersburg 1998, NIST Special Publication* ed W L Wiese and P J Mohr (Gaithersburg: NIST)
- [48] Theodosiou C E, Curtis L J and El-Mekki M 1991 *Phys. Rev. A* **44** 7144
- [49] Dietrich D D, Leavitt J A, Gould H and Marrus R 1980 *Phys. Rev. A* **22** 1109
- [50] Curtis L J 1993 *Phys. Scr.* **48** 559
- [51] Jönsson P, Froese Fischer C and Träbert E 1998 *J. Phys. B: At. Mol. Opt. Phys.* **31** 3497
- [52] Engström L, Denne B, Ekberg J O, Jones K W, Jupén C, Litzén U, Meng W T, Trigueiros A and Martinson I 1981 *Phys. Scr.* **24** 551
- [53] Engström L 1989 *Phys. Scr.* **40** 17
- [54] Träbert E and Heckmann P H 1980 *Phys. Scr.* **22** 489

- [55] Träbert E, Johnson B M, Jones K W, Gregory D C and Kruse T H 1982 *Phys. Lett. A* **87** 336
- [56] Träbert E, Wolf A, Linkemann J and Tordoir X 1999 *J. Phys. B: At. Mol. Opt. Phys.* **32** 537
- [57] Doerfert J, Träbert E, Wolf A, Schwalm D and Uwira O 1997 *Phys. Rev. Lett.* **78** 4355
- [58] Träbert E, Knystautas E J, Saathoff G and Wolf A 2005 *J. Phys. B: At. Mol. Opt. Phys.* **38** 2395
- [59] Dietrich D D, Leavitt J A, Bashkin S, Conway J G, Gould H, MacDonald D, Marrus R, Johnson B M and Pegg D J 1978 *Phys. Rev. A* **18** 208
- [60] Hutton R *et al* 1997 *Phys. Scr.* **55** 431
- [61] Kunze H-J 1972 *Space Sci. Rev.* **13** 565
- [62] Träbert E 1990 *Phys. Scr.* **41** 675
- [63] Chen M H, Cheng K T and Johnson W R 2001 *Phys. Rev. A* **64** 042507
- [64] Bergström I, Bromander J, Buchta R, Lundin L and Martinson I 1969 *Phys. Lett. A* **28** 721
- [65] Andersen T, Jessen K A and Sørensen G 1969 *Phys. Rev.* **188** 76
- [66] Knystautas E J, Barrette L, Neveu B and Drouin R 1971 *J. Quant. Spectrosc. Radiat. Transfer* **11** 75
- [67] Barrette L, Knystautas E J, Neveu B and Drouin R 1970 *Phys. Lett. A* **32** 435
- [68] Kernahan J A, Livingston A E and Pinnington E H 1974 *Can. J. Phys.* **52** 1895
- [69] Bickel W S, Berry H G, Désesquelles J and Bashkin S 1969 *J. Quant. Spectrosc. Radiat. Transfer* **9** 1145
- [70] Knystautas E J 1975 *J. Phys. B: At. Mol. Opt. Phys.* **8** 2001
- [71] Pinnington E H, Irwin D J G, Livingston A E and Kernahan J A 1974 *Can. J. Phys.* **52** 1961
- [72] Irwin D J G, Livingston A E and Kernahan J A 1973 *Can. J. Phys.* **51** 1948
- [73] Martinson I, Gaupp A and Curtis L J 1974 *J. Phys. B: At. Mol. Opt. Phys.* **7** L463
- [74] Denne B, Pegg D J, Ishii K, Alvarez E, Hallin R, Pihl J and Sjödin R 1979 *J. Physique Coll.* **40** C1–C183
- [75] Livingston A E, Serpa F G, Zacarias A S, Curtis L J, Berry H G and Blundell S A 1991 *Phys. Rev. A* **44** 7820
- [76] Pegg D J, Griffin P M, Alton G D, Elston S B, Forester J P, Suter M, Thoe R S, Vane C R and Johnson B M 1978 *Phys. Scr.* **18** 18
- [77] Forester J P, Pegg D J, Griffin P M, Alton G D, Elston S B, Hayden H C, Thoe R S, Vane C R and Wright J J 1978 *Phys. Rev. A* **18** 1476
- [78] Ishii K *et al* 1978 *Phys. Scr.* **18** 57
- [79] Zou Y, Hutton R, Feili D, Neacsu C, Ma X, Schartner K-H and Mokler P H 2005 *Nucl. Instrum. Methods B* **235** 192
- [80] Irving R E, Henderson M, Curtis L J, Martinson I and Bengtsson P 1999 *Can. J. Phys.* **77** 137
- [81] Bashkin S, McIntyre L C, von Buttlar H, Ekberg J O and Martinson I 1985 *Nucl. Instrum. Methods B* **9** 593
- [82] Reistad N, Hutton R, Nilsson A E, Martinson I and Mannervik S 1986 *Phys. Scr.* **34** 151
- [83] Engström L, Denne B, Ekberg J O, Jones K W, Jupén C, Litzén U, Meng W T, Trigueiros A and Martinson I 1981 *Phys. Scr.* **24** 551
- [84] Knystautas E J, Buchet-Poulizac M C, Buchet J P and Druetta M 1979 *J. Opt. Soc. Am.* **69** 474
- [85] Tordoir X, Biémont E, Garnir H P, Dumont P-D and Träbert E 1999 *Eur. Phys. J. D* **6** 1
- [86] Bhattacharya N, Bapat B, Rangwala S A, Kumar S V K and Krishnakumar E 1998 *Eur. Phys. J. D* **2** 125
- [87] Buchet J P, Buchet-Poulizac M C, Denis A, Désesquelles J, Druetta M, Grandin J P, Huet M, Husson X and Lecler D 1984 *Phys. Rev. A* **30** 309